# Chapter 11-1 Detailed Quantitative Analysis

The goal is to relate transistor performance parameters ($\gamma$, $\alpha_T$ , $\beta_{dc}$ etc. ) to doping, lifetimes, base-widths etc.

## Assumptions:

pnp transistor, steady state, low-level injection.

Only drift and diffusion, no external generations

One dimensional etc.

General approach is to solve minority carrier diffusion equations for each of the three regions:

$$\frac{\partial \Delta p}{\partial t} = D_{\mathrm{p}} \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_{\mathrm{p}}} + G_L \quad \text{and} \quad \frac{\partial \Delta n}{\partial t} = D_{\mathrm{n}} \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_{\mathrm{n}}} + G_{\mathrm{L}}$$

# General Quantitative Analysis

Under steady state and when $\boldsymbol{G}_L = 0$,

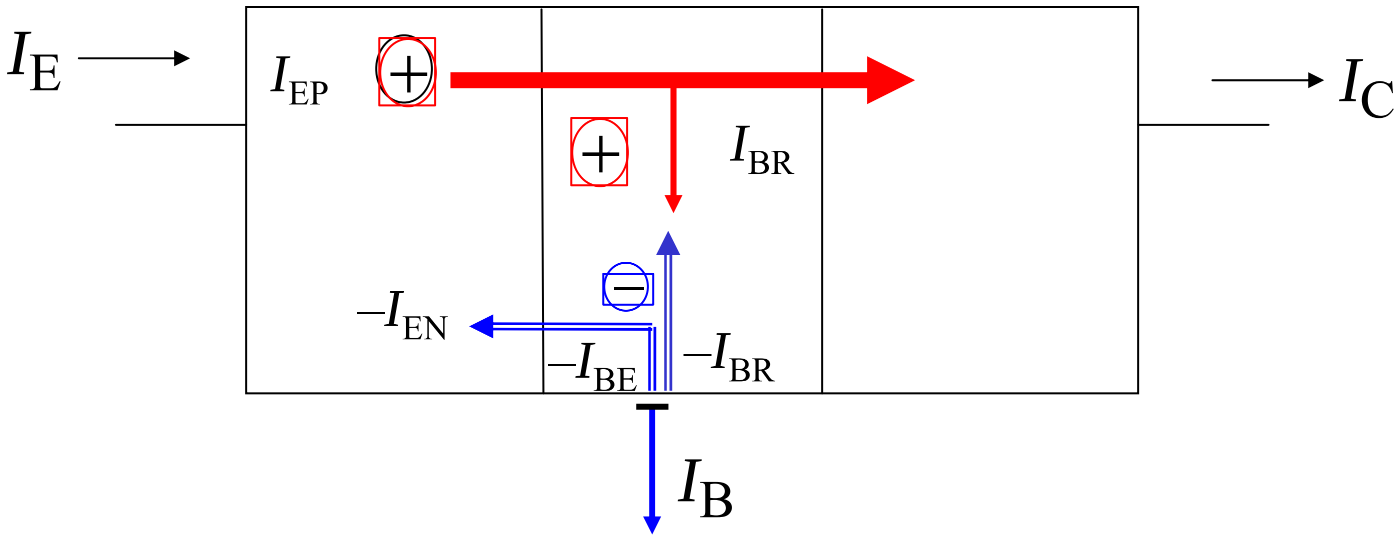
$$D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} = 0 \qquad \text{and} \qquad D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} = 0$$

For the **base** in pnp, we are interested only in holes.

$$D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} = 0$$

The rigorous analysis is carried out in chapter 11, but we are going to take a more simplified approach.

# Review: Operational Parameters




Injection Efficiency : $\gamma = I_{EP}/(I_{EP} + I_{EN})$

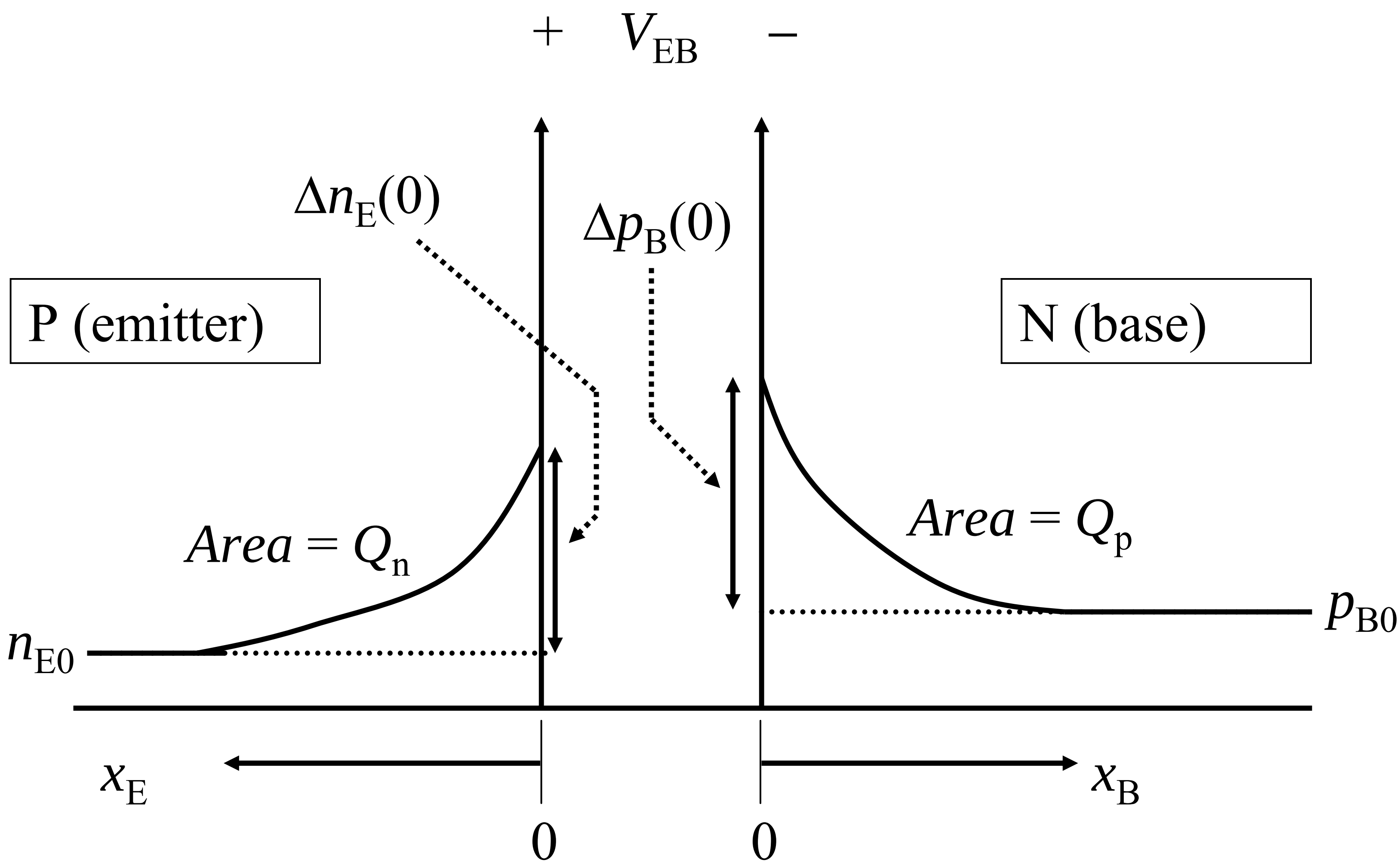
Base transport factor : $\alpha_T = I_C / I_{EP}$

Collector to emitter current gain: $\alpha_{DC} = \alpha_T \gamma$

Collector to base current gain: $\beta_{DC} = \alpha_{DC} / (1 - \alpha_{DC})$

These parameters can be related to device parameters such as doping, lifetimes, diffusion lengths, etc.

# Review of P-N Junction Under Forward Bias



# Review of P-N Junction Under Forward Bias (cont.)

$$I_n = q\,A\,D_E\;\, d\Delta n/dx_E \quad = -\,(q\,A\,D_E/L_E)\;\,\Delta n_E(0)$$

$$I_p = -\,q\,A\,D_B\;\, d\Delta p/dx_B \;\; = \;\; (q\,A\,D_B/L_B)\;\,\Delta p_B(0)$$

Total current

$I \;=\; I_P + (-\,I_N)$ ("–" because $x_E$ and $x_B$ point in opposite directions)

$$= (q\,A\,D_B/L_B)\;\Delta p_B(0) + (q\,A\,D_E/L_E)\;\Delta n_E\,(0)$$

$$= (q\,A\,D_B/L_B)\;p_{B0}[\exp\,(q\,V_{EB}\,/\,kT) - 1] + $$
$$+ \,(q\,A\,D_E/L_E)\;n_{E0}[\exp\,(q\,V_{EB}\,/kT) - 1]$$

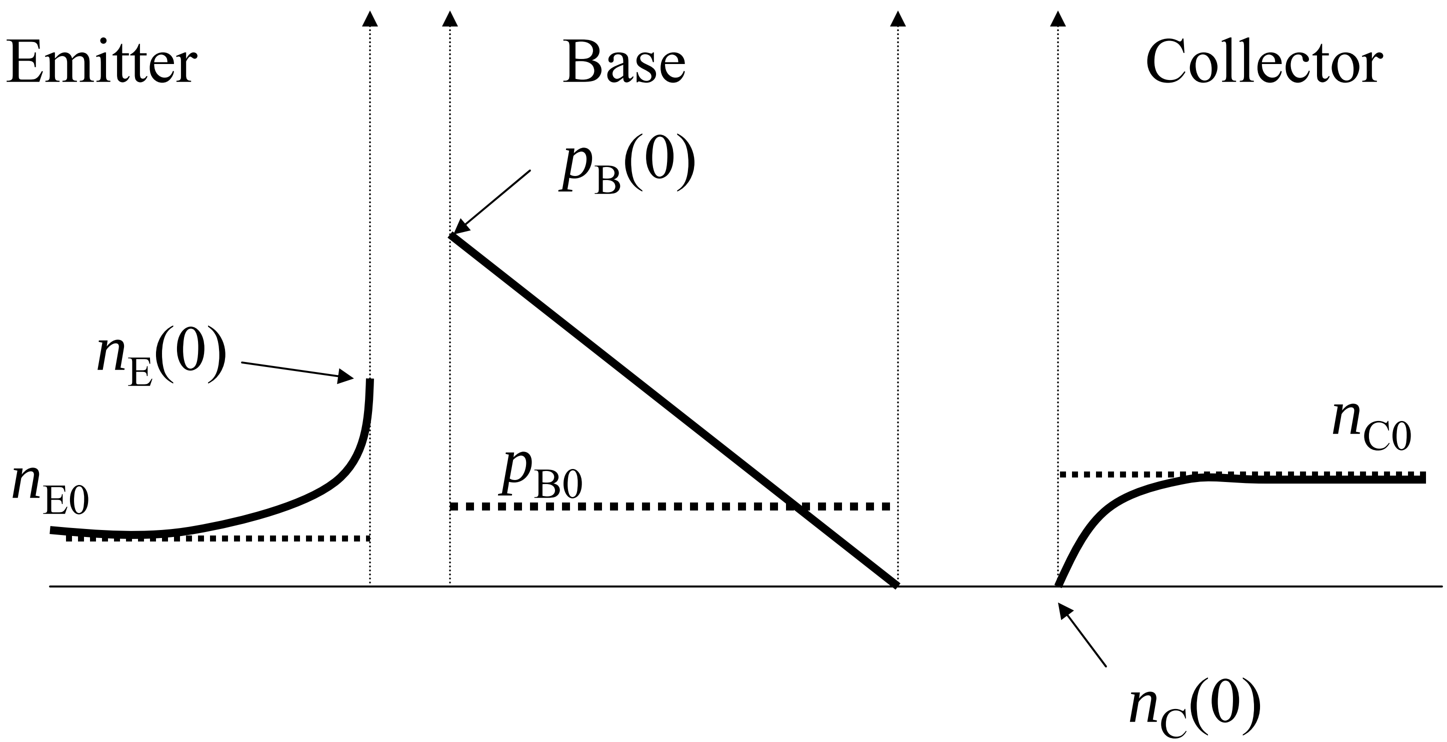
$$\approx (q\,A\,D_B/L_B)\;p_{B0}\,\exp\,(q\,V_{EB}/kT) + (q\,A\,D_E/L_E)\;n_{E0}\,\exp\,(q\,V_{EB}/kT)$$

**Note ! $I_p$ and $I_n$ can also be calculated based on the fact that $Q_p$ has to be replaced every $\tau_B$ seconds**

**➔ $I_p = Q_p/\tau_B$ and $I_n = Q_n/\tau_E$ and $I_E = I_P + I_N$**

# Simplified Analysis

Consider the carrier distribution in a forward active pnp transistor



# Simplified Analysis (cont.)

$n_{E0}$, $p_{B0}$ and $n_{C0}$ = equilibrium concentration of minority carriers in emitter, base and collector

$n_E(0)$, $p_B(0)$ and $n_C(0)$ = minority carrier concentration under forward active conditions at the edge of the respective depletion layers

$\Delta n_E(0)$, $\Delta p_B(0)$ and $\Delta n_C(0)$ = Excess carrier concentration at the edge of the depletion layers

# Simplified Analysis (cont.)

$$\Delta n_E(0) = n_E(0) - n_{E0} = n_{E0} \; [\exp(q\, V_{EB} / kT) - 1]$$

$$\Delta p_B(0) = p_B(0) - p_{B0} = p_{B0} \; [\exp(\mathrm{q}\, V_{EB} / kT) - 1]$$

By taking the slopes of these minority carrier distribution at the depletion layer edges and multiplying it by "*qAD*", we can get hole and electron currents.

Note that $I_n = q\, A\, D_n\, (\mathrm{d}n/\mathrm{d}x)$ and $I_p = -\, q\, A\, D_p\, (\mathrm{d}p\, /\, \mathrm{d}x)$

# Calculation of Currents

**Collector current, $I_C$**

$I_c = q\,A\,D_B\,(\mathrm{d}p/\mathrm{d}x_B)$ (slope must be taken at end of base)

$= q\,A\,D_B\,[p_B(0) - 0] / W_B$

$= q\,A\,D_B\;p_B(0) / W_B$

$I_c = q\,A\,(D_B/W_B)\,p_{B0}\exp(qV_{EB} / kT)$ ---- **(A)**

(only hole current if we neglect the small reverse saturation current of reverse biased C-B junction)

# Calculation of Currents (cont.)

**Emitter Current, $I_E$**

$I_E$ is made up of two components, namely $I_{EP}$ and $I_{EN}$

$$I_{EP} = I_c + \text{current lost in base due to recombination}$$
$$= I_c + \text{excess charge stored in base}/\tau_B$$
$$= I_c + q\,A\,W_B\,\Delta p_B(0) / (2\,\tau_B)$$
$$\approx q\,A\,(D_B/W_B)\;p_{B0}\;[\exp(qV_{EB}/kT)]$$
$$+\, q\,A\,[W_B/(2\,\tau_B)]\;p_{B0}\;[\exp(qV_{EB}/kT)] \quad \text{--- } \mathbf{(B)}$$

[ Assuming $\exp(qV_{EB}/kT) - 1 \approx \exp(qV_{EB}/kT)$ when $V_{EB}$ is positive, i.e forward biased. ]

# Calculation of Currents (cont.)

**Emitter Current (cont.)**

$I_{EN}$ corresponds to electron current injection from base to emitter since E-B junction is forward biased.

$$I_{EN} = qA\,(D_E / L_E)\, n_{E0}\,[\exp(q\,V_{EB} / kT) - 1\,]$$
$$\approx qA\,(D_E / L_E)\, n_{E0}\,[\exp(q\,V_{EB} / kT)] \text{ ----- } \mathbf{(C)}$$

# Calculation of Currents (cont.)

**Base Current, $I_B$**

-supplies electrons for recombination in base
-supplies electrons for injection to emitter.

$$I_B = qA\, p_{B0} \,[W_B / (2\tau_B)]\, [\exp(qV_{EB}/kT)]$$
$$+$$
$$qA(D_E / L_E)\, n_{E0} \exp(qV_{EB}/kT)$$

( recombination) + (electron injection to emitter)

Now we can find transistor parameter easily.

# Calculation of Currents (cont.)

**Base transport factor**, $\alpha_{\mathbf{T}}$

$$\alpha_T = I_C / I_{EP}$$

$$= \frac{\frac{qAD_B}{W_B} p_{B0} \exp\left(\frac{qV_{EB}}{kT}\right)}{\frac{qAD_B}{W_B} p_{B0} \exp\left(\frac{qV_{EB}}{kT}\right) + \frac{qAW_B}{2\tau_B} p_{B0} \exp\frac{qV_{EB}}{kT}} = \frac{1}{1+\frac{W_B^2}{2L_B^2}}$$

(same as eq. 11.42 in text)

**Emitter injection efficiency,** $\gamma$

$$\gamma = I_{EP} / [\ I_{EP} + I_{EN}\ ]$$
$$= 1 / [\ 1 + I_{EN} / I_{EP}\ ]$$
$$= 1 / [\ 1 + \mathbf{(C)} / \mathbf{(B)}\ ] = \frac{1}{1+\frac{(D_E n_{E0} / L_E)}{(D_B p_{B0} / W_B)}}$$

# Calculation of Currents (cont.)

$$\gamma = \frac{1}{1 + \frac{D_E W_B n_{E0}}{D_B L_E p_{B0}}} = \frac{1}{1 + \frac{D_E W_B N_B}{D_B L_E N_E}}$$

(Eq 11.41 in textbook)

➔ $n_{E0} = n_i^{\,2} / N_E$ … doping in emitter

➔ $p_{B0} = n_i^{\,2} / N_B$ … doping in base

$$\alpha_{dc} = \gamma\ \alpha_T$$

$$\beta_{DC} = \alpha_{DC} / (1 - \alpha_{DC})$$