Chapter 16-2. MOS electrostatic: Quantitative analysis

In this class, we will

Derive analytical expressions for the charge density, electric field and the electrostatic potential.

Expression for the depletion layer width

Describe delta depletion solution

Derive gate voltage relationship

- Gate voltage required to obtain inversion

Electrostatic potential, $\phi(x)$

Define a new term, $\phi(x)$ taken to be the potential inside the semiconductor at a given point x. [*The symbol ϕ instead of V used in MOS work to avoid confusion with externally applied voltage, V*]

$$\phi(x) = \frac{1}{q}[E_{\mathrm{i}}(\mathrm{bulk}) - E_{\mathrm{i}}(x)]$$

Potential at any point x

$$\phi_{\mathrm{S}} = \frac{1}{q}[E_{\mathrm{i}}(\mathrm{bulk}) - E_{\mathrm{i}}(\mathrm{surface})]$$

Surface potential

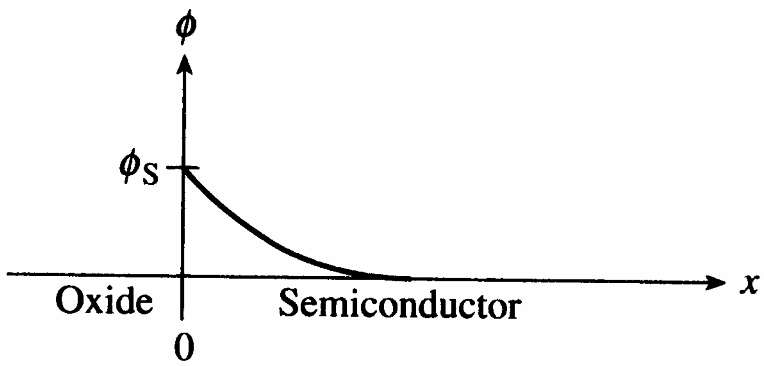
$$\phi_{\mathrm{F}} = \frac{1}{q}[E_{\mathrm{i}}(\mathrm{bulk}) - E_{\mathrm{F}}]$$

$|\phi_{\mathrm{F}}|$ related to doping concentration

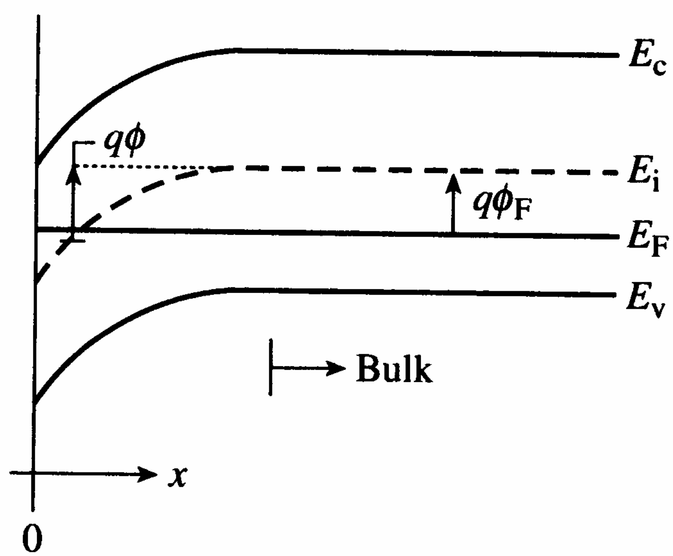
$\phi_{\mathrm{F}} > 0$ means p-type

$\phi_{\mathrm{F}} < 0$ means n-type

Electrostatic parameters



(a)



(b)

ϕ_S is positive if the band bends downward

$\phi_S = 2\phi_F$ at the depletion-inversion transition point

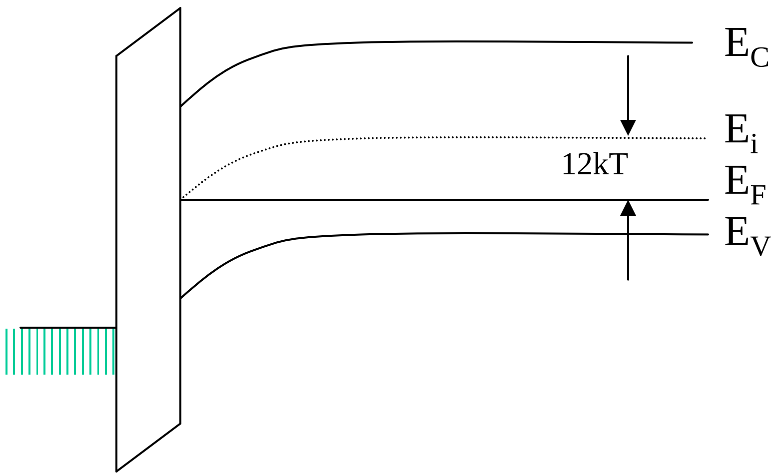
Example 1

Consider the following ϕ_F and ϕ_S parameters. Indicate whether the semiconductor is p-type or n-type, specify the biasing condition, and draw the energy band diagram at the biasing condition.

(i) $\phi_F = 12\ kT/q$; $\phi_S = 12\ kT/q$

$\phi_F = +12\ kT/q$ means that $E_i - E_F$ in the semiconductor is 12 kT (a positive value); So, p-type. $N_A = n_i \exp\left[(E_i - E_F) / kT\right]$

$\phi_S = 12\ kT/q$ means E_i (bulk) – E_i(surface) = 12 kT; i.e. the band bends downward near the surface.

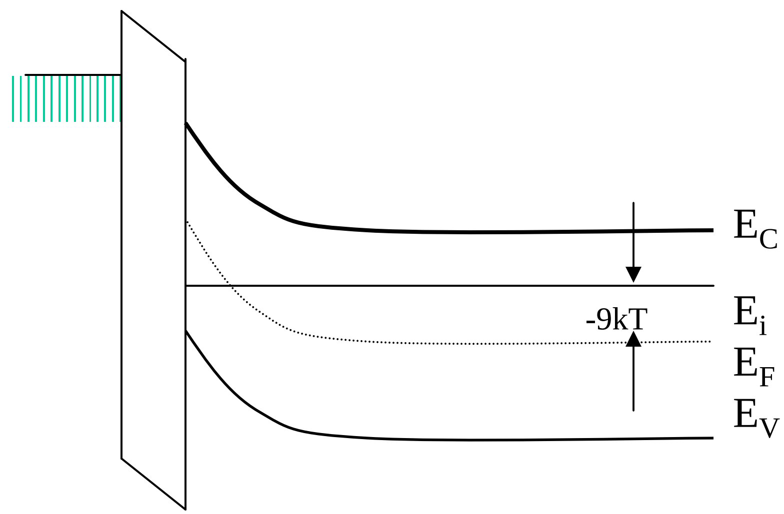


Example 1 (continued)

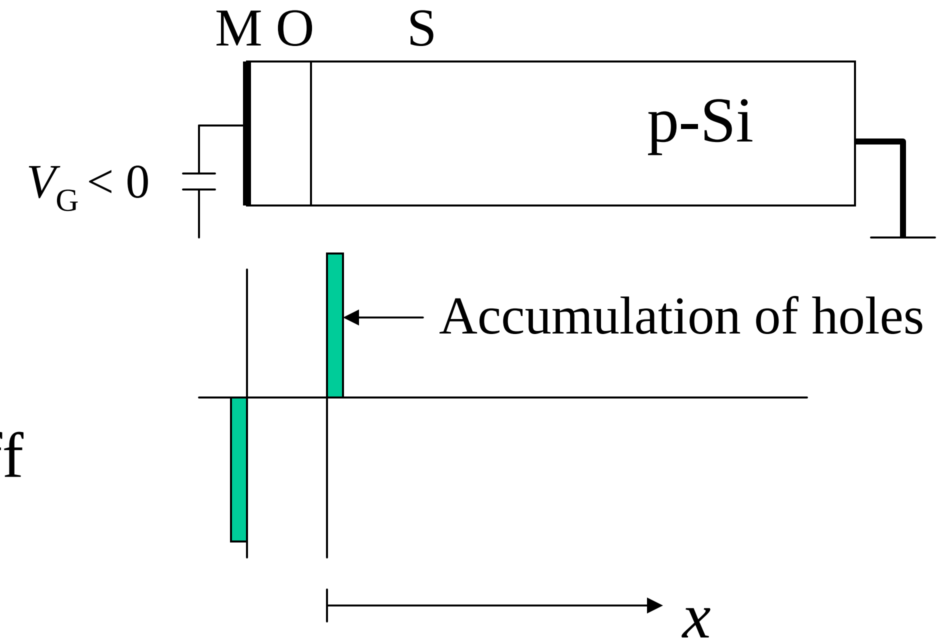
(ii) $\phi_F = -9\ kT/q$; $\phi_S = -18\ kT/q$

here $\phi_F = -9\ kT/q$ means $[E_i(\text{bulk}) - E_F] = -9\ kT$; i.e., E_i is below E_F. Thus the semiconductor is n-type.

$\phi_S = -18\ kT/q$ means that $E_i\ (\text{bulk}) - E_i(\text{surface}) = -18\ kT$; So band bends upwards near the surface. The surface is "inverted" since the surface has the same number of holes as the bulk has electrons.



Delta-depletion solution



Consider p-type silicon

Accumulation condition

The accumulation charges are mobile holes, and appear close to the surface and fall-off rapidly as x increases. **Assume that the free carrier concentration at the oxide-semiconductor interface is a δ-function**.

Charge on metal = $-Q_M$

Charge on semiconductor = – (charge on metal)

$|Q_{Accumulation}| = |Q_M|$

Delta depletion solution (cont.)

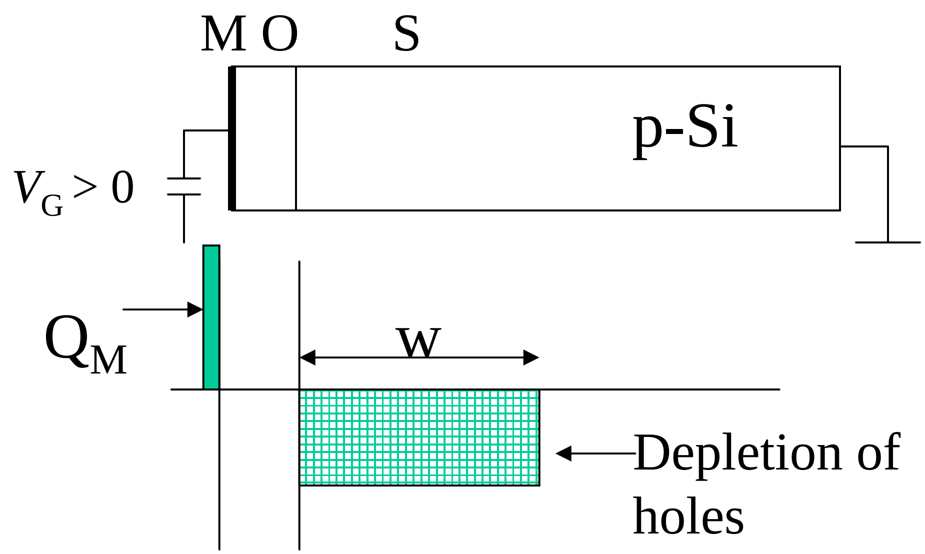
Consider p-type Si, depletion condition

Apply V_G such that $\phi_s < 2\,\phi_F$

Charges in Si are immobile ions - results in depletion layer similar to that in pn junction or Schottky diode.

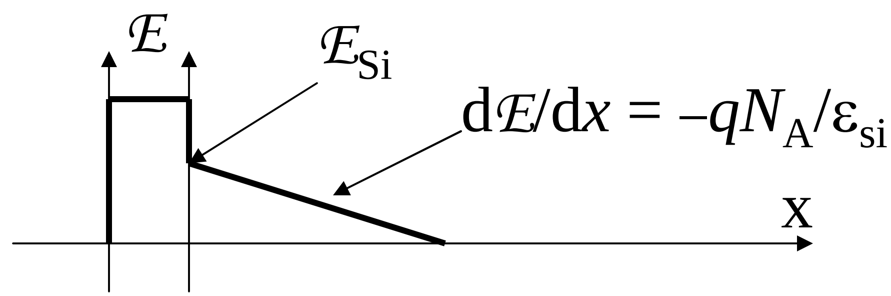
$$|q\,N_A\,A\,W| = |Q_M|$$

(−) (+)



If surface potential is ϕ_s (with respect to the bulk), then the depletion layer width W will be

$$W = \left[\frac{2\varepsilon_{Si}}{qN_A}\phi_S\right]^{\frac{1}{2}} \quad \text{and} \quad \mathcal{E}_{Si} = \left|\frac{qN_A}{\varepsilon}\right| W$$



At the start of inversion, $\phi_s = 2\,\phi_F$ and

$$W = W_T = \left[\frac{2\varepsilon_{Si}}{qN_A}2\phi_F\right]^{1/2}$$

Depletion layer width, W and $\mathcal{E}$-field

For a p$^+$n junction, or a MS (n-Si) junction, the depletion layer width is given by:

$$W = \left(\frac{2\varepsilon_{\mathrm{Si}}}{qN_{\mathrm{D}}} V_{\mathrm{bi}} \right)^{1/2}$$

Where V_{bi} is related to the amount of band bending. V_{bi} in Volts is numerically equal to the amount of band bending in eV.

$$\mathcal{E}_{\max} - \frac{qN_{\mathrm{D}}}{\varepsilon_{\mathrm{Si}}} W = -\left[\frac{2qN_{\mathrm{D}}}{\varepsilon_{\mathrm{Si}}} V_{\mathrm{bi}} \right]^{1/2}$$

For MOS, the same equation applies, except that V_{bi} is replaced by ϕ_{s}.

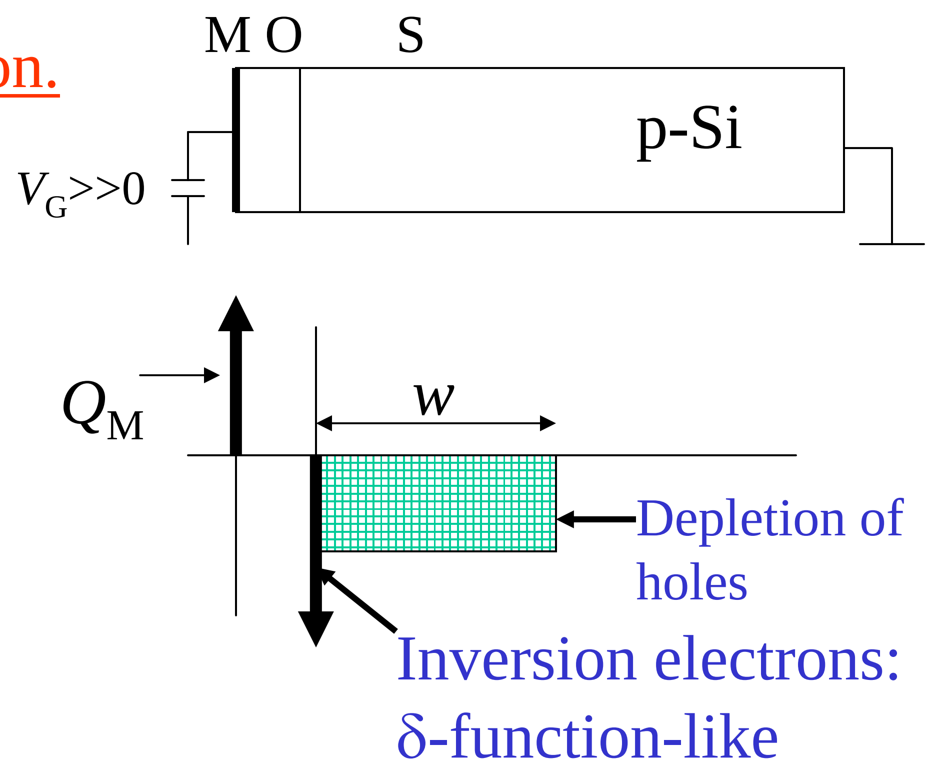
$$\mathcal{E}_{\max}(\text{in Si}) = -\left[\frac{2qN_{\mathrm{D}}}{\varepsilon_{\mathrm{Si}}} |\phi_{\mathrm{s}}| \right]^{1/2} \quad \text{or} \quad \left[\frac{2qN_{\mathrm{A}}}{\varepsilon_{\mathrm{Si}}} |\phi_{\mathrm{s}}| \right]^{1/2}$$

n-type p-type

Delta depletion solution (cont.)

Consider p-Si, strong inversion.

Once inversion charges appear, they remain close to the surface since they are mobile. Any additional voltage to the gate results in extra Q_M in gate and get compensated by extra inversion electrons in semiconductor.

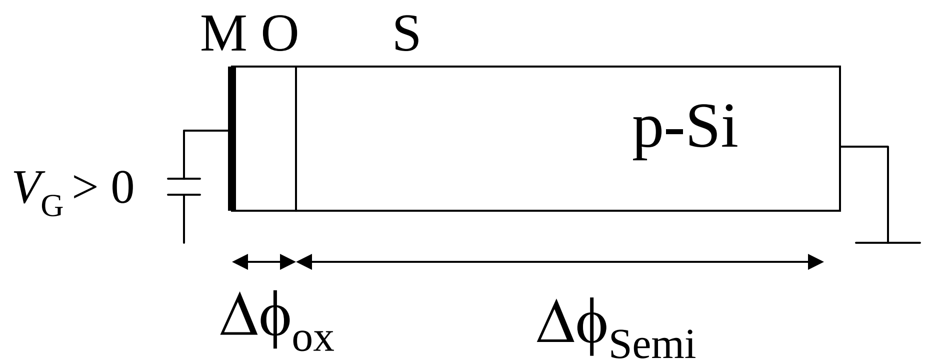


So, depletion layer does not have to increase to balance the charge on the metal. Electrons appear as δ-function near the surface. Maximum depletion layer width $W = W_T$

Gate voltage relationship

Applied gate voltage will be equal to the voltage across the oxide plus the voltage across the semiconductor. Consider p-type Si.

$$V_G = \Delta\phi_{ox} + \Delta\phi_{Semi}$$



$$\Delta\phi_{Semi} = \phi(x = 0) - \phi(\text{bulk})$$
$$= \phi_S$$
$$\Delta\phi_{ox} = x_{ox}\,\mathcal{E}_{ox}$$

Since the interface does not have any charges up to inversion, we can say that $\varepsilon_{ox}\,\mathcal{E}_{ox} = \varepsilon_{Si}\,\mathcal{E}_{Si}$

$$\mathcal{E}_{ox} = (\varepsilon_{Si} / \varepsilon_{ox})\,\mathcal{E}_{Si}$$

Gate voltage relationship (cont.)

$$\mathcal{E}_{\mathrm{Si}} = \left|\frac{qN_{\mathrm{A}}}{\varepsilon_{\mathrm{Si}}}\right| W = \left|\frac{qN_{\mathrm{A}}}{\varepsilon_{\mathrm{Si}}}\right| \left(\frac{2\varepsilon_{\mathrm{Si}}}{qN_{\mathrm{A}}}\phi_{\mathrm{s}}\right)^{1/2} \qquad \text{for} \quad 0 < \phi_{\mathrm{s}} < 2\phi_{\mathrm{F}}$$

$$= \left[\frac{2qN_{\mathrm{A}}}{\varepsilon_{\mathrm{Si}}}\phi_{\mathrm{s}}\right]^{1/2}$$

$$V_{\mathrm{G}} = \phi_{\mathrm{s}} + x_{\mathrm{ox}}\mathcal{E}_{\mathrm{ox}}$$

$$= \phi_{\mathrm{s}} + x_{\mathrm{ox}}\frac{\varepsilon_{\mathrm{Si}}}{\varepsilon_{\mathrm{ox}}}\mathcal{E}_{\mathrm{Si}}$$

$$= \phi_{\mathrm{s}} + x_{\mathrm{ox}}\frac{\varepsilon_{\mathrm{Si}}}{\varepsilon_{\mathrm{ox}}}\left(\frac{2qN_{\mathrm{A}}}{\varepsilon_{\mathrm{Si}}}\phi_{\mathrm{s}}\right)^{1/2} \qquad \text{for} \quad 0 \le \phi_{\mathrm{s}} \le 2\phi_{\mathrm{F}}$$

Gate-voltage relationship (Alternative method)

Consider p-type silicon

$V_G = \Delta\phi_{ox} + \Delta\phi_{Semi}$

$\Delta\phi_{ox} = Q_M/C_{ox} = -Q_s/C_{ox}$ where C_{ox} is oxide capacitance and Q_s is the depletion layer charge in semiconductor

$Q_s = -q\,A\,N_A W$
$C_{ox} = \varepsilon_{ox} A / x_{ox}$

$$\Delta\phi_{ox} = \frac{q\,A\,N_A W}{(\varepsilon_{ox} A / x_{ox})} = \frac{\varepsilon_{Si}}{\varepsilon_{ox}} x_{ox} \frac{qN_A}{\varepsilon_{Si}} W$$

$$V_G = \phi_s + \frac{\varepsilon_{Si}}{\varepsilon_{ox}} x_{ox} \frac{qN_A}{\varepsilon_{Si}} W = \phi_s + \frac{\varepsilon_{Si}}{\varepsilon_{ox}} x_{ox} \left(\frac{2qN_A}{\varepsilon_{Si}} \phi_s \right)^{1/2}$$

(same as before)