

## Crib sheet 3: (MOS devices, Schottky junctions, CMOS)

---

### Electrostatic potentials for MOS capacitors

$$\phi(x) = \frac{1}{q} [E_i(\text{bulk}) - E_i(x)]$$

Potential at position  $x$

$$\phi_S = \frac{1}{q} [E_i(\text{bulk}) - E_i(\text{surface})]$$

Potential at semiconductor / oxide interface

$$\phi_F = \frac{1}{q} [E_i(\text{bulk}) - E_F]$$

Bulk potential in semiconductor ( $\phi_F$  depends on doping concentration)

### Gate-Voltage relationship

$$V_G = \phi_S \pm \frac{\epsilon_{Si}}{\epsilon_{ox}} x_{ox} \sqrt{\frac{2qN_{A/D}}{\epsilon_{Si}} |\phi_S|} \quad \text{+/- and } N_A/N_D \text{ for NMOS/PMOS}$$

Threshold voltage  $V_G = V_T'$  if  $\phi_S = 2\phi_F$  ( $V_T'$  doping dependent, see  $\phi_F$  above)

**Depletion layer width:** 
$$W = \sqrt{\frac{2\epsilon_{Si} |\phi_S|}{qN_{A/D}}} \quad (W = W_T \text{ if } \phi_S = 2\phi_F)$$

**MOS-Capacitances:** 
$$C_{ox} = \frac{\epsilon_{ox}}{x_{ox}}; \quad C_S = \frac{\epsilon_{Si}}{W}; \quad C_G = \frac{C_S C_{ox}}{C_S + C_{ox}}$$

**I-V relationships for NMOS devices:**  $Z =$  gate width,  $L =$  gate length,  $A =$  gate area  $= Z \times L$

$$I_{DSat} = \frac{Z\mu C_{ox}}{2L} (V_G - V_T)^2 \quad \text{for } V_D > V_{DSat} \quad \text{and } V_G \geq V_T \quad \text{with } V_{DSat} = V_G - V_T$$

$$I_D = \frac{Z\mu C_{ox}}{L} \left[ (V_G - V_T)V_D - \frac{V_D^2}{2} \right] \quad \text{for } 0 \leq V_D \leq V_{DSat} \quad \text{and } V_G \geq V_T$$

### Small-signal parameters and cut-off frequency:

Below pinch-off: ( $V_D \leq V_{DSat}$ ) 
$$g_d = \frac{Z\mu C_{ox}}{L} (V_G - V_T - V_D) \quad g_m = \frac{Z\mu C_{ox}}{L} V_D$$

Under saturation: ( $V_D > V_{DSat}$ ) 
$$g_d = 0 \quad g_m = \frac{Z\mu C_{ox}}{L} (V_G - V_T)$$

Cut-off frequency: 
$$f_t = \frac{g_m}{2\pi C_G} \approx \frac{g_m}{2\pi C_{ox} A}$$

**Relationships for real MOS devices:**

Flatband voltage  $V_{FB} = \frac{1}{q} \Phi_{MS} + \left(-\frac{Q_i}{C_{ox}}\right)$  with  $\Phi_{MS} = \Phi_M - \Phi_S$

Real threshold voltage:  $V_T = V_{FB} + V_T'$  ( $V_T'$  = threshold voltage of ideal device)

**Relationships for MS (Schottky) junctions (n-type semiconductor):**

Schottky barrier height:  $\Phi_B = \Phi_M - \chi$  (independent of applied voltage)

Schottky-diode built-in potential:  $qV_{bi} = \Phi_B - (E_c - E_F) = \Phi_B - \left[\frac{E_G}{2} - (E_F - E_i)\right]$

**Ion implantation**

Ion implantation profile:  $N(x) = \frac{i\text{ondose}}{\sqrt{2\pi}\Delta R_p} \exp\left[-\frac{1}{2}\left(\frac{x - R_p}{\Delta R_p}\right)^2\right]$  [*iondose*] = # of ions/cm<sup>2</sup>

$V_T$ -shift by ion-implantation:  $|\Delta V_T| = \left|\frac{q \text{ iondose}}{C_{ox}}\right|$

Boron ions in Si → acceptors:  $\Delta V_T > 0$  (positive shift)

Phosphorous ions in Si → donors:  $\Delta V_T < 0$  (negative shift)

**CMOS schematic for p-type starting substrate**

(For n-type starting substrate, change conductivity types appropriately)

