Final Exam, Fall 2004, Solutions

ECSE-6960, Physical Foundations of Solid-State Devices

1. (a) Calculate the ionization energy for hydrogenic donors in InAs, E_d.
   (b) Calculate the ionization energy for hydrogenic acceptors in InAs, E_a.
   (c) What is the temperature at which the acceptor activation energy is equal to kT?
   (d) Calculate the percentage of acceptors ($N_A = 10^{17}$ cm^{-3}) that are ionized at that temperature.
   (e) Explain the result obtained under (d).

Solution

(a) The ionization energy of hydrogenic donors can be calculated by Eq. (15.39)

$$E_d = \frac{e^4 m_e^*}{2(4\pi\varepsilon\hbar)^2} = \frac{m_e^*/m_0}{\varepsilon_r{}^2} E_{Ryd} = \frac{m_e^*/m_0}{\varepsilon_r{}^2} \ 13.6 \text{ eV} .$$

InAs has effective electron mass of $m_e^* = 0.022\ m_{0,}$ and relative dielectric constant, $\varepsilon_r = 15.1$. Therefore, the ionization energy of hydrogenic donors in InAs is

$$E_d = \frac{m_e^*/m_0}{\varepsilon_r{}^2} \ 13.6 \text{ eV} = \frac{0.022}{15.1^2} \ 13.6 \text{ eV} = 0.00131 \text{ eV} = 1.31 \text{ meV}$$

(b) Eq. (15.39) can also be applied to acceptors to calculate the ionization energy of hydrogenic acceptor by using the hole effective mass instead of electron effective mass. InAs has heavy-hole effective mass of $m_{hh}{}^* = 0.40\ m_e$, and relative dielectric constant, $\varepsilon_r = 15.1$. Therefore, the ionization energy of hydrogenic acceptor in InAs is

$$E_a = \frac{m_e^*/m_0}{\varepsilon_r{}^2} \ 13.6 \text{ eV} = \frac{0.40}{15.1^2} \ 13.6 \text{ eV} = 0.02386 \text{ eV} = 23.86 \text{ meV}$$

(c) To ionize all acceptors, the thermal energy must be larger than the ionization energy. Therefore, the temperature, at which the acceptor activation energy is equal to kT, is

$$kT = 23.86 \text{ me}V \Rightarrow T = \frac{23.86 \text{ meV}}{k} = \frac{23.86\times10^{-3}\times1.6\times10^{-19} \text{ J}}{1.38\times10^{-23} \text{ J/K}} = 276.4 \text{ K}$$

(d) The concentration of ionized acceptor can be written (Eq. 14.19) as

$$p \approx \left(\frac{1}{g} N_A N_v\right)^{1/2} \exp\left(-\frac{E_a}{2kT}\right).$$

Therefore, the percentage of acceptors, which are ionized at temperature T, is

$$\frac{p}{N_A} \approx \left(\frac{1}{g} N_A N_v\right)^{1/2} \exp\left(-\frac{E_a}{2kT}\right) \Big/ N_A = \left(\frac{1}{g}\frac{N_v}{N_A}\right)^{1/2} \exp\left(-\frac{E_a}{2kT}\right)$$

InAs has effective density of state at valence band edge of $N_V = 6.4 \times 10^{18}$ cm^{-3}. Addtionally, $N_V = 10^{17}$ cm^{-3}, $E_a = 23.86$ meV, $T = 276.4$ K and $g = 4$ for acceptors. Therefore,

$$\frac{p}{N_A} \approx \left(\frac{1}{g}\frac{N_v}{N_A}\right)^{1/2} \exp\left(-\frac{E_a}{2kT}\right) = \left(\frac{1}{4}\frac{6.4\times10^{18} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}}\right)^{1/2} \exp\left(-\frac{1}{2}\right) > 100\%$$

The percentage of acceptors that are ionized is larger than 100%, which cannot occur. Therefore, $p/N_A = 100\%$ or $p = N_A$.

(e) The temperature $T = 276.4$ K is so high that all of the acceptors are ionized.

2. Consider a bulk semiconductor with two scattering mechanisms, ionized impurity scattering with mobility μ_{II}, and phonon scattering with mobility μ_{Ph}. It is $\mu_{II} = 500\ \text{cm}^2 / (\text{Vs})\ (T / 300\ \text{K})^{3/2}$ and $\mu_{Ph} = 250\ \text{cm}^2 / (\text{Vs})\ (T / 300\ \text{K})^{-3/2}$
   (a) Which of the two scattering mechanisms dominates at room temperature?
   (b) Which of the two scattering mechanisms dominates at very low temperature (< 50 K)?
   (c) At which temperature are carriers scattered equally strongly by both scattering mechanisms?
   (d) What are the mobilities: (i) μ, (ii) μ_{II} and (iii) μ_{Ph} at this temperature?
   (e) What is the mobility of carriers at room temperature?
   (f) What would be the mobility if the bulk material discussed above would be made into a modulation-doped semiconductor?
   (g) Explain the temperature dependence of μ_{II}.
   (h) Explain the temperature dependence of μ_{Ph}.

Solution

(a) Phonon scattering dominates at room temperature since $\mu_{Ph} < \mu_{II}$ or $\mu_{Ph}^{-1} > \mu_{II}^{-1}$, i.e. the "hindrance" ($\mu^{-1}$) is greater for phonon scattering than for ionized impurity scattering.

(b) Impurity scattering mechanism dominates at very low temperature.

(c) $\mu_{II} = \mu_{Ph} \Rightarrow 500\ \text{cm}^2 / (\text{Vs})\ (T / 300\ \text{K})^{3/2} = 250\ \text{cm}^2 / (\text{Vs})\ (T / 300\ \text{K})^{-3/2} \Rightarrow T = 238\ \text{K}$

(d) $\mu^{-1} = \mu_{II}^{-1} + \mu_{Ph}^{-1} \Rightarrow \mu = 176.8\ \text{cm}^2 / (\text{Vs})$
for $T = 238$ K, $\mu_{II} = \mu_{Ph} = 353.5\ \text{cm}^2 / (\text{Vs})$

(e) For $T = 300$ K, $\mu^{-1} = (\mu_{II}^{-1} + \mu_{Ph}^{-1}) = (250\ \text{cm}^2 / (\text{Vs}))^{-1} + (500\ \text{cm}^2 / (\text{Vs}))^{-1} \Rightarrow$
$\mu = 166.7\ \text{cm}^2 / (\text{Vs})$

(f) In the general case, $\mu^{-1} = \mu_{II}^{-1} + \mu_{Ph}^{-1}$. For modulation-doped semiconductor, $\mu_{II} \rightarrow \infty$, and then $\mu = \mu_{Ph} = 250\ \text{cm}^2 / (\text{Vs})\ (T / 300\ \text{K})^{3/2}$.

(g) As the temperature increases, the carriers will have higher kinetic energy and move faster, so that carriers are less influenced by the Coulomb field of ionized impurities (carries feel interaction for a shorter time).

(h) As the temperature increases, phonons will have more energy and larger vibrations around the equilibrium position, which will further hinder the movement of carriers and reduce the phonon mobility.

3. Discuss the advantages and disadvantages of three different methods that are used to calculate the energy levels in a semiconductor quantum well.

Solution

(a) ***Analytical solution***:
Advantages:
(i) High accuracy, no any approximation

Disadvantages:
(i) Not applicable to many cases, which limits the applicability of this method

(b) ***WKB method***:
Advantages:
(i) Can be applied to quantum wells with complicated potential form
Disadvantages:
(i) Only good for the potentials do not change rapidly
(ii) It is an approximation instead of exact solution

(c) ***Variational method***:
Advantages:
(i) Can be applied to quantum wells with complicated potential form
(ii) Has high accuracy if the trial function is good
Disadvantages:
(i) Strongly dependent on quality of trial function
(ii) Difficulty in finding a good trial function

(d) ***Numerical solution with computer***:
Advantages:
(i) Can be widely applied to many cases
Disadvantages:
(i) Need a computer and a program

4. Consider and electron in a one-dimensional periodic structure along the x direction with lattice constant (period) a = 5 Å = 0.5 nm. Consider further that the electron occupies the lowest band of the periodic structure. The lowest band has a width of $2\Delta E_0 = 20$ meV.
   (a) What is the group velocity of the electron at $k = 0$, $k = \pi / (2a)$, and $k = \pi / a$.
   (b) Consider that the electron has a k value of $k = \pi / (2a)$ and that the electron is subjected to an electric field pointing along the *positive* x direction with magnitude 1000 V/cm. How much time passes until the electron has the group velocity zero?
   (c) Consider that the electron has a k value of $k = \pi / (2a)$ and that the electron is subjected to an electric field pointing along the *negative* x direction with magnitude 1000 V/cm. How much time passes until the electron has the group velocity zero?

Solution

(a) Let us assume that the dispersion relation of the electron is given by $E = E_0 - \Delta E_0 \cos(ka)$ where $\Delta E_0 = 10$ meV and a = 5 Å. The group velocity v_{gr} is given by

$$v_{gr} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a}{\hbar}\Delta E_0 \sin(ka)$$

For $k = 0$,

$$v_{gr} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a}{\hbar}\Delta E_0 \sin(ka) = \frac{0.5\times10^{-9}\,\text{m}\; 1.6\times10^{-19}\,\text{C}\; 10\times10^{-3}\,\text{eV}}{1.05\times10^{-34}\ \text{J s}} \sin(0) = 0\ \frac{\text{m}}{\text{s}}$$

For $k = \pi/(2a)$,

$$v_{gr} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a}{\hbar}\Delta E_0 \sin(ka)$$

$$= \frac{0.5\times10^{-9}\ \text{m} \times 1.6\times10^{-19}\ \text{C} \times 10\times10^{-3}\ \text{eV}}{1.05\times10^{-34}\ \text{J s}}\sin(\pi/2) = 7.6\times10^{3}\ \text{m/s}$$

For $k = \pi/a$,

$$v_{gr} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a}{\hbar}\Delta E_0 \sin(ka)$$

$$= \frac{0.5\times10^{-9}\ \text{m} \times 1.6\times10^{-19}\ \text{C} \times 10\times10^{-3}\ \text{eV}}{1.05\times10^{-34}\ \text{J s}}\sin(\pi) = 0\ \text{m/s}$$

(b) From Part (a), we have known that the group velocity will be zero for $k = 0$, and $k = \pi / a$. From $k = \pi/2a$, it will take $\Delta k = \pi/2a$ for the group velocity to go to zero under both positive and negative electric field.
For the positive case,

$$\frac{dk}{dt} = -\frac{1}{\hbar}e\mathcal{E} \Rightarrow \Delta k = -\frac{1}{\hbar}e\mathcal{E}\Delta t$$

Thus,

$$\Delta t = -\frac{\hbar}{e\mathcal{E}}\Delta k = \frac{1.05\times10^{-34}\ \text{J s}}{1.6\times10^{-19}\ \text{C} \times 100000\ \text{V/m}}\frac{\pi}{0.5\times2\times10^{-9}\ \text{m}} = 2.1\times10^{-11}\ \text{s}$$

(c) Similarly to Part (b)

$$\Delta t = -\frac{\hbar}{e\mathcal{E}}\Delta k = \frac{1.05\times10^{-34}\ \text{J s}}{1.6\times10^{-19}\ \text{C} \times 100000\ \text{V/m}}\frac{\pi}{0.5\times2\times10^{-9}\ \text{m}} = 2.1\times10^{-11}\ \text{s}$$

5. In the mid infrared spectral region, inter-subband transitions are used in quantum-well infrared photodetectors or "QWIP" detectors. In n-type detectors, the lowest state in a conduction band quantum well is populated with electrons. Upon illumination, electrons are lifted to the first excited state of the quantum well from where they tunnel out of the quantum well, driven by an electric field. These electrons constitute the photocurrent.
   (a) Calculate the detection wavelength of an n-type GaAs quantum-well photo-detector with a well width of 100 Å. The infinite well approximation can be used in the calculation.
   (b) Assume that the tunneling probability from the first excited state out of the well was determined to be $T = 10^{-4}$. What is the time it takes the electron to tunnel out of the well?
   (c) Suggest an application for detectors in this spectral region.
   (d) Calculate the detection wavelength if the structure is p-type doped.
   (e) Is it possible to use inter-subband transitions in semiconductor quantum wells for the visible spectral range? Explain your answer.

Solution

(a) Infinite quantum well has electron energy levels given by

$$E_n = \frac{\hbar^2}{2m^*}\left[\frac{(n+1)\pi}{L}\right]^2 \quad (n = 0, 1, 2, ...).$$

Therefore, the energy difference between the ground state and first excited energy level is,

$$\Delta E = \frac{\hbar^2}{2m^*}\left[\frac{(1+1)\pi}{L}\right]^2 - \frac{\hbar^2}{2m^*}\left[\frac{(0+1)\pi}{L}\right]^2 = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2$$

GaAs has effective electron mass of $m^* = 0.067\, m_e$, and the well width is $L = 100$ Å. Hence,

$$\Delta E = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 = \frac{3\times\left(1.055\times10^{-34}\ \text{J s}\right)^2}{2\times0.067\times9.1\times10^{-31}\ \text{kg}}\left(\frac{\pi}{100\ \text{Å}}\right)^2$$

$$= \frac{3\times\left(1.055\times10^{-34}\ \text{J s}\right)^2}{2\times0.067\times9.1\times10^{-31}\ \text{kg}}\left(\frac{\pi}{100\times10^{-10}\ \text{m}}\right)^2 = 2.698\times10^{-20}\ \text{J} = 0.168\ \text{eV}$$

Therefore, the wavelength that yield such photon energy is,

$$h\nu = \Delta E \Rightarrow$$

$$\lambda = \frac{c_0}{\nu} = \frac{c_0}{\Delta E / h} = \frac{3\times10^8\ \text{m/s}}{0.168\ \text{eV}/(6.626\times10^{-34}\ \text{J s})}$$

$$= \frac{3\times10^8\ \text{m/sec}}{0.168\times1.6\times10^{-19}\ \text{J}/(6.626\times10^{-34}\ \text{J s})} = 7.387\ \mu\text{m}$$

(b) The lifetime of the first excited electron in the well is,

$$\Delta\tau = \frac{\pi\hbar}{2E_1T} = \frac{\pi\hbar}{2T}\frac{2m^*}{\hbar^2}\left[\frac{L}{2\pi}\right]^2 = \frac{1}{T}\frac{m^*}{\hbar}\frac{L^2}{4\pi}$$

GaAs has effective electron mass of $m^* = 0.067\, m_e$, the well width is $L = 100$ Å and the tunneling probability, $T = 10^{-4}$. Hence,

$$\Delta\tau = \frac{1}{T}\frac{m^*}{\hbar}\frac{L^2}{4\pi} = \frac{1}{10^{-4}}\frac{0.067\times9.1\times10^{-31}\ \text{kg}}{1.055\times10^{-34}\ \text{J s}}\frac{\left(100\times10^{-10}\ \text{m}\right)^2}{4\pi} = 4.606\times10^{-11}\text{s}$$

(c) Because QWIP detectors are very sensitive in mid infrared, they can be used in thermal detection, such as thermal imaging camera system.

(d) If the structure is p-type doped, the effective heavy-hole mass of GaAs is $m^* = 0.45\, m_e$. Therefore,

$$\Delta E = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 = \frac{3\hbar^2}{2\times0.45m_e}\left(\frac{\pi}{100\ \text{Å}}\right)^2$$

$$= \frac{3\left(1.055\times10^{-34}\ \text{J s}\right)^2}{2\times0.45\times9.1\times10^{-31}\ \text{kg}}\left(\frac{\pi}{100\times10^{-10}\ \text{m}}\right)^2 = 0.025\ \text{eV}$$

Hence the detection wavelength becomes,

$$h\nu = \Delta E \Rightarrow$$

$$\lambda = \frac{c_0}{\nu} = \frac{c_0}{\Delta E / h} = \frac{3 \times 10^8 \text{ m/s}}{0.025 \text{ eV}/(6.626 \times 10^{-34} \text{ J s})}$$

$$= \frac{3 \times 10^8 \text{ m/s}}{0.025 \times 1.6 \times 10^{-19} \text{ J} / (6.626 \times 10^{-34} \text{ J s})} = 49.64 \ \mu\text{m}$$

(e) The smallest photon energy for visible light is the one for wavelength, $\lambda = 800$ nm,

$$\Delta E = h\nu = \frac{hc_0}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m/sec}}{800 \times 10^{-9} \text{ m}}$$

$$= 2.4831 \times 10^{-19} \text{ J} = 1.551 \text{ eV}$$

Using n-type GaAs with effective electron mass of $m^* = 0.067\, m_e$, to have so huge inter-subband transition, the width of the quantum well has to be,

$$\Delta E = \frac{3\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 \Rightarrow$$

$$L = \pi\sqrt{\frac{3\hbar^2}{2m^*\Delta E}} = \pi\hbar\sqrt{\frac{3}{2m^*\Delta E}} = \pi\hbar\sqrt{\frac{3}{2 \times 0.067 \times m_e \times \Delta E}}$$

$$= \pi\left(1.055 \times 10^{-34} \text{ J s}\right)\sqrt{\frac{3}{2 \times 0.067 \times 9.1 \times 10^{-31} \text{ kg} \times 1.551 \text{ eV}}}$$

$$= \pi\left(1.055 \times 10^{-34} \text{ J s}\right)\sqrt{\frac{3}{2 \times 0.067 \times 9.1 \times 10^{-31} \text{ kg} \times 1.551 \times 1.6 \times 10^{-19} \text{ J}}}$$

$$= \pi\left(1.055 \times 10^{-34} \frac{\text{kg m}^2}{\text{s}^2}\text{s}\right) \times 9.945 \times 10^{24} \frac{\text{s}}{\text{kg m}} = 3.3 \times 10^{-9} \text{ m} = 33 \text{ Å}$$

To detect the visible light, the n-type GaAs quantum well has to be smaller than 33 Å.