

Final Exam, Fall 2007, Solutions

ECSE-6920 – Physical Foundations of Solid-State Devices

- Note:** (i) Put your name on paper, show your work, underline results, and always show units.
(ii) Textbook, manuscript, excerpts, and calculators are allowed.

1. Consider a hypothetical semiconductor, GaAs, which we assume contains only one single ionized donor impurity and no other impurities. Consider further a free electron that is located at a distance of $d = 150 \text{ \AA} = 15 \text{ nm}$ from the donor impurity.
- (a) What is the numerical value of the potential, caused by the donor impurity at the electron’s location?
(b) Is the potential repulsive or attractive?
(c) What is the numerical value of the potential for $d \rightarrow \infty$?

Next consider n-type GaAs with a Si doping concentration of $N_D = 1 \times 10^{17} \text{ cm}^{-3}$. Consider one specific ionized donor impurity and the potential around this impurity. A specific free electron is located at a distance of $d = 150 \text{ \AA} = 15 \text{ nm}$ from the donor impurity.

- (d) What is the numerical value of the potential, caused by the specific donor impurity and the free electrons at the specific electron’s location?
(e) Which electron, the specific electron considered under (a) or the specific electron under (d) will be scattered more strongly by the donor impurity?
(f) As the temperature increases, will the scattering of electrons become stronger or weaker?
(g) What is the name of the scattering mechanism?
(h) As the temperature increases, will the potential of the impurity in doped GaAs ($N_D = 1 \times 10^{17} \text{ cm}^{-3}$) become more similar or less similar to the potential of the impurity in undoped GaAs?

Solution:

- (a) The numerical value of the potential, caused by the donor impurity at the electron’s location can be calculated as,

$$V(d) = \frac{e}{4\pi\epsilon_r\epsilon_0 d} = \frac{1.6022 \times 10^{-19} \text{ C}}{4\pi \times 13.1 \times 8.8542 \times 10^{-12} \frac{\text{As}}{\text{Vm}} \times 15 \times 10^{-9} \text{ m}}$$

$$= 7.33 \times 10^{-3} \text{ V} = 7.33 \text{ mV}$$

- (b) Attractive, because the two charges are of opposite polarity.
(c) Zero, because $\lim_{r \rightarrow \infty} \frac{1}{r} = 0$.
(d) Because the electron concentration is non-degenerate we use the Debye screening radius,

$$r_D = \sqrt{\frac{\epsilon_r\epsilon_0 kT}{(e^2 n)}} = \sqrt{\frac{13.1 \times 8.8542 \times 10^{-12} \frac{\text{As}}{\text{Vm}} \times 25.86 \times 10^{-3} \times 1.6022 \times 10^{-19} \text{ CV}}{((1.6022 \times 10^{-19})^2 \times 1 \times 10^{17} \times 10^6 \text{ m}^{-3})}}$$

$$= 13.7 \times 10^{-9} \text{ m}$$

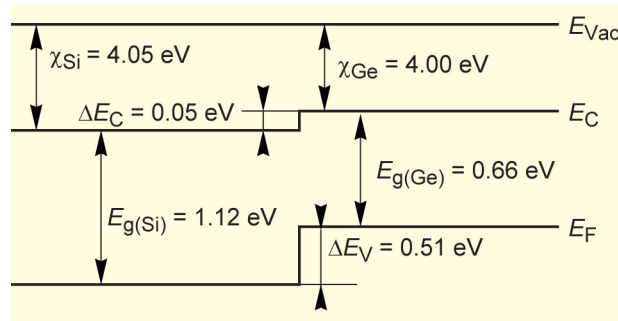
$$V(d) = \frac{e}{4\pi\epsilon_r\epsilon_0 d} e^{-\frac{r}{r_D}} = \frac{1.6022 \times 10^{-19} \text{ C}}{4\pi \times 13.1 \times 8.8542 \times 10^{-12} \frac{\text{As}}{\text{Vm}} \times 15 \times 10^{-9} \text{ m}} e^{-1.1}$$

$$= 2.45 \times 10^{-3} \text{ V} = 2.45 \text{ mV}$$

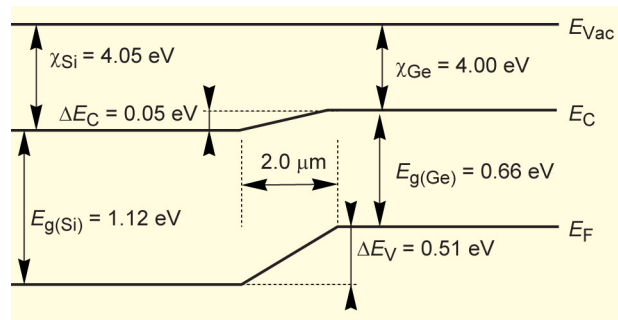
- (e) The electron considered under (a) will be scattered more strongly by the donor impurity, because the potential causing the scattering is larger.
 - (f) Weaker, because at higher T , electrons have a higher thermal velocity so that the interaction time is shorter.
 - (g) Ionized impurity scattering.
 - (h) More similar, because r_{Debye} becomes larger at higher T , so that screening becomes weaker.
2. Consider an abrupt heterostructure between Si and Ge in which the conduction and valence band discontinuities, ΔE_C and ΔE_V , are determined by the “electron-affinity rule”.
- (a) Draw a band diagram and label all relevant features.
 - (b) What is the numerical value of the conduction band discontinuity, ΔE_C , and the valence band discontinuity, ΔE_V , of the heterostructure?
 - (c) Next consider the heterostructure to not be abrupt but be graded in a region of thickness $2.0 \mu\text{m}$. So in this region, the chemical composition of the graded material can be expressed as $\text{Si}_x\text{Ge}_{1-x}$. Draw a band diagram and label all relevant features.
 - (d) What are the numerical values of the quasi-electric fields in the conduction and valence band in the composition-graded region?
 - (e) As a result of the electric field, will electrons in the graded region drift towards Si or Ge?
 - (f) As a result of the electric field, will holes in the graded region drift towards Si or Ge?
 - (g) What is the numerical value of the time it would take a hole to drift across the entire graded region assuming a hole mobility of $400 \text{ cm}^2/(\text{V s})$.
 - (h) Calculate the numerical value of the diffusion constant of the hole, based on the Einstein relationship.
 - (i) What is the distance the hole would diffuse during the time calculated under (g)?
 - (j) Comparing the answers of (g) and (i), which distance, the distance diffused or the distance drifted, is larger? What do you conclude from this comparison?

Solution:

(a) Band diagram of abrupt heterostructure between Si and Ge.


 (b) $\Delta E_C = 0.05 \text{ eV}$ and $\Delta E_V = 0.51 \text{ eV}$.

(c) Band diagram of graded heterostructure between Si and Ge.



Note: The electron affinity rule predicts a “staggered lineup”. However the true lineup is “straddled”. This discrepancy shows that the “electron affinity rule” has limited applicability.

 (d) The quasi-electric field in the conduction band, $|\mathcal{E}_C|$

$$= \frac{\Delta E_C}{e \Delta z} = \frac{0.05 \text{ eV} \times 1.6022 \times 10^{-19} \text{ C}}{1.6022 \times 10^{-19} \text{ C} \times 2 \times 10^{-6} \text{ m}} = 25 \times 10^3 \frac{\text{V}}{\text{m}}$$

 The quasi-electric field in the conduction band, $|\mathcal{E}_V|$

$$= \frac{\Delta E_V}{e \Delta z} = \frac{0.51 \text{ eV} \times 1.6022 \times 10^{-19} \text{ C}}{1.6022 \times 10^{-19} \text{ C} \times 2 \times 10^{-6} \text{ m}} = 255 \times 10^3 \frac{\text{V}}{\text{m}}$$

(e) Electrons in the graded region drift towards Si.

(f) Holes in the graded region drift towards Ge.

(g) The time it would take a hole to drift across the entire graded region is,

$$t = \frac{d}{v} = \frac{d}{\mu E} = \frac{2 \times 10^{-6} \text{ m}}{400 \times 10^{-4} \frac{\text{m}^2}{\text{Vs}} \times 255 \times 10^3 \frac{\text{V}}{\text{m}}} = 1.96 \times 10^{-10} \text{ s}$$

(h) Diffusion constant of the hole is

$$D = \frac{\mu kT}{e} = \frac{400 \times 10^{-4} \frac{\text{m}^2}{\text{Vs}} \times 25.86 \times 10^{-3} \times 1.6022 \times 10^{-19} \text{C}}{1.6022 \times 10^{-19} \text{C}}$$

$$= 1.0344 \times 10^{-3} \frac{\text{m}^2}{\text{s}} = 10.344 \text{ cm}^{-2}/\text{s}$$

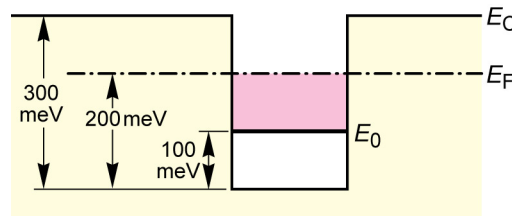
(i) Distance the hole would diffuse during the time calculated under (g) is

$$L_D = \sqrt{D \times t} = \sqrt{1.0344 \times 10^{-3} \frac{\text{m}^2}{\text{s}} \times 1.96 \times 10^{-10} \text{s}} = 0.45 \times 10^{-6} \text{ m} = 0.45 \mu\text{m}$$

(j) Distance drifted is larger. Drift is a directed process. Diffusion is a random process. For long times, the directed process (drift) will dominate.

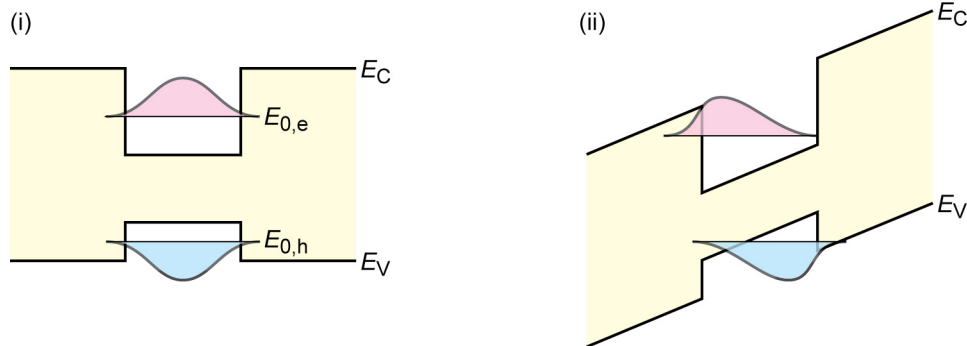
3. (a) At low temperature, the Fermi-Dirac distribution can be considered as a step-function. Explain why!
 (b) At high degeneracy, the Fermi-Dirac distribution can be considered as a step-function. Explain why!

Consider the quantum well structure shown in the figure below. Assume the quantum well material is GaAs.



- (c) Is the carrier concentration in the quantum well degenerate or non-degenerate?
 (d) What is the numerical value of the density of states in the GaAs quantum well?
 (e) What is the carrier density (per unit area) in the GaAs quantum well?
 (f) What would the carrier density in the quantum well need to be for the quantum well to overflow with carriers (i.e. the Fermi level reach the conduction band of the quantum-barrier material)?

Next consider electron-hole recombination in the two quantum well structures shown in the figure below.



- (g) Which of the two structures, (i) or (ii) has a greater probability of radiative recombination?
 (h) Justify your answer using perturbation theory!

Solution:

- (a) Fermi-Dirac distribution is given by

$$f_F(E) = \left[1 + \exp\left(\frac{E - E_F}{kT}\right) \right]^{-1}$$

As $T \rightarrow 0$, $f_F(E) \rightarrow 0$ for $E > E_F$ and $f_F(E) \rightarrow 1$ for $E < E_F$. Therefore at low temperature, the Fermi-Dirac distribution can be considered as a step-function.

- (b) At high degeneracy, the Fermi-Dirac distribution can be considered as a step-function, because the energy interval for which $f(E)$ goes from 1 to 0 (which is $\sim kT$) is much smaller than $E_F - E_C$ (which is $\sim 10kT$).
 (c) Degenerate, because the Fermi level in the quantum well is above the conduction band.
 (d) The density of states in the GaAs quantum well is

$$\begin{aligned} \rho_{\text{DOS}}^{2\text{D}}(E) &= \frac{m^*}{\pi \hbar^2} = \frac{0.067 \times 9.1094 \times 10^{-31} \text{ kg}}{\pi \times (1.0546 \times 10^{-34} \text{ Js})^2} = 1.75 \times 10^{36} \text{ m}^{-2} \text{ J}^{-1} \\ &= 2.8 \times 10^{13} \text{ cm}^{-2} \text{ eV}^{-1} \end{aligned}$$

- (e) The carrier density (per unit area) in the GaAs quantum well is

$$n = \rho_{\text{DOS}}^{2\text{D}}(E) \times (E_F - E_0) = 2.8 \times 10^{13} \text{ cm}^{-2} \text{ eV}^{-1} \times 100 \times 10^{-3} \text{ eV} = 2.8 \times 10^{12} \text{ cm}^{-2}$$

- (f) The carrier density in the quantum well for the quantum well to overflow with carriers is

$$\begin{aligned} n_{\text{overflow}} &= \rho_{\text{DOS}}^{2\text{D}}(E) \times (E_C - E_0) = 2.8 \times 10^{13} \text{ cm}^{-2} \text{ eV}^{-1} \times 200 \times 10^{-3} \text{ eV} \\ &= 5.6 \times 10^{12} \text{ cm}^{-2} \end{aligned}$$

- (g) Structure (i) has a greater probability of radiative recombination.
 (h) Perturbation theory predicts that

$$W_{C \rightarrow V} \propto \langle \psi_C(x) | H' | \psi_V(x) \rangle$$

This equation shows that a spatial overlap of electron and hole wave functions must exist for the transition to occur. Therefore, a larger spatial overlap, shown in (i), has higher probability than the one shown in (ii).

Room temperature properties of Si, Ge, and GaAs

<i>Quantity</i>	<i>Symbol</i>	<i>Si</i>	<i>Ge</i>	<i>GaAs</i>	<i>(Unit)</i>
Crystal structure		D	D	Z	–
Gap: Direct (<i>D</i>) / Indirect (<i>I</i>)		<i>I</i>	<i>I</i>	<i>D</i>	–
Lattice constant	$a_0 =$	5.43095	5.64613	5.6533	Å
Bandgap energy	$E_g =$	1.12	0.66	1.42	eV
Intrinsic carrier concentration	$n_i =$	1.0×10^{10}	2.0×10^{13}	2.0×10^6	cm^{-3}
Effective DOS at CB edge	$N_c =$	2.8×10^{19}	1.0×10^{19}	4.4×10^{17}	cm^{-3}
Effective DOS at VB edge	$N_v =$	1.0×10^{19}	6.0×10^{18}	7.7×10^{18}	cm^{-3}
Electron mobility	$\mu_n =$	1500	3900	8500	$\text{cm}^2/(\text{Vs})$
Hole mobility	$\mu_p =$	450	1900	400	$\text{cm}^2/(\text{Vs})$
Electron diffusion constant	$D_n =$	39	101	220	cm^2/s
Hole diffusion constant	$D_p =$	12	49	10	cm^2/s
Electron affinity	$\chi =$	4.05	4.0	4.07	V
Minority carrier lifetime	$\tau =$	10^{-6}	10^{-6}	10^{-8}	s
Electron effective mass	$m_e^* =$	$0.98 m_e$	$1.64 m_e$	$0.067 m_e$	–
Heavy hole effective mass	$m_{hh}^* =$	$0.49 m_e$	$0.28 m_e$	$0.45 m_e$	–
Relative dielectric constant	$\epsilon_r =$	11.9	16.0	13.1	–
Refractive index near E_g	$\bar{n} =$	3.3	4.0	3.4	–
Absorption coefficient near E_g	$\alpha =$	10^3	10^3	10^4	cm^{-1}

- D = Diamond. Z = Zinblende. W = Wurtzite. DOS = Density of states. VB = Valence band. CB = Conduction band
- The Einstein relation relates the diffusion constant and mobility in a non-degenerately doped semiconductor: $D = \mu (kT/e)$
- Minority carrier diffusion lengths are given by $L_n = (D_n \tau_n)^{1/2}$ and $L_p = (D_p \tau_p)^{1/2}$
- The mobilities and diffusion constants apply to low doping concentrations ($\approx 10^{15} \text{ cm}^{-3}$). As the doping concentration increases, mobilities and diffusion constants decrease.
- The minority carrier lifetime τ applies to doping concentrations of 10^{18} cm^{-3} . For other doping concentrations, the lifetime is given by $\tau = B^{-1} (n + p)^{-1}$, where $B_{\text{Si}} \approx 5 \times 10^{-14} \text{ cm}^3/\text{s}$, $B_{\text{Ge}} \approx 5 \times 10^{-13} \text{ cm}^3/\text{s}$, and $B_{\text{GaAs}} = 10^{-10} \text{ cm}^3/\text{s}$.

Physical constants

a_B	=	0.5292 Å	Bohr radius	$(a_B = 0.5292 \times 10^{-10} \text{ m})$
ϵ_0	=	$8.8542 \times 10^{-12} \text{ A s/(V m)}$	absolute dielectric constant	
e	=	$1.6022 \times 10^{-19} \text{ C}$	elementary charge	
c	=	$2.9979 \times 10^8 \text{ m/s}$	velocity of light in vacuum	
E_{Ryd}	=	13.606 eV	Rydberg energy	
g	=	9.8067 m/s^2	acceleration on earth at sea level due to gravity	
G	=	$6.6873 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$	gravitational constant	$(F = G M m / r^2)$
h	=	$6.6261 \times 10^{-34} \text{ J s}$	Planck constant	$(h = 4.1356 \times 10^{-15} \text{ eV s})$
\hbar	=	$1.0546 \times 10^{-34} \text{ J s}$	$\hbar = h/(2\pi)$	$(\hbar = 6.5821 \times 10^{-16} \text{ eV s})$
k	=	$1.3807 \times 10^{-23} \text{ J/K}$	Boltzmann constant	$(k = 8.6175 \times 10^{-5} \text{ eV/K})$
μ_0	=	$1.2566 \times 10^{-6} \text{ V s/(A m)}$	absolute magnetic constant	
m_e	=	$9.1094 \times 10^{-31} \text{ kg}$	free electron mass	
N_{Avo}	=	$6.0221 \times 10^{23} \text{ mol}^{-1}$	Avogadro number	
$R = k N_{\text{Avo}}$	=	$8.3145 \text{ J K}^{-1} \text{ mol}^{-1}$	ideal gas constant	

Note:

The *dielectric permittivity* of a material is given by $\epsilon = \epsilon_r \epsilon_0$ where ϵ_r and ϵ_0 are the *relative* and *absolute* dielectric permittivity, respectively.

The *magnetic permeability* of a material is given by $\mu = \mu_r \mu_0$ where μ_r and μ_0 are the *relative* and *absolute* magnetic permeability, respectively.

Useful conversions

$$\begin{aligned}
 1 \text{ eV} &= 1.6022 \times 10^{-19} \text{ C V} = 1.6022 \times 10^{-19} \text{ J} \\
 E &= h\nu = hc/\lambda = 1239.8 \text{ eV} / (\lambda/\text{nm}) \\
 kT &= 25.86 \text{ meV} \quad (\text{at } T = 300 \text{ K}) \\
 kT &= 25.25 \text{ meV} \quad (\text{at } T = 20 \text{ }^\circ\text{C} = 293.15 \text{ K})
 \end{aligned}$$