Midterm Exam, Fall 2008

ECSE-6920 – Physical Foundations of Solid-State Devices, Prof. Schubert

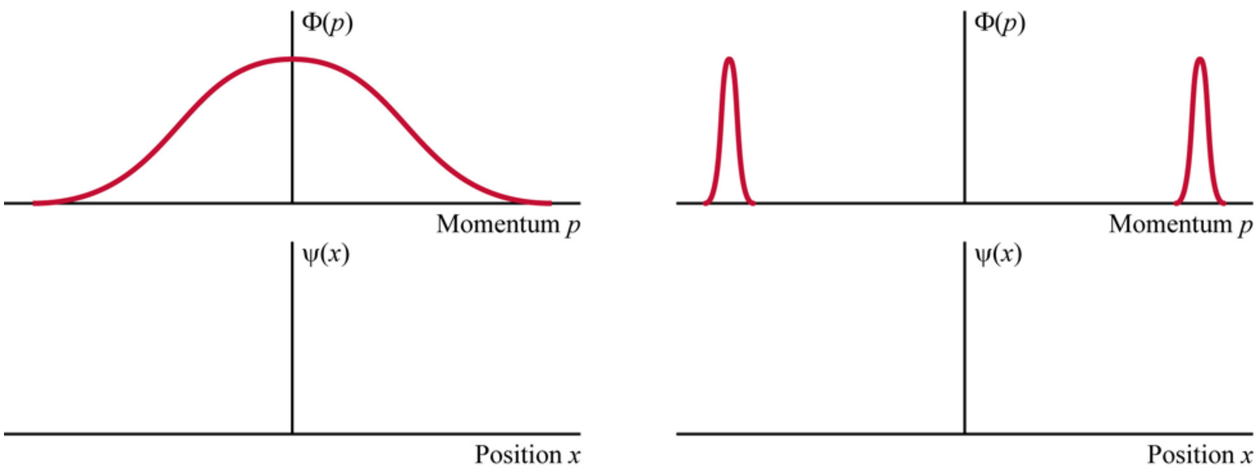
Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, excerpts, and calculators are allowed.

1. Assume that an electron, which is in the ground state of an atom, is moving at a velocity of 1×10^6 m/s around the atom's core in a circular orbit.

(a) Calculate the de Broglie wavelength of the electron.

(b) Calculate the Bohr radius of this atom from the quantities calculated in (a).

2. Given is a momentum-space wave function, $\Phi(p)$, as shown in the figure below.
   (a) Sketch the corresponding real-space wave function $\psi(x)$.
   (b) Explain your choice of $\psi(x)$.



3. Assume that the dispersion relation of the conduction band of a semiconductor within the first Brillouin zone is cosine-function shaped with (i) a minimum at $k = 0$, (ii) an amplitude of 25 meV, and (iii) a full-period width of $2\pi / a = 2\pi / 5$ Å $= 2\pi / 0.5$ nm. Assume further that the semiconductor has an energy gap of $E_g = 2.5$ eV.
   (a) Sketch the dispersion relation of the conduction band within the first Brillouin zone.
   (b) Calculate the effective mass of electrons near the minimum at $k = 0$.
   (c) Calculate the photon momentum of light emitted by the semiconductor.
   (d) Assume that an electron-hole recombination event occurs with the electron having a momentum equal to the photon momentum and a hole having a momentum of $p = 0$. Under these conditions, is momentum conservation satisfied?
   (e) Express the electron momentum as a percentage of the momentum the electron would have at the Brillouin zone edge.
   (f) What meaningful conclusion can be drawn from the results of this exercise?

4. Assume a perturbation hamiltonian operator has the following function: $H' = A_0 \exp(\mathrm{i}\, \omega_0\, t)$.

   (a) ***First***, we assume that A_0 is a constant that does ***not*** depend on x, i.e. A_0 = constant. Assume further that this perturbation acts on an occupied quantum mechanical state j, possessing even-function symmetry. Next we consider the possibility of a quantum mechanical transition between this state j and another state m. Can such a transition occur?
   (b) Justify your answer given under (a).

   (c) ***Second***, we assume that $A_0(x)$ has the following spatial dependence: $A_0(x) = x \exp(-x^2)$. Is $A_0(x)$ an even-symmetry or an odd-symmetry function?
   (d) Could this perturbation excite an electron from the valence band to the conduction band of a semiconductor? (Justify your answer)

5. Consider an infinite quantum well having its left-hand-side wall at $x = -(1/2)\, L_{QW}$ and its right-hand-side wall at $x = +(1/2)\, L_{QW}$.
   (a) Make a careful qualitative plot the wave functions $\psi_0(x)$, $\psi_1(x)$, $\psi_2(x)$, and $\psi_3(x)$.

   (b) Make a careful qualitative plot of the product $\psi_0(x)\ \psi_1(x)$.
   (c) Is the area under the curve, i.e. $\int_{-0.5 L_{QW}}^{0.5 L_{QW}} \psi_0(x)\ \psi_1(x)\, \mathrm{d}x$, zero, or non-zero?
   (d) Make a careful qualitative plot of the product $\psi_0(x)\ \psi_2(x)$.
   (e) Is the area under the curve, i.e. $\int_{-0.5 L_{QW}}^{0.5 L_{QW}} \psi_0(x)\ \psi_2(x)\, \mathrm{d}x$, zero, or non-zero?
   (f) Make a careful qualitative plot of the product $\psi_1(x)\ \psi_3(x)$.
   (g) Is the area under the curve, i.e. $\int_{-0.5 L_{QW}}^{0.5 L_{QW}} \psi_1(x)\ \psi_3(x)\, \mathrm{d}x$, zero, or non-zero?
   (h) Make a careful qualitative plot of the product $\psi_3(x)\ \psi_3(x)$.
   (i) Is the area under the curve, i.e. $\int_{-0.5 L_{QW}}^{0.5 L_{QW}} \psi_3(x)\ \psi_3(x)\, \mathrm{d}x$, zero, or non-zero?