Final Exam, Fall Semester 2009

ECSE-6220 – Physical Foundations of Solid-State Devices, Prof. E. F. Schubert

Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, table of constants, materials parameters, excerpts, and calculators are allowed. If you need the value of any constant, please do not hesitate to ask.

1. GaN has a very different electron and hole transport characteristics. Whereas holes are quite immobile, electrons are quite mobile. The root cause of the different characteristics is the effective mass of the carriers. Assume that the hole and electron effective masses in GaN are $m_h^* = 0.8\ m_e$ and $m_e^* = 0.2\ m_e$, respectively.
   (a) Calculate the donor and acceptor ionization energy in GaN based on the hydrogen-atom model.
   (b) Assume n-type GaN with a donor concentration of 10^{17} cm^{-3}. What is the fraction of donors ionized at room temperature?
   (c) Assume p-type GaN with an acceptor concentration of 10^{17} cm^{-3}. What is the fraction of acceptors ionized at room temperature?
   (d) What is the ratio of electron concentration in the n-type GaN to hole concentration in the p-type GaN?
   (e) Based on the effective masses, estimate the ratio of electron mobility in GaN to the hole mobility in GaN.
   (f) Based on the electron-mobility-to-hole-mobility ratio and the electron-concentration-to-hole-concentration ratio, calculate the n-type-to-p-type conductivity ratio in GaN.

(a) Donor ionization energy,

$$E_d = \left[\frac{m_e^*/m_0}{\epsilon_r^2}\right] 13.6\ \text{eV} = \left[\frac{0.2}{8.9^2}\right] 13.6\ \text{eV} = 34.34\ \text{eV}$$

Acceptor ionization energy,

$$E_a = \left[\frac{m_h^*/m_0}{\epsilon_r^2}\right] 13.6\ \text{eV} = \left[\frac{0.8}{8.9^2}\right] 13.6\ \text{eV} = 137.36\ \text{eV}$$

(b) Ionized donors at room temperature

$$N_D^+ = n \approx \left(\frac{1}{g} N_D N_C\right)^{\frac{1}{2}} \exp\left(-\frac{E_d}{2kT}\right)$$

$$\approx \left(\frac{1}{2} \times 1 \times 10^{17} \times 2.3 \times 10^{18}\right)^{\frac{1}{2}} \exp\left(-\frac{34.34}{2 \times 25.86}\right) \approx 1.75 \times 10^{17}$$

$$\approx 1 \times 10^{17}\ \text{cm}^{-3}$$

100% donors are ionized. (Note that the equation is an approximation. More than 100% ionization is not possible.)

(c) Ionized acceptors at room temperature

$$N_A^+ = p \approx \left(\frac{1}{g} N_A N_V\right)^{\frac{1}{2}} \exp\left(-\frac{E_a}{2kT}\right)$$

$$\approx \left(\frac{1}{4} \times 1 \times 10^{17} \times 1.8 \times 10^{19}\right)^{\frac{1}{2}} \exp\left(-\frac{137.36}{2 \times 25.86}\right) \approx 4.71 \times 10^{16}\ \text{cm}^{-3}$$

47.1% acceptors are ionized.

(d) The ratio of electron and hole concentration in the p-type and n-type sample is 2.1.

(e) According to Drude model the mobility is inversely proportional to the effective mass.

$$\mu = \frac{e\tau}{m^*}$$

Therefore, the ratio of electron mobility in GaN to the hole mobility in GaN is 4.

(f) Conductivity is given by,

$$\sigma = q(\mu_n n + \mu_p p)$$

For n-type GaN

$$\sigma \approx q\mu_n n$$

For p-type GaN

$$\sigma \approx q\mu_p p$$

Since $n = 2.1\ p$ and $\mu_n = 4\ \mu_p$, the ratio of n-type to p-type conductivity is 8.4. (In practice, typical values are 100 or greater.)

2. Assume that an electron in GaN propagates along the positive x direction and tunnels through a thin AlGaN layer which has a tunnel barrier height of 50 meV (the tunnel barrier has a constant height with respect to the coordinate x). The WKB approximation is frequently used to calculate the tunneling probability through a barrier. It is suggested that the WKB approximation also be used in the present exam question. (Use $m_{e,barrier}^* = m_{e,GaN}^* = 0.2\ m_e$.)
   (a) Re-write the WKB approximation so that it has the form $T = \exp(-x / L_T)$.
   (b) Give an expression for L_T.
   (c) Calculate the numerical value of L_T.
   (d) What is the thickness of the tunnel barrier if the tunnel probability through the barrier is $T = e^{-1}$?
   (e) What is the thickness of the tunnel barrier if the tunneling probability is $T = e^{-3}$?
   (f) What is the thickness of the tunnel barrier if the tunneling probability is $T = 10^{-25}$?

(a) Tunneling probability using zero-order WKB approximation,

$$T = e^{-\int_{x=0}^{x=L_B} 2\hbar^{-1}\sqrt{2m^*[U(x)-E]}dx}$$

For a constant tunnel barrier height,

$$T = e^{-2\hbar^{-1}\sqrt{2m^*[U-E]}\int_{x=0}^{L_B} dx} = e^{-L_B/\left(2\hbar^{-1}\sqrt{2m^*[U-E]}\right)^{-1}} = e^{-L_B/L_T}$$

(b) $L_T = \left(2\hbar^{-1}\sqrt{2m^*[U-E]}\right)^{-1}$

(c) $L_T = \frac{1.0546\times10^{-34}}{2\times\sqrt{2\times0.2\times9.1094\times10^{-31}\times[50\times10^{-3}\times1.6022\times10^{-19}]}} = 9.78\times10^{-10}\ \text{m} = 9.78\ \text{Å}$

(d) Thickness of the tunnel barrier for tunneling probability $T = e^{-1}$,

$$L_B = L_T = 9.78\ \text{Å}$$

(e) Thickness of the tunnel barrier for tunneling probability $T = e^{-3}$,

$$L_B = 3\times L_T = 3\times 9.78\ \text{Å} = 29.34\ \text{Å}$$

(f) Thickness of the tunnel barrier for tunneling probability $T = 10^{-25}$,

$$e^{-L_B/9.78\times10^{-10}} = 10^{-25}$$

$$-\frac{L_B}{9.78\times10^{-10}} = \ln 10^{-25}$$

$$L_B = -\ln 10^{-25}\times 9.78\times10^{-10} = 562.98\ \text{Å}$$

3. A potential perturbation in GaAs is caused by a positive elementary coulomb charge ($Q = +1.602 \times 10^{-19}$ C) that is located at $x = 0$.
   (a) Assume that the semiconductor is intrinsic. What is the distance on the x axis at which the potential caused by the coulomb charge has decreased to 1 mV.
   (b) Assume that the semiconductor is p-type doped at 1×10^{16} cm^{-3}. What is the potential caused by the charge on the x axis at a distance calculated in (a). (Assume $p = N_A$)
   (c) Assume that the semiconductor is n-type doped at 1×10^{19} cm^{-3}. What is the potential caused by the charge on the x axis at a distance calculated in (a). (Assume $n = N_D$)

(a) Coulomb potential

$$V(\mathrm{r}) = \frac{e}{4\pi\epsilon r}$$

$$r = \frac{e}{4\pi\epsilon V(\mathrm{r})} = \frac{1.6022 \times 10^{-19}}{4\pi \times 13.1 \times 8.8542 \times 10^{-12} \times 1 \times 10^{-3}} = 1.1 \times 10^{-7}\ \mathrm{m} = 110\ \mathrm{nm}$$

(b) Screened Coulomb potential

$$V(\mathrm{r}) = \frac{e}{4\pi\epsilon r} e^{-r/r_s}$$

Calculate r_s using Debye screening radius formula

$$r_{\mathrm{s,D}} = \sqrt{\epsilon kT/(e^2 p)}$$
$$= \sqrt{13.1 \times 8.8542 \times 10^{-12} \times 25.86 \times 10^{-3}/(1.6022 \times 10^{-19} \times 1 \times 10^{16})}$$
$$= 4.33 \times 10^{-8}\ \mathrm{m} = 43.3\ \mathrm{nm}$$

Therefore,

$$V(\mathrm{r}) = \frac{1.6022 \times 10^{-19}}{4\pi \times 13.1 \times 8.8542 \times 10^{-12} \times 1.1 \times 10^{-7}} e^{-110/43.3} = 78.8 \times 10^{-6}\ \mathrm{V}$$
$$= 78.8\ \mu\mathrm{V}$$

(c) Screened Coulomb potential

$$V(\mathrm{r}) = \frac{e}{4\pi\epsilon r} e^{-r/r_s}$$

Calculate r_s using Thomas-Fermi screening radius formula

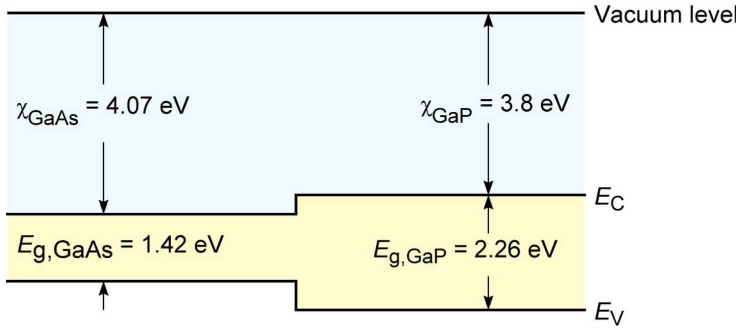
$$r_{\mathrm{s,TF}} = \pi^{2/3} \sqrt{\frac{\epsilon \hbar^2}{e^2 m^* (3n)^{1/3}}}$$
$$= \pi^{2/3} \sqrt{\frac{13.1 \times 8.8542 \times 10^{-12} \times (1.0546 \times 10^{-34})^2}{(1.6022 \times 10^{-19})^2 \times 0.067 \times 9.1094 \times 10^{-31} \times (3 \times 1 \times 10^{19})^{1/3}}}$$
$$= 3.492 \times 10^{-9}\ \mathrm{m} = 3.492\ \mathrm{nm}$$

Therefore,

$$V(\mathrm{r}) = \frac{1.6022 \times 10^{-19}}{4\pi \times 13.1 \times 8.8542 \times 10^{-12} \times 1.1 \times 10^{-7}} e^{-110/3.492} = 2.08 \times 10^{-17}\ \mathrm{V}$$

4. Consider an abrupt heterojunction between GaAs and GaP where the band discontinuities are given by the electron-affinity rule.
   (a) Draw the band diagram and label important quantities.
   (b) Identify the type of band alignment.
   (c) Calculate conduction and valence band discontinuities.
   (d) Calculate the lattice mismatch ($\Delta a/a_{GaAs}$) between GaAs and GaP.
   (e) Calculate the gallium mole fraction in $Ga_xIn_{1-x}P$ that is lattice matched to GaAs.
   (f) Calculate the electron affinity and bandgap of this lattice matched $Ga_xIn_{1-x}P$ (from (e)). Assume linear variation of electron affinity with mole fraction x.
   (g) Calculate the gallium mole fraction in $Ga_xIn_{1-x}P$ that has the same band gap as GaAs. Neglect bowing.
   (h) Draw the band diagram of heterojunction formed by GaAs and this $Ga_xIn_{1-x}P$ (from (g)) and identify the type of band alignment.

(a) Band diagram.



(b) Straddled or Type I.

(c) The band discontinuities are given as,

$$\Delta E_C = \chi_{GaAs} - \chi_{GaP} = 4.07 - 3.8 = 0.27 \text{ eV}$$

$$\Delta E_V = (\chi_{GaP} + E_{g,GaP}) - (\chi_{GaAs} + E_{g,GaAs}) = (3.8 + 2.26) - (4.07 + 1.42) = 0.57 \text{ eV}$$

(d) Lattice mismatch is given as,

$$\frac{\Delta a}{a_{GaAs}} = \frac{(a_{GaAs} - a_{GaP})}{a_{GaAs}} = \frac{5.6533 - 5.4512}{5.6533} = 3.575\%$$

(e) Using Vegard's law,

$$(a_{GaP} \times x) + (a_{InP} \times (1 - x)) = a_{GaAs}$$
$$(5.4512 \times x) + (5.8686 \times (1 - x)) = 5.6533$$
$$\therefore x = 0.5158 = 51.58\%$$

(f) Electron affinity of the alloy can be calculated using,

$$\chi_{Ga_xIn_{1-x}P} = (\chi_{GaP} \times x) + (\chi_{InP} \times (1 - x)) = (3.8 \times 0.5158) + (4.5 \times (1 - 0.5158))$$
$$= 4.14 \text{ eV}$$

Band gap of the alloy can be calculated using,

$$E_{g,Ga_xIn_{1-x}P} = 1.34 + (0.511 \times x) + (0.604 \times x^2) = 1.764 \text{ eV}$$

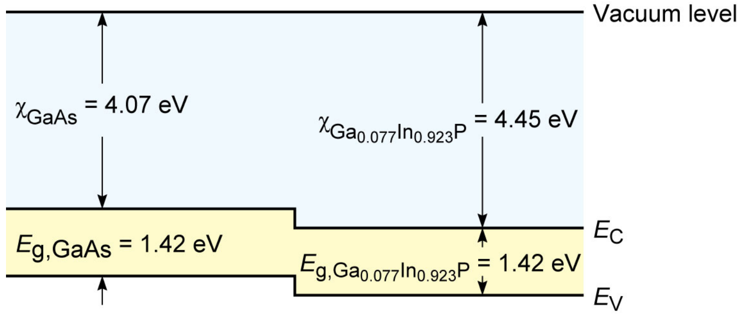
(g) Gallium mole fraction of band-gap matched $Ga_xIn_{1-x}P$ can be calculated as,

$$(E_{g,GaP} \times x) + E_{g,InP} \times (1 - x) = E_{g,Ga_xIn_{1-x}P} = E_{g,GaAs}$$
$$(2.26 \times x) + 1.35 \times (1 - x) = 1.42$$
$$\therefore x = 0.077 = 7.7\%$$

(h) Electron affinity of the alloy can be calculated using,

$$\chi_{\mathrm{Ga_xIn_{1\text{-}x}P}} = (\chi_{\mathrm{GaP}} \times x) + (\chi_{\mathrm{InP}} \times (1 - x)) = (3.8 \times 0.077) + (4.5 \times (1 - 0.077))$$
$$= 4.45 \text{ eV}$$

Band diagram:



Staggered or Type II.