Midterm Exam, Fall 2009

ECSE-6220 – Physical Foundations of Solid-State Devices, Prof. E. F. Schubert

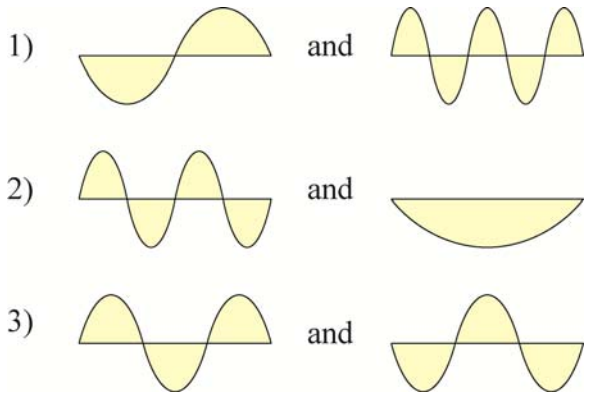
Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, excerpts, and calculators are allowed.

1. The Bohr radius of an electron occupying the ground state of a hydrogen atom is $r_{Bohr} = 0.53$ Å $= 0.053$ nm.
   (a) Calculate the de Broglie wavelength of the electron.
   (b) Calculate the momentum p of the electron.
   (c) Calculate the classical velocity v of the electron.
   (d) Calculate the ratio of the electron velocity to the speed of light (2.99×10^8 m/s).
   (e) Calculate the ratio of the electron velocity to the speed of sound (340 m/s).

2. An electron occupies the ground state of a quantum well in the GaN conduction band. Assume that the quantum well has a width of $L_{QW} = 20$ Å and that the quantum well walls are infinitely high.
   (a) Calculate the ground-state energy.
   (b) Calculate the de Broglie wavelength of the electron.
   (c) Calculate the velocity v of the electron.
   (d) Give two differences between the *classical velocity* of a classical particle and a *quantum-mechanical velocity* (the group velocity) of an electron.

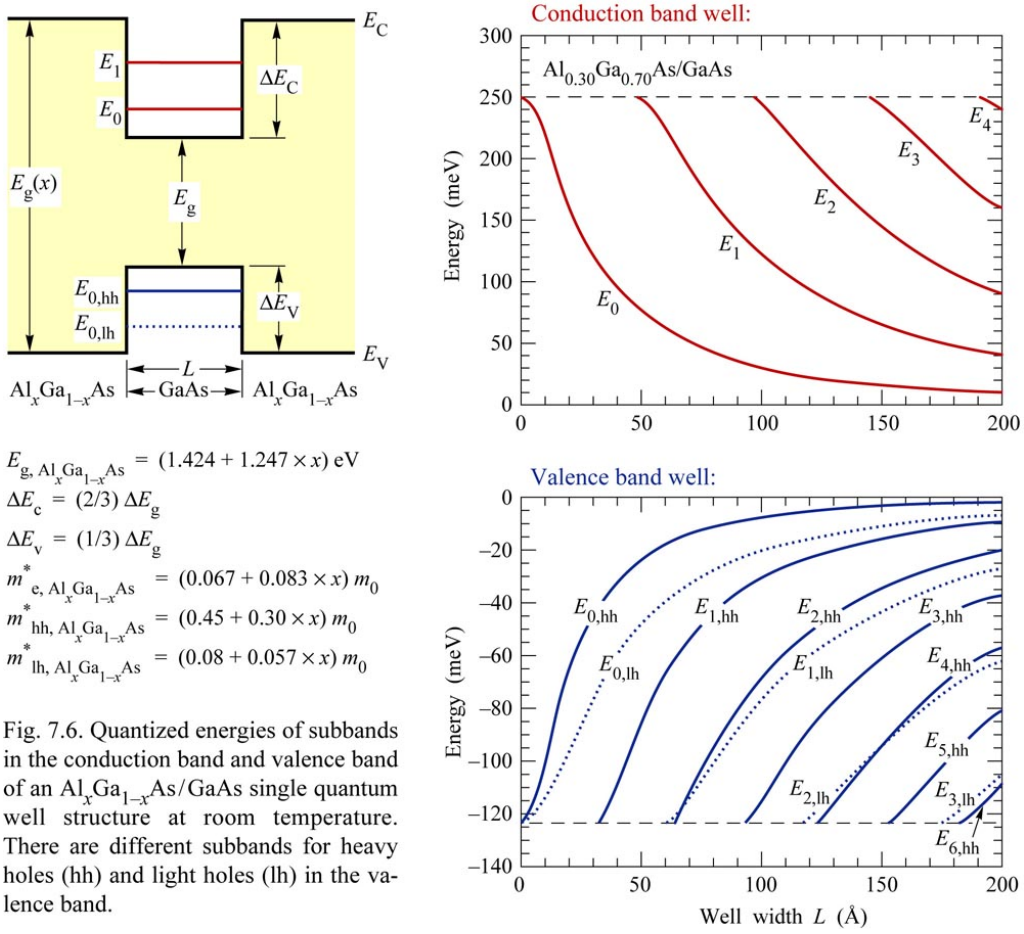
3. Assume a dispersion relation of an electron in a one-dimensional (1D) periodic potential is given by: $E = 2\ \Delta E_0\ [1 - \cos(ka)]$, where $a = 5$ Å $= 0.5$ nm is the lattice constant of a one-dimensional lattice and $\Delta E_0 = 10$ meV.
   (a) Calculate the phase velocity at the points $k = 0$, $k = \pi / (2a)$, and $k = \pi / a$.
   (b) Calculate the group velocity at the points $k = 0$, $k = \pi / (2a)$, and $k = \pi / a$.
   (c) Assume that it takes the electron the time 10^{-10} s to go from $k = 0$ to $k = \pi / a$. What is the electric field that is applied to the 1D periodic potential?

4. Selection rule for optical transitions
   (a) State whether transition between the following pairs of wave functions due to an odd-symmetry perturbation function are allowed or disallowed.



   (b) Is an intra-band optical transition allowed in a bulk semiconductor? Explain.

5. Consider a symmetric finite $Al_{0.30}Ga_{0.70}As$ / GaAs conduction band potential well, as shown in the figure below.
   (a) How many quantized energy states does a 12 nm wide potential well have in the conduction band and the valence band?
   (b) If the width of the well is decreased, will the energy of the quantized states increase or decrease?
   (c) If the width of the well is decreased, will it contain fewer or more quantized energy states?
   (d) Can the well width be adjusted so that it contains zero quantized energy states?



$$E_{g,\,Al_xGa_{1-x}As} = (1.424 + 1.247 \times x)\ \text{eV}$$
$$\Delta E_c = (2/3)\,\Delta E_g$$
$$\Delta E_v = (1/3)\,\Delta E_g$$
$$m^*_{e,\,Al_xGa_{1-x}As} = (0.067 + 0.083 \times x)\,m_0$$
$$m^*_{hh,\,Al_xGa_{1-x}As} = (0.45 + 0.30 \times x)\,m_0$$
$$m^*_{lh,\,Al_xGa_{1-x}As} = (0.08 + 0.057 \times x)\,m_0$$

Fig. 7.6. Quantized energies of subbands in the conduction band and valence band of an $Al_xGa_{1-x}As/GaAs$ single quantum well structure at room temperature. There are different subbands for heavy holes (hh) and light holes (lh) in the valence band.

6. An electron with effective mass $m^* = 0.5\ m_0$ tunnels through a barrier that is 100 meV high and 10 nm wide.
   (a) Calculate the tunneling probability of the electron.
   (b) What is the tunneling probability if only the barrier height is doubled?
   (c) What is the tunneling probability if only the barrier width is doubled?
   (d) What is the tunneling probability if only the electron effective mass is doubled?
   (e) Which of the three parameters (barrier height, barrier width, electron effective mass) has the strongest effect on the tunneling probability?
   (f) Which of the three parameters has the weakest effect on the tunneling probability?