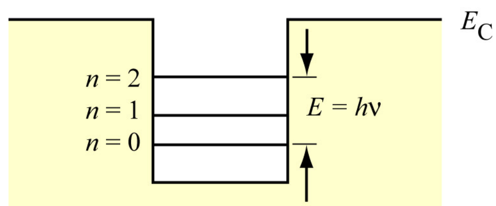
Midterm Exam, Fall 2004, Solutions

ECSE-6960, Physical Foundations of Solid-State Devices

Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, excerpts, and calculators are allowed.

1. Consider an $Al_xGa_{1-x}As$ / GaAs / $Al_xGa_{1-x}As$ quantum well (QW) structure. The thickness of the GaAs well layer is denoted as L_{QW}. Describe the changes in optical emission energy for an intra-well transition from the 2nd excited state to the ground state if
   (a) m^* decreases
   (b) m^* increases
   (c) Give asymptotic value of question (b) for $m^* \to \infty$
   (d) x decreases
   (e) Give asymptotic value of question (d) for $x \to 0$
   (f) x increases

Solution:



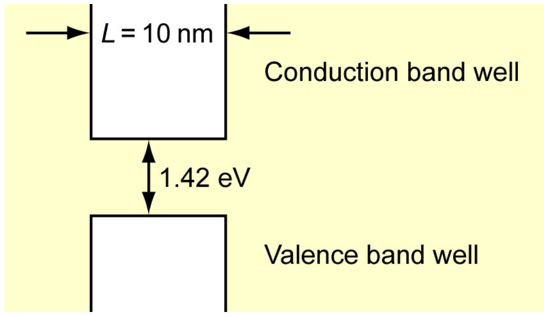
(a) If m^* decreases, the optical emission energy for an intra-well transition from the 2^{nd} excited state to the ground state increases.
(b) If m^* increases, the optical emission energy for an intra-well transition from the 2^{nd} excited state to the ground state decreases
(c) For $m^* \to \infty$, there is no split between the energy levels. The emission energy will be 0.
(d) If x decreases, the barrier height U decreases. As a result, the emission energy due to intra-well transition from the 2^{nd} state to the ground state will decrease.
(e) If $x \to 0$, $U \to 0$, there is no barrier and well. So, there is no split between the energy levels. The emission energy will be 0.
(f) If x increases, the barrier energy U will increase. The emission energy due to the intra-well transition from the 2^{nd} excited state to the ground state will increase.

2. Consider a symmetric QW semiconductor structure. The bandgap energy of the semiconductor forming the well is 1.42 eV. Assume that the barriers are infinitely high ($U_{barrier} = \infty$ eV). Assume further that electrons in the conduction-band well and holes in the valence-band well have effective masses of $m_e^* = 0.067\ m_e$ and $m_h^* = 0.45\ m_e$, where m_e is the free electron mass.
   (a) Calculate the optical transition energy of the structure for a QW thickness of $L_{QW} = 100$ Å $= 10$ nm.
   (b) Assume that the barrier height is now reduced to a *finite* barrier height. Is the optical transition energy going up or down as compared to (a)? Explain your answer.

(c) Does the *infinite* well approximation work better for
- a heavy or light carrier mass?
- a thick or thin QW?

Solution:

(a) The model for this QW semiconductor structure is shown in the following figure



In both of the conduction band and valence band, the quantum wells are square wells with infinitely high barriers. The eigenstate energies in the infinite square well are,

$$E_n = \frac{\hbar^2}{2m}\left[\frac{(n+1)\pi}{L}\right]^2 \qquad (n = 0, 1, 2 \ldots).$$

The ground state's energy level is, ($n = 0$)

$$E_0 = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$$

In the conduction band well, the ground state energy level is given by (the energy level is given with respect to the bottom of the conduction band well)

$$E_0 = \frac{\hbar^2}{2m_e^*}\left(\frac{\pi}{L}\right)^2 = 0.056\,\text{eV}.$$

In the valence band well, the ground state energy level is given by (the energy level is given with respect to the bottom of the valence band well)

$$E_0 = \frac{\hbar^2}{2m_e^*}\left(\frac{\pi}{L}\right)^2 = 0.0084\,\text{eV}$$

The optical transition energy of this structure is,

$$E = h\nu = 1.42\text{ eV} + 0.056\text{ eV} + 0.0084\text{ eV} = 1.4844\text{ eV}$$

(b) If the barrier height is reduced to a finite barrier height, the ground-state energy of the electron and hole decreases. Therefore the optical transition energy decreases.

(c) The ground-state energy level of a quantum well with infinitely high barriers is given by

$$E_0 = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$$

Inspection of the equation yields that a heavier carrier mass yields a smaller energy value. Hence, once the infinitely high barrier reduced to a finite height, the energy level decreases by a smaller amount for a carrier with heavy mass compared with a carrier with a lighter mass. Therefore, the infinite barrier quantum well approximation works better for the heavier carrier mass.

For a similar reason, a thick QW yields a smaller quantum energy values. Therefore, the infinite barrier quantum well approximation works better for the thick QW compared with the case of thin QW.

3. Assume that an electron is propagating in a periodic potential under the influence of a constant electric field of 100 V/cm. Assume that the dispersion relation of the electron is given by $E = E_0 - \Delta E_0 \cos(ka)$ where ΔE_0 = 20 meV and a = 20 Å. Consider an electron having an initial k value of 0.
   (a) What is the acceleration of the electron at $t = 0$?
   (b) Give an estimate as to how long a time (after $t = 0$) the acceleration of the electron is a constant.
   (c) At which point(s) of the $E(k)$ relation does the group velocity change linearly with time?

Solution:

(a) Similar to the exercise given in the manuscript, the group velocity v_{gr} is given by

$$v_{gr} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a}{\hbar}\Delta E_0 \sin(ka) = \frac{a}{\hbar}\Delta E_0 \sin(C\,a\,t)\,.$$

The acceleration can be expressed as

$$a_{acceleration} = \frac{dv_{gr}}{dt} = \frac{Ca^2}{\hbar}\Delta E_0 \cos(C\,a\,t)\,.$$

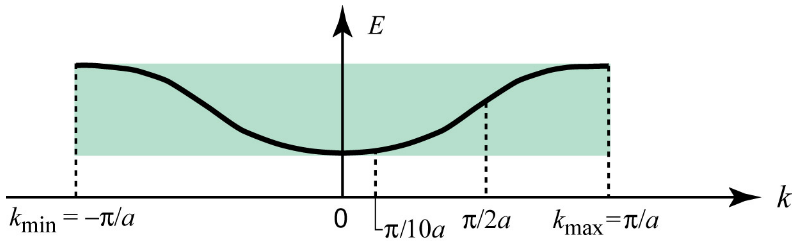
We know that C is a function of external electric field:

$$C = -\frac{e\mathcal{E}}{\hbar}\,.$$

Thus, the acceleration at $t = 0$ is given by:

$$a_{acceleration} = \frac{e\mathcal{E}}{\hbar}\frac{a^2}{\hbar}\Delta E_0 \cos\left(\frac{e\mathcal{E}}{\hbar}a\,0\right) = \frac{a^2 e\mathcal{E}}{\hbar^2}\Delta E_0$$

$$= \frac{(2\times10^{-9}\,\text{m})^2\;1.6\times10^{-19}\,\text{C}\;\;10000\,(\text{V}/\text{m})\;2\times10^{-3}\,\text{eV}}{(1.05\times10^{-34}\,\text{Js})^2} = 1.86\times10^{13}\,\frac{\text{m}}{\text{s}^2}$$



(b) As shown in the figure, we assume that the band is parabolic for a maximum k value of $\pi/10a$. This is 10% of the first Brillion zone. From the acceleration theorem,

$$\frac{dk}{dt} = -\frac{1}{\hbar} e\mathcal{E} \quad \text{and therefore} \quad \Delta k = -\frac{1}{\hbar} e\mathcal{E}\Delta t$$

$$\text{Thus } \Delta t = -\frac{\hbar}{e\mathcal{E}} \Delta k = \frac{1.05\times 10^{-34}\,\text{Js}}{1.6\times 10^{-19}\,\text{C}\;10000\;\text{V/m}} \frac{\pi}{10\times 2\times 10^{-9}\,\text{m}} = 3.3\times 10^{-12}\,\text{s}$$

(c) The question implies the acceleration is a constant. This occurs when $E(k)$ is parabolic, i.e. when the effective mass of the electron is a constant. This is the case for $k = -\pi/a$, 0, π/a, as shown in the figure.

4. Assume that a finite quantum well problem is to be solved by the variational method.
   (a) Suggest a suitable trial wave function for the first excited state of the potential well (including formula).
   (b) Sketch your trial function for different trial parameters.
   (c) Explain all parameters of your trial wave function.

Solution:

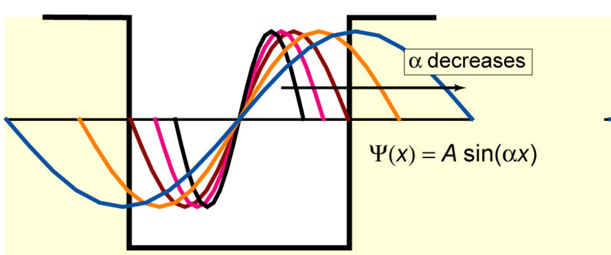
(a) Here we suggest the two following trial functions:

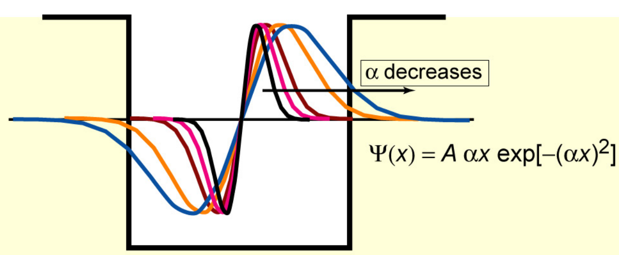
$$\psi(x) = A\sin(\alpha x)$$

$$\psi(x) = A\,\alpha x\,\exp\left[-(\alpha x)^2\right]$$

where A is the normalization parameter, and α is the trial parameter.

(b) The figure shown as below is with different trial parameters α.





(c) The parameters have the following meaning: The variable A is the normalization parameter of the trial wave function and the variable α is the trial parameter.

5. (a) What is the dimension (the units) of the 2D density of states?
   (b) What is the density-of-states in the lowest subband of an n-type GaAs quantum well?
   (c) What is the total number of states in a 1 cm^2 large sample within the first 100 meV of the lowest subband (E_0)?

Solution:

(a) 2D density-of-states is given by

$$g(E) = \frac{m}{\pi\hbar^2}$$

Therefore the units of 2D density-of-states are: $\dfrac{s^2}{kg\ m^4} = m^{-2}\ J^{-1}$

Very frequently used units are $cm^{-2}\ eV^{-1}$

(b) In an n-type GaAs quantum well, the effective electron mass is $m^* = 0.067\, m_e$. The density-of-states of the lowest subband is a 2D state density. Therefore, the state density per area is,

$$g(E) = \frac{m^*}{\pi\hbar^2} = \frac{0.067 m_e}{\pi\hbar^2} = 1.747\times 10^{36}\ \frac{s^2}{kg\cdot m^4} = 1.747\times 10^{36}\, m^{-2}\cdot J^{-1}$$

$$= 2.799\times 10^{13}\, cm^{-2}\ eV^{-1}$$

(c) The total number of states in a 1 cm^2 large sample within the first 100 meV of the lowest subband (E_0) is,

$$n_{2D} = \text{Area} \int_0^{100\text{ meV}} g(E)\, dE = 1\, cm^2 \int_0^{100\text{ meV}} \frac{m^*}{\pi\hbar^2} dE = 1\, cm^2\ \frac{0.067 m_e}{\pi\hbar^2} \int_0^{100\text{ meV}} dE$$

$$= 1\, cm^2\ 1.747\times 10^{36} m^{-2}\ J^{-1} \int_0^{100\text{ meV}} dE = 1\, cm^2\ 1.747\times 10^{36} m^{-2}\ J^{-1}\ 100\, meV$$

$$= 10^{-4} m^2\ 1.747\times 10^{36} m^{-2}\ J^{-1}\ 100\, 10^{-3}\ 1.6\times 10^{-19}\ CV = 2.799\times 10^{12}$$

Thus, there are 2.799×10^{12} carriers within the area of 1 cm^2 within the first 100 meV of the subband.