

**Solution for Final Exam, Fall 2006**

ECSE-6968 – Physical Foundations of Solid-State Devices

1. The composition  $x$  of an  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer is graded from  $x = 0$  to 0.30 over a distance of 500 nm. Assume that the bandgap energy varies with composition according to  $E_g = 1.424 \text{ eV} + 1.247 \text{ eV}$  and that 2/3 of the band discontinuity occur in the conduction band.
  - (a) What is the quasi-electric field in the composition-graded region?
  - (b) How long would it take an electron with mobility of  $2000 \text{ cm}^2/\text{Vs}$  to traverse the graded region?
  - (c) What is the diffusion constant of the electrons at 300 K?
  - (d) What is the time needed for carriers to diffuse over the thickness of the graded region?

**Solution:**

- (a) GaAs has  $E_g = 1.424 \text{ eV}$ ;  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  has  $E_g = 1.798 \text{ eV}$ . Thus  $\Delta E_g = 0.374 \text{ eV}$ .  
 The conduction band discontinuity is  $\Delta E_C = (2/3) \Delta E_g = 0.249 \text{ eV}$ . Therefore, the quasi-electric field is,  $\mathcal{E}_C = \Delta E_C / (e \Delta z) = 0.249 \text{ V} / 500 \text{ nm} = 4.98 \times 10^3 \text{ V/cm}$ .  
 The valence band discontinuity is  $\Delta E_V = (1/3) \Delta E_g = 0.125 \text{ eV}$ . Therefore, the quasi-electric field is,  $\mathcal{E}_V = \Delta E_V / (e \Delta z) = 0.125 \text{ V} / 500 \text{ nm} = 2.49 \times 10^3 \text{ V/cm}$ .
- (b) The drift velocity of an electron in the conduction band is  $v = \mu \mathcal{E}_C = 2000 \text{ cm}^2/(\text{V s}) \times 4.98 \times 10^3 \text{ V/cm} = 9.96 \times 10^6 \text{ cm/s}$ . Therefore, the drift time for the graded composition region is  $t = \Delta z / v = 500 \text{ nm} / (9.96 \times 10^6 \text{ cm/s}) = 5.02 \times 10^{-12} \text{ s}$ .
- (c) According to the Einstein relation, the diffusion constant is related to the mobility by
 
$$D_e = \mu k T / e = 2000 \text{ cm}^2 / (\text{V s}) \times 0.026 \text{ V} = 52 \text{ cm}^2 / \text{s}.$$
- (d) With  $L_D = (D_e \tau)^{1/2} \rightarrow (\Delta z)^2 = L_D^2 = D_e \tau \rightarrow \tau = L_D^2 / D_e = (500 \text{ nm})^2 / (52 \text{ cm}^2/\text{s}) = 4.8 \times 10^{-11} \text{ s}$ . Comparison of the results shows that (directed) drift is faster than (un-directed, random) diffusion.

2. The measured electron mobility in a selectively doped  $\text{Al}_x\text{Ga}_{1-x}\text{N}/\text{GaN}$  heterostructure at room temperature is  $1800 \text{ cm}^2/\text{Vs}$ . An equivalent uniformly doped GaN sample has a mobility of  $600 \text{ cm}^2/\text{Vs}$ . Answer the following questions; explain your conclusions.
  - (a) What is the ionized impurity scattering mobility in the two structures?
  - (b) Which scattering mechanism dominates for the two structures?
  - (c) Assuming  $T \rightarrow \infty$ , which scattering mechanism will dominate for the two structures?
  - (d) Assuming  $T \rightarrow 0$ , which scattering mechanism will dominate for the two structures?

**Solution:**

- (a) In the selectively doped  $\text{AlGaN}/\text{GaN}$  heterostructure, ionized impurity scattering mobility is very high ( $\mu_{\text{II}} \rightarrow \infty$ ), since the two-dimensional electron gas exists in the undoped side of the interface. Therefore the mobility for selectively doped  $\text{AlGaN}/\text{GaN}$  heterostructures is given by the phonon scattering mobility,  $\mu_{\text{ph}} = 1800 \text{ cm}^2/(\text{V s})$ .  
 In the equivalent uniformly doped GaN, the phonon scattering mobility should be same,  $\mu_{\text{ph}} = 1800 \text{ cm}^2/(\text{V s})$ . Its actual mobility is  $\mu = 600 \text{ cm}^2/(\text{V s})$ . Therefore, the ionized

impurity scattering mobility is, according to Matthiessen’s rule:

$$\mu_{II}^{-1} = \mu^{-1} - \mu_{ph}^{-1} = (600 \text{ cm}^2 / (\text{V s}))^{-1} - (1800 \text{ cm}^2 / (\text{V s}))^{-1} = (900 \text{ cm}^2 / (\text{V s}))^{-1}$$

Hence the ionized impurity scattering mobility is,  $\mu_{II} = 900 \text{ cm}^2 / (\text{V s})$ .

- (b) Phonon scattering dominates in the selectively doped AlGaIn/GaN heterostructure. Both phonon scattering and ionized impurity scattering are present in uniformly doped GaN. The calculation shows that ionized impurity scattering is stronger, i.e. it dominates.
- (c) When  $T \rightarrow \infty$ , phonon scattering dominates in both, the selectively doped AlGaIn/GaN heterostructure and uniformly doped GaN.
- (d) When  $T \rightarrow 0$ , ionized impurity scattering dominates in the uniformly doped GaN. In the selectively doped AlGaIn/GaN heterostructure, for  $T \rightarrow 0$ , both phonon scattering and ionized impurity scattering are minimized so that the mobility becomes very high.

3. How would the electron density,  $n_{2\text{DEG}}$ , in a selectively doped  $\text{Al}_x\text{Ga}_{1-x}\text{N}/\text{GaN}$  heterostructure change when ...

- (a) ...increasing the Al content in  $\text{Al}_x\text{Ga}_{1-x}\text{N}$ ?
- (b) ...increasing the doping concentration?
- (c) ...increasing the spacer layer thickness?
- (d) ...increasing  $\Delta E_C$ ?

**Solution**

- (a) The electron density,  $n_{2\text{DEG}}$ , increases with increasing Al content in the AlGaIn. As the Al content increases, the barrier height increases. Therefore, the electron density increases due to the high higher energy that electrons gain when transferring from the AlGaIn to the GaN.
- (b) The electron density,  $n_{2\text{DEG}}$ , increases with increasing doping concentration in the AlGaIn.
- (c) The electron density,  $n_{2\text{DEG}}$ , decreases with increasing the spacer layer thickness. As the spacer layer thickness increases, the energy drop in the spacer layer increases.
- (d) The electron density,  $n_{2\text{DEG}}$ , increases with increasing  $\Delta E_C$ . This case is equivalent to the one of increasing Al content.

4. An electron with effective mass  $m^* = 0.45 m_0$  tunnels through a barrier that is  $U_0 = 50 \text{ meV}$  high. Due to a design requirement, the tunneling probability must be  $10^{-6}$  or less.

- (a) Calculate the minimum barrier thickness  $L_B$  (give numerical value).
- (b) Give a formula that describes the tunneling probability through a rectangular barrier.
- (c) Write the formula in the following form:  $T = \exp(-L_B / L_e)$ .
- (d) Can you explain the physical meaning of  $L_e$ ?
- (e) Calculate  $L_e$  (give mathematical formula and numerical value).

**Solution:**

(a) The tunneling probability can be calculated from Eq. (18.1) in the textbook

$$T = \exp\left(-\int_{x=0}^{L_B} 2 \hbar^{-1} \sqrt{2 m^* [U(x) - E]} dx\right) = \exp\left(-2 \hbar^{-1} \sqrt{2 m^* U_0} L_B\right)$$

Therefore, the minimum barrier thickness  $L_B$  that yields the tunneling probability of  $T \leq 10^{-6}$  is obtained by solving the equation for  $L_B$ .

$$L_B = \frac{-\ln(T) \hbar}{2 \sqrt{2 m^* U_0}} = \frac{-\ln(10^{-6}) \hbar}{2 \sqrt{2 \times 0.45 m_0 \cdot 50 \text{ meV}}} = 9.0 \text{ nm}$$

(b) The tunneling probability can be calculated from Eq. (18.1)

$$T = \exp\left(-\int_{x=0}^{L_B} 2 \hbar^{-1} \sqrt{2 m^* [U(x) - E]} dx\right)$$

For a rectangular barrier,  $U(x) - E$  is a constant, say,  $U$ . Thus

$$T = \exp\left(-2 \hbar^{-1} \sqrt{2 m^* U} L_B\right)$$

(c) The tunneling formula can be re-written with form

$$T = \exp\left(-2 \hbar^{-1} \sqrt{2 m^* U} L_B\right) = \exp\left(-\frac{L_B}{\hbar / (2 \sqrt{2 m^* U})}\right) = \exp\left(-\frac{L_B}{L_e}\right)$$

Therefore,  $L_e = \hbar / (2 \sqrt{2 m^* U})$ .

(d)  $L_e$  is the tunneling length at which the tunneling probability has decreased to  $e^{-1}$ .

(e)  $L_e = \hbar / (2 \sqrt{2 m^* U}) = \hbar / (2 \sqrt{2 \cdot 0.45 m_0 \cdot 50 \text{ meV}}) = 6.5 \text{ \AA}$

5. Consider electrons and holes in a quantum well (QW) structure. The Hamiltonian operator of an optical transition has the form  $H'(x) = e \mathcal{E} x$ .

(a) What is the symmetry of p-orbital-like valence band states and s-orbital-like conduction band states?

(b) Which of the QW electron states,  $E_{0e}, E_{1e}, E_{2e}, E_{3e}$ , has even symmetry?

(c) Which of the QW hole states,  $E_{0h}, E_{1h}, E_{2h}, E_{3h}$ , has even symmetry?

(d) What is the symmetry (even or odd symmetry) of the Hamiltonian operator of an optical transition?

(e) Which interband (conduction-to-valence-band) transitions are possible for an electron in the  $E_{0e}$  state?

(f) Which intraband (within conduction band) transitions are possible for an electron in the  $E_{0e}$  state?

(g) Do the answers to (e) and (f) change in the presence of a static electric field? Explain your answer.

**Solution:**

(a) The p-orbital-like valence-band state is an odd-symmetry function. The s-orbital-like conduction-band state is an even symmetry function.

(b) The envelope wave functions with energy  $E_{0e}, E_{2e}$  have even symmetry.

- (c) The envelope wave functions with energy  $E_{0h}, E_{2h}$  have even symmetry.
- (d) The Hamiltonian operator of an optical excitation,  $H' = e \mathcal{E} x$ , has odd symmetry.
- (e) The allowed interband transitions for an electron in the  $E_{0e}$  state are

$$E_{0e} \rightarrow E_{0h}, \quad E_{0e} \rightarrow E_{2h}, \quad E_{0e} \rightarrow E_{4h} \dots \quad (\text{even symmetry states})$$

**Explanation:** The matrix element for a transition between the conduction and valence band is given by

$$H'_{CV} = \int \psi_C(x) H'(x) \psi_V(x) dx$$

where  $H'(x) = e \mathcal{E} x$ , and  $\psi_C(x)$  is the product of the (s- and p-type) atomic wave function and the quantum-state envelope wave functions.

For the transition  $E_{0e} \rightarrow E_{0h}$ , the transition matrix element is non-zero, because  $H'_{CV} = \int$  even function ( $\psi_{0e}$ )  $\times$  even function ( $\psi_{0e}$ , s-orbital)  $\times$  odd function ( $H'$ )  $\times$  even function ( $\psi_{0h}$ )  $\times$  odd function ( $\psi_{0h}$ , p-orbital)  $dx = \int$  even symmetry function  $dx \neq 0$ .

For the transition  $E_{0e} \rightarrow E_{1h}$ , the transition matrix element is zero, because  $H'_{CV} = \int$  even function ( $\psi_{0e}$ )  $\times$  even function ( $\psi_{0e}$ , s-orbital)  $\times$  odd function ( $H'$ )  $\times$  odd function ( $\psi_{1h}$ )  $\times$  even function ( $\psi_{1h}$ , p-orbital)  $dx = \int$  odd symmetry function  $dx = 0$ .

- (f) The allowed intraband transitions for an electron in the  $E_{0e}$  state are

$$E_{0e} \rightarrow E_{1e}, \quad E_{0e} \rightarrow E_{3e}, \quad E_{0e} \rightarrow E_{5e} \dots \quad (\text{odd symmetry states})$$

**Explanation:** The matrix element for a transition within the conduction band is given by

$$H'_{CC} = \int \psi_C(x) H'(x) \psi_C^*(x) dx$$

where  $H' = e \mathcal{E} x$ ,  $\psi_C(x)$ ,  $\psi_C^*(x)$  are the product of the s-type atomic wave function and the quantum-state envelope wave functions.

For the transition  $E_{0e} \rightarrow E_{1e}$ , the transition matrix element is non-zero, because  $H'_{CC} = \int$  even function ( $\psi_{0e}$ )  $\times$  even function ( $\psi_{0e}$ , s-orbital)  $\times$  odd function ( $H'$ )  $\times$  odd function ( $\psi_{1e}$ )  $\times$  even function ( $\psi_{1e}$ , s-orbital)  $dx = \int$  even-symmetry function  $\neq 0$ .

For the transition  $E_{0e} \rightarrow E_{2e}$ , the transition matrix element is zero, because  $H'_{CC} = \int$  even function ( $\psi_{0e}$ )  $\times$  even function ( $\psi_{0e}$ , s-orbital)  $\times$  odd function ( $H'$ )  $\times$  even function ( $\psi_{1e}$ )  $\times$  even function ( $\psi_{1e}$ , s-orbital)  $dx = \int$  odd-symmetry function  $= 0$ .

- (g) In the presence of a static electric field, the allowed transitions are still allowed. But additional, originally disallowed transitions become allowed in the presence of a static electric field. This is because in the presence of a static electric field, wave functions no longer are of strict even or odd symmetry. As a result, the integral that is used to calculate the transition matrix element is not zero, i.e.  $H'_{jm} \neq 0$ .