Final Exam, Fall 2007

ECSE-6920 – Physical Foundations of Solid-State Devices

Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, excerpts, and calculators are allowed.

1. Consider a hypothetical semiconductor, GaAs, which we assume contains only one single ionized donor impurity and no other impurities. Consider further a free electron that is located at a distance of $d = 150$ Å = 15 nm from the donor impurity.

   (a) What is the numerical value of the potential, caused by the donor impurity at the electron's location?
   (b) Is the potential repulsive or attractive?
   (c) What is the numerical value of the potential for $d \rightarrow \infty$?

   Next consider n-type GaAs with a Si doping concentration of $N_D = 1 \times 10^{17}$ cm^{-3}. Consider one specific ionized donor impurity and the potential around this impurity. A specific free electron is located at a distance of $d = 150$ Å = 15 nm from the donor impurity.

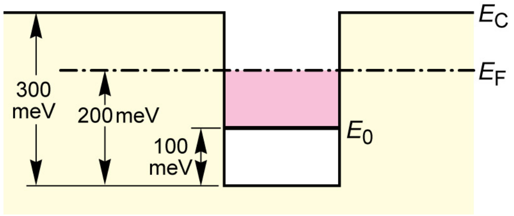
   (d) What is the numerical value of the potential, caused by the specific donor impurity and the free electrons at the specific electron's location?
   (e) Which electron, the specific electron considered under (a) or the specific electron under (d) will be scattered more strongly by the donor impurity?
   (f) As the temperature increases, will the scattering of electrons become stronger or weaker?
   (g) What is the name of the scattering mechanism?
   (h) As the temperature increases, will the potential of the impurity in doped GaAs ($N_D = 1 \times 10^{17}$ cm^{-3}) become more similar or less similar to the potential of the impurity in undoped GaAs?

2. Consider an abrupt heterostructure between Si and Ge in which the conduction and valence band discontinuities, ΔE_C and ΔE_V, are determined by the "electron-affinity rule".

   (a) Draw a band diagram and label all relevant features.
   (b) What is the numerical value of the conduction band discontinuity, ΔE_C, and the valence band discontinuity, ΔE_V, of the heterostructure?
   (c) Next consider the heterostructure to not be abrupt but be graded in a region of thickness 2.0 μm. So in this region, the chemical composition of the graded material can be expressed as Si_xGe_{1-x}. Draw a band diagram and label all relevant features.
   (d) What are the numerical values of the quasi-electric fields in the conduction and valence band in the composition-graded region?
   (e) As a result of the electric field, will electrons in the graded region drift towards Si or Ge?
   (f) As a result of the electric field, will holes in the graded region drift towards Si or Ge?
   (g) What is the numerical value of the time it would it take a hole to drift across the entire graded region assuming a hole mobility of 400 cm^2/(V s).
   (h) Calculate the numerical value of the diffusion constant of the hole, based on the Einstein relationship.
   (i) What is the distance the hole would diffuse during the time calculated under (g)?
   (j) Comparing the answers of (g) and (i), which distance, the distance diffused or the distance drifted, is larger? What do you conclude from this comparison?

3. (a) At low temperature, the Fermi-Dirac distribution can be considered as a step-function. Explain why!
   (b) At high degeneracy, the Fermi-Dirac distribution can be considered as a step-function. Explain why!

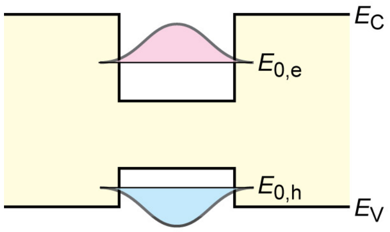
   Consider the quantum well structure shown in the figure below. Assume the quantum well material is GaAs.



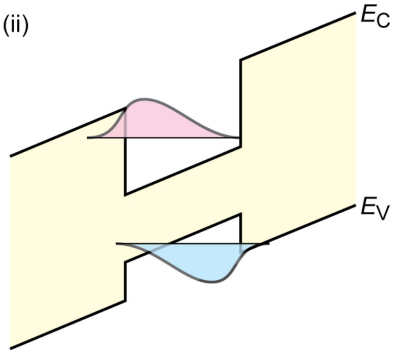
   (c) Is the carrier concentration in the quantum well degenerate or non-degenerate?
   (d) What is the numerical value of the density of states in the GaAs quantum well?
   (e) What is the carrier density (per unit area) in the GaAs quantum well?
   (f) What would the carrier density in the quantum well need to be for the quantum well to overflow with carriers (i.e. the Fermi level reach the conduction band of the quantum-barrier material)?

   Next consider electron-hole recombination in the two quantum well structures shown in the figure below.

(i)



(ii)



   (g) Which of the two structures, (i) or (ii) has a greater probability of radiative recombination?
   (h) Justify your answer using perturbation theory!

Room temperature properties of Si, Ge, and GaAs

Quantity	***Symbol***	***Si***	***Ge***	***GaAs***	***(Unit)***
Crystal structure		D	D	Z	–
Gap: Direct (*D*) / Indirect (*I*)		*I*	*I*	*D*	–
Lattice constant	$a_0 =$	5.43095	5.64613	5.6533	Å
Bandgap energy	$E_\mathrm{g} =$	1.12	0.66	1.42	eV
Intrinsic carrier concentration	$n_\mathrm{i} =$	1.0×10^{10}	2.0×10^{13}	2.0×10^{6}	cm^{-3}
Effective DOS at CB edge	$N_\mathrm{c} =$	2.8×10^{19}	1.0×10^{19}	4.4×10^{17}	cm^{-3}
Effective DOS at VB edge	$N_\mathrm{v} =$	1.0×10^{19}	6.0×10^{18}	7.7×10^{18}	cm^{-3}
Electron mobility	$\mu_\mathrm{n} =$	1500	3900	8500	$\mathrm{cm}^2/(\mathrm{Vs})$
Hole mobility	$\mu_\mathrm{p} =$	450	1900	400	$\mathrm{cm}^2/(\mathrm{Vs})$
Electron diffusion constant	$D_\mathrm{n} =$	39	101	220	cm^2/s
Hole diffusion constant	$D_\mathrm{p} =$	12	49	10	cm^2/s
Electron affinity	$\chi =$	4.05	4.0	4.07	V
Minority carrier lifetime	$\tau =$	10^{-6}	10^{-6}	10^{-8}	s
Electron effective mass	$m_\mathrm{e}^* =$	$0.98\ m_\mathrm{e}$	$1.64\ m_\mathrm{e}$	$0.067\ m_\mathrm{e}$	–
Heavy hole effective mass	$m_\mathrm{hh}^* =$	$0.49\ m_\mathrm{e}$	$0.28\ m_\mathrm{e}$	$0.45\ m_\mathrm{e}$	–
Relative dielectric constant	$\varepsilon_\mathrm{r} =$	11.9	16.0	13.1	–
Refractive index near E_g	$\overline{n} =$	3.3	4.0	3.4	–
Absorption coefficient near E_g	$\alpha =$	10^3	10^3	10^4	cm^{-1}

- D = Diamond. Z = Zincblende. W = Wurtzite. DOS = Density of states. VB = Valence band. CB = Conduction band
- The Einstein relation relates the diffusion constant and mobility in a non-degenerately doped semiconductor: $D = \mu\,(k\,T/e)$
- Minority carrier diffusion lengths are given by $L_\mathrm{n} = (D_\mathrm{n}\,\tau_\mathrm{n})^{1/2}$ and $L_\mathrm{p} = (D_\mathrm{p}\,\tau_\mathrm{p})^{1/2}$
- The mobilities and diffusion constants apply to low doping concentrations ($\approx 10^{15}\ \mathrm{cm}^{-3}$). As the doping concentration increases, mobilities and diffusion constants decrease.
- The minority carrier lifetime τ applies to doping concentrations of $10^{18}\ \mathrm{cm}^{-3}$. For other doping concentrations, the lifetime is given by $\tau = B^{-1}\,(n + p)^{-1}$, where $B_\mathrm{Si} \approx 5 \times 10^{-14}\ \mathrm{cm}^3/\mathrm{s}$, $B_\mathrm{Ge} \approx 5 \times 10^{-13}\ \mathrm{cm}^3/\mathrm{s}$, and $B_\mathrm{GaAs} = 10^{-10}\ \mathrm{cm}^3/\mathrm{s}$.

Physical constants

a_B	=	0.5292 Å	Bohr radius	($a_B = 0.5292 \times 10^{-10}$ m)
ε_0	=	8.8542×10^{-12} As/(Vm)	absolute dielectric constant	
e	=	1.6022×10^{-19} C	elementary charge	
c	=	2.9979×10^{8} m/s	velocity of light in vacuum	
E_{Ryd}	=	13.606 eV	Rydberg energy	
g	=	9.8067 m/s^2	acceleration on earth at sea level due to gravity	
G	=	6.6873×10^{-11} m^3/(kg s^2)	gravitational constant	($F = G\,M\,m/r^2$)
h	=	6.6261×10^{-34} Js	Planck constant	($h = 4.1356 \times 10^{-15}$ eVs)
$\hbar$	=	1.0546×10^{-34} Js	$\hbar = h/(2\pi)$	($\hbar = 6.5821 \times 10^{-16}$ eVs)
k	=	1.3807×10^{-23} J/K	Boltzmann constant	($k = 8.6175 \times 10^{-5}$ eV/K)
μ_0	=	1.2566×10^{-6} Vs/(Am)	absolute magnetic constant	
m_e	=	9.1094×10^{-31} kg	free electron mass	
N_{Avo}	=	6.0221×10^{23} mol^{-1}	Avogadro number	
$R = k\,N_{Avo}$	=	8.3145 J K^{-1} mol^{-1}	ideal gas constant	

Note:

The *dielectric permittivity* of a material is given by $\varepsilon = \varepsilon_r\,\varepsilon_0$ where ε_r and ε_0 are the *relative* and *absolute* dielectric permittivity, respectively.

The *magnetic permeability* of a material is given by $\mu = \mu_r\,\mu_0$ where μ_r and μ_0 are the *relative* and *absolute* magnetic permeability, respectively.

Useful conversions

1 eV = 1.6022×10^{-19} C V = 1.6022×10^{-19} J

$E = h\nu = hc/\lambda = 1239.8\ \text{eV} / (\lambda/\text{nm})$

$kT = 25.86$ meV (at $T = 300$ K)

$kT = 25.25$ meV (at $T = 20$ °C = 293.15 K)