Midterm Exam, Fall 2007

ECSE-6920 – Physical Foundations of Solid-State Devices

Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***) Textbook, manuscript, excerpts, and calculators are allowed.

1. Assume that a one-dimensional wave function is given by

$$\psi = A\left(\frac{3}{16} - \frac{x^2}{L^2} + \frac{x^4}{L^4}\right) \quad \text{for} \quad -L/2 \le x \le L/2$$

$$\psi = 0 \quad \text{for} \quad x < -L/2 \text{ and } x > L/2$$

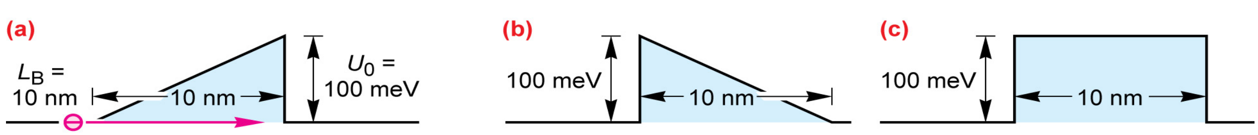
   (a) Schematically sketch the wave function. Explain the purpose of the constant A.
   (b) Are ψ and ψ' continuous?
   (c) What is $\psi(x = 0)$?
   (d) What is $\psi(x = L/2)$?
   (e) For $A = 1 / (L^{1/2})$, is the wave function normalized? Give ***yes*** or ***no*** answer and explain.
   (f) Calculate the position expectation value.
   (g) Assume that the potential energy surrounding the wave function is given by

$$U = 0 \quad \text{for} \quad -L/2 \le x \le L/2$$

$$U = +U_0 \quad \text{for} \quad x < -L/2 \text{ and } x > L/2$$

   What is the expectation value of the potential energy?
   (h) Is the expectation value of the total energy zero or non-zero? (Explain)

2. An electron with effective mass $m^* = 0.5\ m_0$ tunnels through a barrier, shown in ***Figure*** (a) below that is triangular in shape with U_0 = 100 meV high and L_B = 100 Å = 10 nm wide.



   (a) Give a formula for the tunneling barrier potential as a function of x.
   (b) What is the tunneling probability of the electron calculated by using the WKB approximation?
   (c) If a free electron with mass m_0 were to tunnel through the barrier, what would be its tunnel probability?
   (d) Is the tunneling probability of the barrier shown in ***Figure*** (b) the same as the one shown in ***Figure*** (a)? If not, is it higher or lower?
   (e) Is the tunneling probability of the barrier shown in ***Figure*** (c) the same as the one shown in ***Figure*** (a)? If not, is it higher or lower?

3. Given are the dispersion relations (energy band structures) of (a) gallium arsenide and (b) silicon.
   (a) Identify a direct and an indirect band-gap semiconductor material. (Explain)
   (b) Which material has a smaller conduction-band electron effective mass, Si or GaAs?
   (c) Can an effective mass become negative? (Explain)
   (d) If so, show one region in the E-versus-k diagram, in which the electron effective mass is negative? (Mark location in figure)
   (e) Highlight the range of k values in the figure for which the materials have an electron effective mass and hole effective mass that can be assumed to be a constant.
   (f) A dispersion relation is given by $E = \alpha k^2$, where $\alpha = 5 \times 10^{-39}$ J m^2. Calculate the numerical value of the effective mass.
   (g) For E = 100 meV, calculate the particle velocity.
   (h) For E = 100 meV, calculate wave number k.
   (i) For E = 100 meV, calculate the wavelength λ.
   (j) If the above E is purely kinetic, calculate the group velocity.