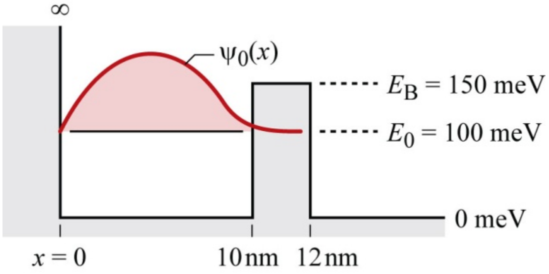
Final Exam, Fall 2008

ECSE-6920 – Physical Foundations of Solid-State Devices, Prof. Schubert

Note: (***i***) Put your name on paper, show your work, underline results, and always show units.
(***ii***)Textbook, manuscript, excerpts, and calculators are allowed.

1. Consider the diagram shown below and assume that $\psi_0(x)$ is the wave function of an electron and that $\psi_0(x)$ is essentially confined to the well region, that is, $\int_0^{L_{QW}} \psi_0^*(x)\ \psi_0(x) \approx 1$.



(a) Calculate the approximate value of the expectation value of the potential energy.
(b) Give the approximate numerical value of the electron's kinetic energy.
(c) Assuming that the effective electron mass is $m_e^* = 0.1 \times m_e$, calculate the velocity of the electron.
(d) The electron oscillates between the two walls. Calculate the mean time that elapses between two collisions of the electron with the right-hand side wall.
(e) Calculate the attempt rate, that is, the electron's number of attempts to escape per second.
(f) Calculate the electron's tunneling probability through the quantum barrier (L_{QB} = 20 Å = 2 nm).
(g) Whenever the electron collides with the right-hand side wall, it attempts to escape. Calculate the average time the electron is located inside the well before successfully escaping from the well.
(h) What is the energy uncertainty ΔE of the state?
(i) Does ΔE increase as the "transparency" of the barrier increases?
(j) Make the analogy to an optical Fabry-Perot resonator. Does the line width $\Delta\lambda$ increase as the transparency of the two reflectors increase? Explain.

(a) $\langle U \rangle = \int_{-\infty}^{\infty} \psi_0^*(x) U(x) \psi_0(x) = 0$

(b) $\langle E_{kin} \rangle = \left\langle \frac{p^2}{2m} \right\rangle = \int_{-\infty}^{\infty} \psi_0^*(x) \left[\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \right] \psi_0(x)$

Since, E_{Total} = 100 meV = $E_{\text{kin}} + E_{\text{pot}}$ and E_{pot} = 0 meV
Therefore, E_{kin} = 100 meV

(c) $E_{\text{kin}} = \frac{1}{2} m_e^* v^2 = 100 \text{ meV}$

Therefore $v = \sqrt{\dfrac{2 \times E_{\text{kin}}}{m_e^*}} = 5.93 \times 10^5$ m/s

(d) $t = \dfrac{\text{Distance}}{\text{Velocity}} = \dfrac{2 \times L_{\text{QW}}}{v} = 33.7 \times 10^{-15}\ \text{s} = 33.7\ \text{fs}$

(e) The attempt rate, $A = \dfrac{v}{D} = \dfrac{1}{t} = 2.965 \times 10^{13}\ \#/\text{s}$

(f) The tunneling probability, $T = e^{-\int_{x=0}^{L_B} 2\hbar^{-1}\sqrt{2m^*[U(x)-E]}\,dx} = 0.234$

(g) $\Delta\tau = \dfrac{1}{AT} = 144 \times 10^{-15}\ \text{s} = 144\ \text{fs}$

(h) The energy uncertainty, $\Delta E = \dfrac{\hbar}{\Delta\tau} = 7.324 \times 10^{-22}\ \text{J} = 4.57\ \text{meV}$

(i) Yes.

(j) An analogy can be given for an electron in a potential well by a photon in a Fabry-Perot cavity in optics. The photon is analogous to the electron, the mirrors of the Fabry-Perot cavity to the potential wells, the mirror reflectivity to the height/energy and thickness of the barriers. A photon will be completely confined and trapped inside the Fabry-Perot cavity with 100% mirror reflectivities, just as the electron inside an infinite potential well. The probability of the photon escaping the cavity depends on the reflectivity of the mirrors, similar to tunneling probability of the electron which depends on the 'transparency' of the barrier. As the mirror reflectivity decreases or transparency increases, the probability of photon escaping the cavity increases, just as tunneling probability increases with thinner barriers. Also, the uncertainty in the photon energy or wavelength increases as the mirror reflectivity increases; similar to how the energy uncertainty increases when barrier 'transparency' decreases.

2. (a) Write down the Einstein relation.

(b) In a semiconductor at $T = 0$ K, can the carrier mobility be $\mu \neq 0$?

(c) In a semiconductor at $T = 0$ K can $D \neq 0$?

(d) Assume an electric field $\mathcal{E}$ is applied to semiconductor. Give the distance x a carrier with mobility μ drifts as function of time as a general function (no numeric values).

(e) Assume a diffusing carrier with diffusion constant D. Give diffusion distance x as a function of time as a general function (no numeric values).

(f) Explain in words the physical origin of the different time dependency functions, diffusion $x_{\text{diffusion}}(t)$ and drift $x_{\text{drift}}(t)$.

(a) $D = \dfrac{kT}{e}\mu$

(b) Yes.

(c) No. There is no thermal energy for the carriers to diffuse. Therefore, $D = 0$.

(d) The drift velocity of a carrier with mobility μ under an electric field $\mathcal{E}$ is given as, $V_{\text{drift}} = \mu\,\mathcal{E}$. Therefore, $x_{\text{drift}} = \mu\,\mathcal{E}\,t$. Since μ and $\mathcal{E}$ are constants, distance x_{drift} travelled as a function of time t shows linear relation.

(e) The diffusion distance x_{diff}, also called as diffusion length L_D, travelled by a carrier with diffusion constant D in time t is given as,

$x_{\text{diff}} = \sqrt{D \times t}$. Therefore, it shows square root relation.

(f) Drift is motion of a carrier under the influence of electric field. The electric field pulls the carrier in one definite direction making the carrier movement much more directed, impeded only by the scattering events. Due to this 'pull', the backward propagation of the carrier is minimized, resulting in a net movement in the specific direction dictated by the electric field. Without any scattering, a carrier would continuously accelerate under the electric field. However, due to the scattering, at each scattering event the carrier velocity reduces and the carrier accelerates back until the next scattering event. The average drift velocity of the carrier shows a linear relation of distance travelled with the time. Whereas drift is a *directed* motion in an electric field, diffusion is a *random* movement of carriers with thermal energy, called *Brownian motion*. Since there is no 'pull' experience by the carrier in any specific direction, there is a significantly higher probability of a diffusing carrier travelling in a backward direction than a drifting carrier. Due to the random nature of diffusion, it tends to be slower process than drift.

3. Consider the left-hand-side part of the figure below which shows a band occupation in a semiconductor.

(a) What process will occur when the photon is incident on the semiconductor?

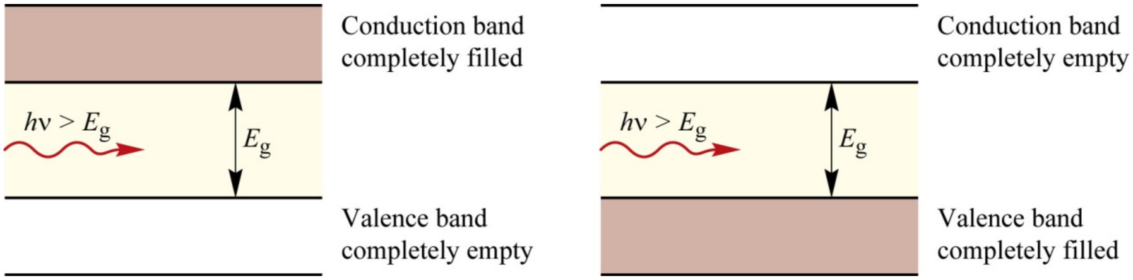
(b) Write the formula for the quantum mechanical probability per unit time for this process.

Next, consider the right-hand-side part of the figure below.

(c) What process will occur when the photon is incident on the semiconductor?

(d) Write the formula for the quantum mechanical probability per unit time for this process.

(e) Are probabilities calculated under (b) and (d) the same or are they different? Explain.



(a) When a photon is incident on the semiconductor with the given band occupation on the left-hand-side of the figure stimulated emission can occur.

(b) The downward transition probability (per atom) is the sum of and induced term proportional to the radiation density, and a spontaneous term. Ignoring the spontaneous emission in the above example, the probability of stimulated emission is given as,

$W_{2\to1} = B_{2\to1}\rho(\nu)$

(c) When a photon is incident on the semiconductor with the given band occupation on the left-hand-side of the figure stimulated/induced absorption can occur.

(d) The probability of the stimulated/induced absorption is given as,

$W_{1\to2} = B_{1\to2}\rho(\nu)$

(e) Using thermal equilibrium considerations, Einstein showed that the probabilities in (b) and (d) are exactly the same.

$W_{2\to1} = W_{1\to2}$

4. Let us assume that GaAs and Si both would have a lattice-atom concentration of about 5×10^{22} atoms per cm^3. For some applications, it is desirable that the electrically active doping concentration be as high as possible. Assume that GaAs is doped with Si occupying exclusively the Ga site. Assume further that Si is doped with P.
   (a) What conductivity type will result when doping the semiconductors?
   (b) What is the concentration of potentially available donor sites in GaAs?
   (c) What is the concentration of potentially available donor sites in Si?
   (d) Based on the above two values of the potentially available sites, which of the two semiconductors would you expect to allow for a higher doping concentration?
   (e) Do you happen to know which of the two semiconductors, GaAs or Si, allows for higher electrically active dopant concentrations and thus higher carrier concentrations?

(a) GaAs doped with Si and Si doped with P, both will results in n-type conductivity.

(b) Since GaAs crystal has equal number of Ga and As atoms, 2.5×10^{22} atoms per cm^3 are potentially available donor sites in GaAs.

(c) 5.0×10^{22} atoms per cm^3 are potentially available donor sites in Si.

(d) Si.

(e) Si. At room temperature, donors in both GaAs and Si are close to 100% ionized. Therefore Si shows higher carrier concentration because it allows for much higher doping concentration.