

Density of states

The concentration of neutral impurities, ionized impurities, and free carriers in a doped semiconductor depends on a large number of parameters such as the impurity atom concentration, the free carrier mass, the bandgap energy, and the dielectric constant. The interdependences of the free majority and minority carrier concentration, the impurity concentration, impurity ionization energy as well as some other constants and materials parameters are given by *semiconductor statistics*. Semiconductor statistics describes the probabilities that a set of electronic states are either vacant or populated.

Electronic states include localized impurity states as well as delocalized conduction and valence band states. In the simplest case, an impurity has a single state with no degeneracy ($g_0 = 1$). However, an impurity may have a degenerate ground state ($g_0 > 1$) as well as excited levels which may need to be considered. The states in the bands and their dependence on energy are described by the *density of states*. In semiconductor heterostructures, the free motion of carriers is restricted to two, one, or zero spatial dimensions. In order to apply semiconductor statistics to such systems of reduced dimensions, the density of states in quantum wells (two dimensions), quantum wires (one dimension), and quantum dots (zero dimensions), must be known. The density of states in such systems will also be calculated in this chapter.

12.1 Density of states in bulk semiconductors (3D)

Carriers occupy either localized impurity states or delocalized continuum states in the conduction band or valence band. In the simplest case, each impurity has a single, non-degenerate state. Thus, the density of impurity states equals the concentration of impurities. The energy of the impurity states is the same for all impurities (of the same species) as long as the impurities are sufficiently far apart and do not couple. The density of continuum states is more complicated and will be calculated in the following sections. Several cases will be considered including (i) a spherical, single-valley band, (ii) an anisotropic band, (iii) a band with multiple valleys, and (iv) the density of states in a semiconductor with reduced degrees of freedom such as quantum wells, quantum wires, and quantum boxes. Finally the *effective* density of states will be calculated.

Single-valley, spherical, and parabolic band

The simplest band structure of a semiconductor consists of a single valley with an isotropic (*i. e.* spherical), parabolic dispersion relation. This situation is closely approximated by, for example, the conduction band of GaAs. The electronic density of states is defined as the number of electron states per unit volume and per unit energy. The finiteness of the density of states is a result of the *Pauli principle*, which states that only two electrons of opposite spin can occupy one volume element in phase space. The *phase space* is defined as a six-dimensional space composed of real space and momentum space. We now define a ‘volume’ element in phase space to consist of a range of positions and momenta of a particle, such that the position and momentum of the particle are distinguishable from the positions and momenta of other particles. In order to be distinguishable, the range of positions and momenta must be equal or exceed the range given by the *uncertainty relation*. The volume element in phase space is then given by

$$\Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z = (2\pi\hbar)^3. \quad (12.1)$$

The ‘volume’ element in phase space is $(2\pi\hbar)^3$. For systems with only one degree of freedom, Eq. (12.1) reduces to the one-dimensional Heisenberg uncertainty principle $\Delta x \Delta p_x = 2\pi\hbar$. The Pauli principle states that two electrons of opposite spin occupy a ‘volume’ of $(2\pi\hbar)^3$ in phase space. Using the de Broglie relation ($p = \hbar k$) the ‘volume’ of phase space can be written as

$$\Delta x \Delta y \Delta z \Delta k_x \Delta k_y \Delta k_z = (2\pi)^3. \quad (12.2)$$

The **density of states** per unit energy and per unit volume, which is denoted by $\rho_{\text{DOS}}(E)$, allows us to determine the total number of states per unit volume in an energy band with energies E_1 (bottom of band) and E_2 (top of band) according to

$$N = \int_{E_1}^{E_2} \rho_{\text{DOS}}(E) dE. \quad (12.3)$$

Note that N is the total number of states per unit volume, and $\rho_{\text{DOS}}(E)$ is the density of states per unit energy per unit volume. To obtain the density of states per unit energy dE , we have to determine how much unit-volumes of k -space is contained in the energy interval E and $E + dE$, since we already know that one unit volume of k -space can contain two electrons of opposite spin.

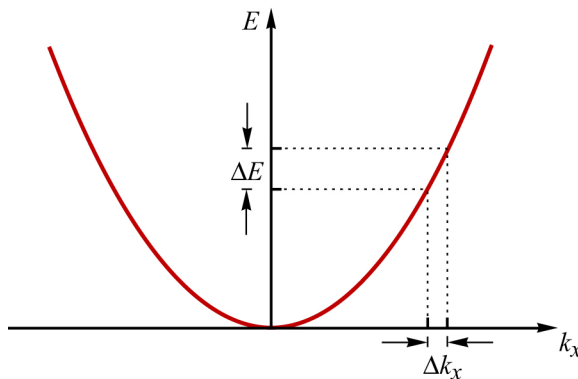


Fig. 12.1. Parabolic dispersion relation with a k -space interval Δk_x and a corresponding energy interval $\Delta E = (\partial E / \partial k_x) \Delta k_x$.

In order to obtain the volume of k -space included between two energies, the *dispersion relation* will be employed. A one-dimensional, parabolic dispersion relation $E = E(k_x)$ is shown in **Fig. 12.1**. For a given dE one can easily determine the corresponding length in k -space, as illustrated in **Fig. 12.1**. The k -space length associated with an energy interval dE is simply given by the slope of the dispersion relation. While the one-dimensional dispersion relations can be illustrated easily, the three-dimensional dispersion relation cannot be illustrated in three-dimensional space. To circumvent this difficulty, *surfaces of constant energy in k -space* are frequently used to illustrate a three-dimensional dispersion relation. As an example, the constant energy surface in k -space is illustrated in **Fig. 12.2** for a spherical, single-valley band. A large separation of the constant energy surfaces, *i. e.* a large Δk for a given ΔE , indicates a weakly curved dispersion and a large effective mass.

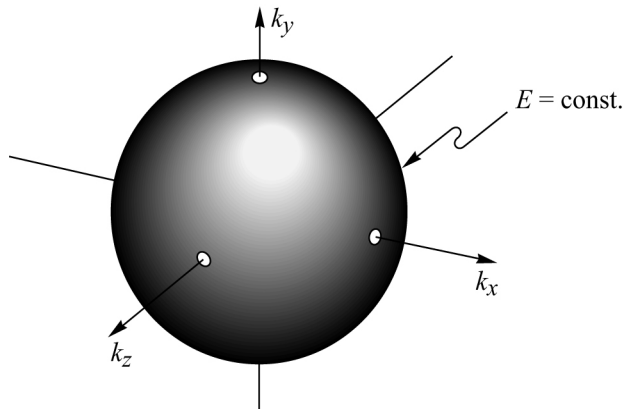


Fig. 12.2. Constant energy surface for a single-valley, isotropic band.

In order to obtain the volume of k -space enclosed between two constant energy surfaces, which correspond to energies E and $E + dE$, we (first) determine dk associated with dE and (second) integrate over the entire constant energy surface. The ‘volume’ of k -space enclosed between the two constant energy surfaces shown in **Fig.** 12.3 is thus given by

$$V_{k\text{-space}}(E) = dE \int_{\text{Surface}} \frac{\partial k}{\partial E(k)} ds \quad (12.4)$$

where ds is an area element of the constant energy surface. In a three-dimensional k -space we use $\text{grad}_k = (\partial / \partial k_x, \partial / \partial k_y, \partial / \partial k_z)$ and obtain

$$V_{k\text{-space}}(E) = dE \int_{\text{Surface}} \frac{ds}{\nabla_k E(k)} \cdot \quad (12.5)$$

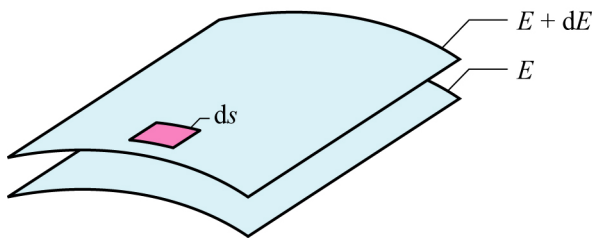


Fig. 12.3. Constant-energy surfaces with energy E and $E + dE$ used to calculate volume in k -space enclosed between the two surfaces.

Since an electron requires a volume of $4\pi^3$ in phase space, the number of states per unit volume is given by

$$N(E) = \frac{1}{4\pi^3} dE \int_{\text{Surface}} \frac{ds}{\nabla_k E(k)} \cdot \quad (12.6)$$

Finally, we obtain the density of states per unit energy and unit volume according to

$$\rho_{\text{DOS}}(E) = \frac{1}{4\pi^3} \int_{\text{Surface}} \frac{ds}{\nabla_k E(k)} \cdot \quad (12.7)$$

In this equation, the surface element ds is always perpendicular to the vector $\text{grad}_k E(k)$. Note

that the surface element ds is in k -space and that ds has the dimension m^{-2} .

Next we apply the expression for the density of states to *isotropic parabolic* dispersion relations of a three-dimensional semiconductor. In this case the surface of constant energy is a sphere of area $4\pi k^2$ and the parabolic dispersion is $E = \hbar^2 k^2 / (2m^*) + E_{\text{pot}}$ where k is the wave vector. Insertion of the dispersion in Eq. (12.7) yields the density of states in a semiconductor with a single-valley, isotropic, and parabolic band

$$\rho_{\text{DOS}}^{\text{3D}}(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_{\text{pot}}} \quad (12.8)$$

where E_{pot} is a potential energy such as the conduction band edge or the valence band edge energy, E_C or E_V , respectively.

Single-valley, anisotropic, parabolic band

In an anisotropic single-valley band, the dispersion relation depends on the spatial direction. Such an anisotropic dispersion is found in III–V semiconductors in which the L- or X- point of the Brillouin zone is the lowest minimum, for example in GaP or AlAs. The surface of constant energy is then no longer a sphere, but an ellipsoid, as shown in **Fig. 12.4**. The three main axes of the ellipsoid may have different lengths, and thus the three dispersion relations are curved differently. If the main axes of the ellipsoid align with a cartesian coordinate system, the dispersion relation is

$$E = \frac{\hbar^2 k_x^2}{2m_x^*} + \frac{\hbar^2 k_y^2}{2m_y^*} + \frac{\hbar^2 k_z^2}{2m_z^*} . \quad (12.9)$$

The vector $\text{grad}_k E$ is given by $\text{grad}_k E = (\hbar^2 k_x / m_x^*, \hbar^2 k_y / m_y^*, \hbar^2 k_z / m_z^*)$. Since the vector $\text{grad}_k E$ is perpendicular on the surface element, the *absolute* values of ds and $\text{grad}_k E$ can be taken for the integration. Integration of Eq. (12.7) with the dispersion relation of Eq. (12.9) yields the density of states in an anisotropic semiconductor with parabolic dispersion relations, *i. e.*

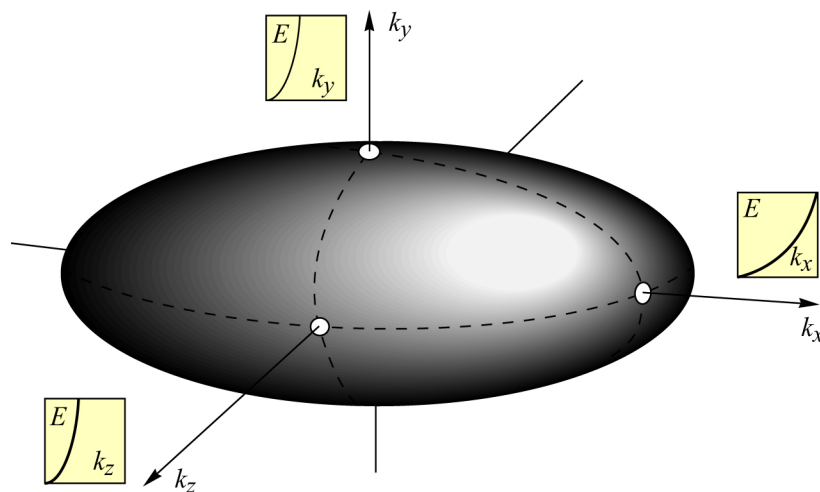


Fig. 12.4. Ellipsoidal constant energy surface with a weakly curved dispersion relation along the k_x axis and strongly curved dispersion along the k_y and k_z axis.

$$\rho_{\text{DOS}}(E) = \frac{\sqrt{2}}{\pi^2 \hbar^3} \sqrt{m_x^* m_y^* m_z^*} \sqrt{E - E_{\text{pot}}} . \quad (12.10)$$

If the main axes of the constant-energy ellipsoid do not align with the k_x , k_y , and k_z axes of the coordinate system then m_x^* , m_y^* , and m_z^* can be formally replaced by m_1^* , m_2^* , and m_3^* .

Frequently, the constant energy surfaces are rotational ellipsoids, that is, two of the main axes of the ellipsoid are identical. The axes are then denoted as the transversal and the longitudinal axes for the short and long axes, respectively. Such a rotational ellipsoid is schematically shown in **Fig.** 12.4. A relatively light mass is associated with the (short) transversal axis, while a relatively heavy mass is associated with the (long) longitudinal axis. If the masses are denoted as m_t^* and m_l^* for the transversal and the longitudinal mass, respectively, Eq. (12.10) can be modified according to

$$\rho_{\text{DOS}}(E) = \frac{\sqrt{2}}{\pi^2 \hbar^3} \sqrt{m_l^* m_t^{*2}} \sqrt{E - E_{\text{pot}}} . \quad (12.11)$$

The anisotropic masses m_x^* , m_y^* , m_z^* , m_l^* , and m_t^* are frequently used to define a **density-of-states effective mass**. This mass is given by

$$m_{\text{DOS}}^* = (m_x^* m_y^* m_z^*)^{1/3} \quad (12.12a)$$

$$m_{\text{DOS}}^* = (m_l^2 m_t)^{1/3} \quad (12.12b)$$

The density of states is then given by

$$\rho_{\text{DOS}}(E) = \frac{\sqrt{2}}{\pi^2 \hbar^3} (m_{\text{DOS}}^*)^{3/2} \sqrt{E - E_{\text{pot}}} . \quad (12.13)$$

Note that for isotropic semiconductors the effective mass coincides with the density-of-states effective mass.

Multiple valleys

At several points of the Brillouin zone, several equivalent minima occur. For example, eight equivalent minima occur at the L-point as schematically shown in **Fig.** 12.5. Each of the valleys can accommodate carriers, since the minima occur at different k_x , k_y , and k_z values, *i. e.* the Pauli principle is not violated. The density of states is thus obtained by multiplication with the number of equivalent minima, that is

$$\rho_{\text{DOS}}(E) = \frac{M_c \sqrt{2}}{\pi^2 \hbar^3} \sqrt{m_1^* m_2^* m_3^*} \sqrt{E - E_{\text{pot}}} \quad (12.14)$$

where M_c is the number of equivalent minima and m_1^* , m_2^* , and m_3^* are the effective masses for motion along the three main axes of the ellipsoid.

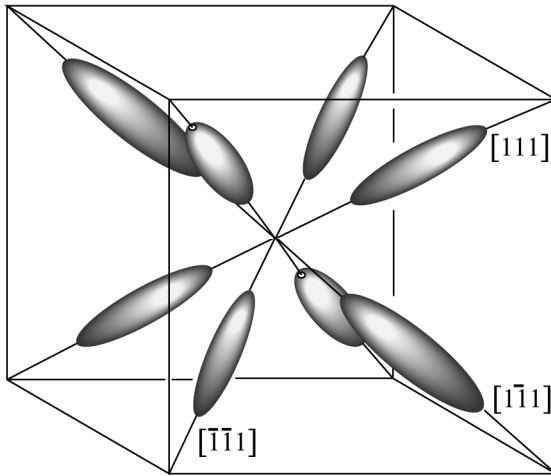


Fig. 12.5. Constant energy surface for the L -point of the Brillouin zone. The band structure consists of eight equivalent rotational ellipsoids.

12.2 Density of states in semiconductors with reduced dimensionality (2D, 1D, 0D)

Semiconductor heterostructure allows one to change the band energies in a controlled way and confine charge carriers to two (2D), one (1D), or zero (0D) spatial dimensions. Due to the confinement of carriers, the dispersion relation along the confinement direction is changed. The change in dispersion relation results in a change in the density of states.

Confinement of a carrier in one spatial dimension, *e. g.* the z -direction results in the formation of quantum states for motion along this direction. Consider the ground state in a quantum well of width L_z with infinitely high walls. The ground-state energy is obtained from the solution of Schrödinger's equation and is given by

$$E_0 = \frac{\hbar^2}{2m^*} \left(\frac{\pi}{L_z} \right)^2. \quad (12.15)$$

The particle in the quantum well can assume a range of momenta in the z -direction; the range is given by the uncertainty principle, *i. e.*

$$\Delta k_z = \frac{\Delta p_z}{\hbar} = \frac{2\pi}{L_z}. \quad (12.16)$$

The dispersion relation for motion along the confinement (z -) direction is thus given by

$$E = E_0 \quad \text{for entire range of } k_z. \quad (12.17)$$

The dispersion is flat, *i. e.* constant for all values of k_z . The z -component of the vector $\text{grad}_k E$ (see Eq. 12.7) is therefore zero and need not be considered.

We next consider the x - and y -direction and recall that the Schrödinger equation is separable for the three spatial dimensions. Thus, the kinetic energy in the xy -plane is given by

$$E = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2) \quad (12.18)$$

for a parabolic dispersion.

The surface of constant energy for the dispersion relation given by Eq. (12.18) is shown in

Fig. 12.6, and is a circle around $k_x = k_y = 0$. The density of states of such a 2D electron system is obtained by similar considerations as for the 3D case. The reduced phase space now consists only of the xy -plane and the k_x and k_y coordinates. Correspondingly, the two-dimensional density of states is the number of states per *unit-area* and unit-energy. The volume of k -space between the circles of constant energy is given by Eq. (12.5). The equation is evaluated most conveniently in polar coordinates in which $k_r = (k_x^2 + k_y^2)^{1/2}$ is the radial component of the k -vector. The surface integral reduces to a line integral and the total length of the circular line is $2\pi k_r$. The volume of k -space then obtained is

$$V_{k\text{-space}}^{2D}(E) = dE \int_{\text{Surface}} \frac{ds}{\nabla_k E(k)} = \frac{2\pi m^*}{\hbar^2}. \quad (12.19)$$

Since two electrons of opposite spin require a volume element of $(2\pi)^2$ in phase space, the density of states of a 2D electron system is given by

$$\rho_{\text{DOS}}^{2D}(E) = \frac{m^*}{\pi \hbar^2} \quad (E \geq E_0) \quad (12.20)$$

where E_0 is the ground state of the quantum well system. For energies $E \geq E_0$, the 2D density of states is a constant and does not depend on energy. If the 2D semiconductor has more than one quantum state, each quantum state has a state density of Eq. (12.20). The total density of states can be written as

$$\rho_{\text{DOS}}^{2D}(E) = \frac{m^*}{\pi \hbar^2} \sum_n \sigma(E - E_n) \quad (12.21)$$

where E_n are the energies of quantized states and $\sigma(E - E_n)$ is the step function.

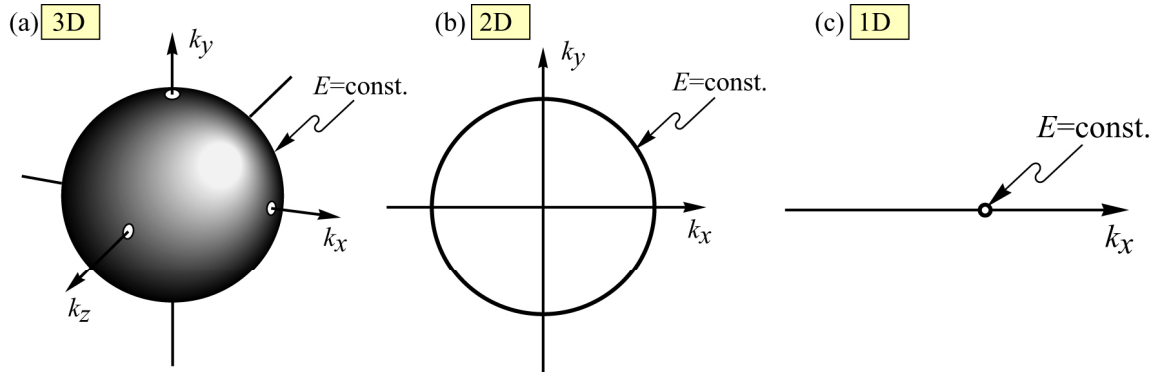


Fig. 12.6. Constant energy surfaces of a (a) 3-dimensional, (b) 2-dimensional, and (c) 1-dimensional system. The surfaces are a sphere, a circle, and a point for 3D, 2D, and 1D systems, respectively.

We next consider a one-dimensional (1D) system, the quantum wire, in which only one direction of motion is allowed, *e. g.* along the x -direction. The dispersion relation is then given by $E = \hbar^2 k_x^2 / (2m^*)$. The ‘volume’ (*i. e.* length-unit) in k -space is obtained in analogy to the

three-dimensional and two-dimensional case according to Eq. (12.5). The ‘surface’ integral reduces to a single point in k -space, *i. e.* the point $k = k_x$. Thus, the volume of k -space is given by

$$V_{k\text{-space}}^{\text{1D}}(E) = \int_{\text{Surface}} \frac{\delta(k_x - k_{x0}) ds}{\nabla_k E(k_x)} = \sqrt{\frac{m^*}{2\hbar^2(E - E_0)}} \quad (E \geq E_0). \quad (12.22)$$

The volume in phase space of two electrons with opposite spin is given by 2π and thus the 1D density of states is given by

$$\rho_{\text{DOS}}^{\text{1D}}(E) = \frac{1}{\pi\hbar} \sqrt{\frac{m^*}{2(E - E_0)}} \quad (E \geq E_0). \quad (12.23)$$

Note that the density of states in a 3-, 2- and 1-dimensional system has a functional dependence on energy according to $E^{1/2}$, E^0 , and $E^{-1/2}$, respectively. For more than one quantized state, the 1D density of states is given by

$$\rho_{\text{DOS}}^{\text{1D}}(E) = \frac{1}{\pi\hbar} \sum_n \sqrt{\frac{m^*}{2(E - E_n)}} \sigma(E - E_n) \quad (12.24)$$

where E_n are the energies of the quantized states of the wire.

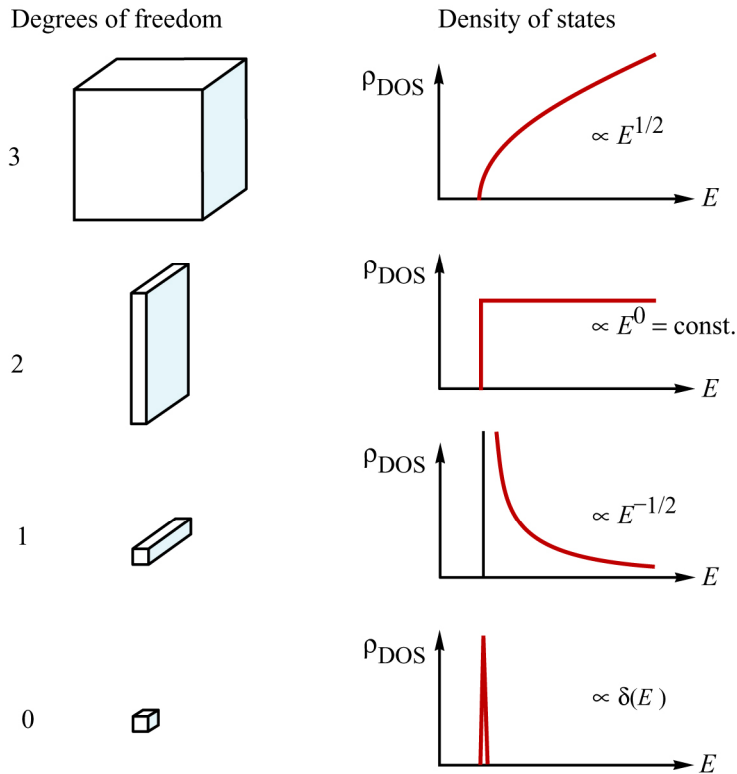


Fig. 12.7. Electronic density of states of semiconductors with 3, 2, 1, and 0 degrees of freedom for electron propagation. Systems with 2, 1, and 0 degrees of freedom are referred to as quantum wells, quantum wires, and quantum boxes, respectively.

Finally, we consider the density of states in a zero-dimensional (0D) system, the quantum box. No free motion is possible in such a quantum box, since the electron is confined in all three spatial dimensions. Consequently, there is no k -space available which could be filled up with

electrons. Each quantum state of a 0D system can therefore be occupied by only two electrons. The density of states is therefore described by a δ -function.

$$\rho_{\text{DOS}}^{0\text{D}}(E) = 2 \delta(E - E_0) \quad (12.25)$$

For more than one quantum state, the density of states is given by

$$\rho_{\text{DOS}}^{0\text{D}}(E) = \sum_n 2 \delta(E - E_n) . \quad (12.26)$$

The densities of states for one quantized level for a 3D, 2D, 1D, and 0D electron system are schematically illustrated in **Fig. 12.7**.

12.3 Effective density of states in 3D, 2D, 1D, and 0D semiconductors

The *effective density of states* is introduced in order to simplify the calculation of the population of the conduction and valence band. The basic simplification made is that all band states are assumed to be located directly at the band edge. This situation is illustrated in **Fig. 12.8** for the conduction band. The 3D density of states has square-root dependence on energy. The effective density of states is δ -function-like and occurs at the bottom of the conduction band.

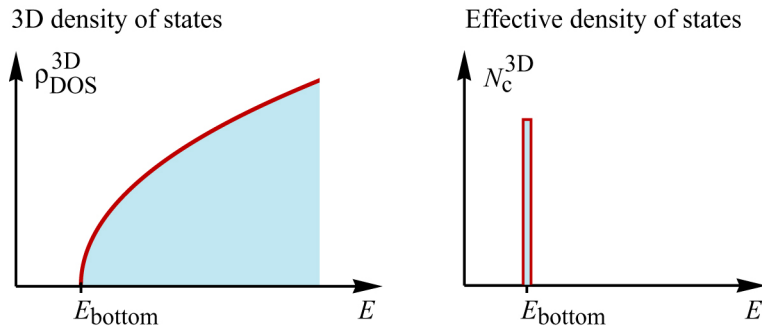


Fig. 12.8. Energy dependent density of states, $\rho_{\text{DOS}}^{3\text{D}}$, and *effective* density of states, $N_c^{3\text{D}}$, at the bottom of the conduction band.

An electronic state can be either occupied by an electron or unoccupied. Quantum mechanics allows us to attribute to the state a probability of occupation. The total electron concentration in a band is then obtained by integration over the product of state density and the probability that the state is occupied, that is

$$n = \int_{E_{\text{bottom}}}^{E_{\text{top}}} \rho_{\text{DOS}}(E) f(E) dE \quad (12.27)$$

where $f(E)$ is the (dimensionless) probability that a state of energy E is populated (see Sect. on *semiconductor statistics*). The limits of the integration are the bottom and the top energy of the band, since the electron concentration in the entire band is of interest.

As will be shown in a subsequent section, the probability of occupation, $f(E)$, is given by the Maxwell–Boltzmann distribution (see Sect. on *Maxwell–Boltzmann distribution*). The Maxwell–Boltzmann distribution, also frequently referred to as the Boltzmann distribution, is given by

$$f_{\text{B}}(E) = \exp\left(-\frac{E - E_{\text{F}}}{k T}\right) \quad (12.28)$$

where E_F is the Fermi energy (for a definition of the Fermi energy the reader is again referred to the next section). Using Eq. (12.27), the electron concentration can be determined by evaluating the integral.

The *effective* density of states at the bottom of the conduction band is now defined as the density of states which yields, with the Boltzmann distribution, the *same* electron concentration as the true density of states, that is

$$n = \int_{E_{\text{bottom}}}^{E_{\text{top}}} \rho_{\text{DOS}}(E) f_B(E) dE = N_c f_B(E = E_C) \quad (12.29)$$

where N_c is the effective density of states at the bottom of the conduction band and E_C is the energy of the bottom of this band. Strictly speaking, the effective density of states has no physical meaning but is simply a mathematical tool to facilitate calculations. For completeness, Eqs. (12.27) and (12.29) are now given explicitly using the Boltzmann distribution and the density of states of an isotropic three-dimensional semiconductor:

$$n = \int_{E_C}^{\infty} \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_C} e^{-(E - E_F)/kT} dE, \quad (12.30)$$

$$n = N_c e^{-(E_C - E_F)/kT}. \quad (12.31)$$

The upper limit of the integration can be taken to be infinity without loss of accuracy due to the strongly converging Boltzmann factor. Evaluation of the integral in Eq. (12.30) and comparison with Eq. (12.31) yields the effective density of states

$$N_c = \frac{1}{\sqrt{2}} \left(\frac{m^* kT}{\pi \hbar^2} \right)^{3/2} \quad (12.32)$$

Note that the effective density of states given by Eq. (12.32) applies to one minimum in the conduction band. If there are a number of M_c equivalent minima in the conduction band, the corresponding density of states must be multiplied by M_c . Furthermore, if the band structure is anisotropic, the effective mass m^* must be replaced by the density-of-states effective mass m_{DOS}^* . For a degenerate valence band with heavy and light holes, the effective density of states is the sum of both effective state densities, that is

$$N_v = \frac{1}{\sqrt{2}} \left(\frac{m_{\text{hh}}^* kT}{\pi \hbar^2} \right)^{3/2} + \frac{1}{\sqrt{2}} \left(\frac{m_{\text{lh}}^* kT}{\pi \hbar^2} \right)^{3/2}. \quad (12.33)$$

The effective density of states in a two-dimensional system (*i. e.* a system with two degrees of freedom) is obtained by the identical procedure as the three-dimensional effective density of states. The equations analogue to Eqs. (12.30) and (12.31) then read

$$n^{2D} = \int_{E_C}^{\infty} \frac{m^*}{\pi \hbar^2} e^{-(E - E_F)/kT} dE, \quad (12.34)$$

$$n^{2D} = N_c^{2D} e^{-(E_C - E_F)/kT} \quad (12.35)$$

where N_c^{2D} is the two-dimensional effective density of states. The carrier concentration n^{2D} represents the number of electrons per unit-area and is also referred to as the 2D density. Evaluation of the integral yields

$$N_c^{2D} = \frac{m^*}{\pi \hbar^2} kT \quad (12.36)$$

Finally, the effective density of states of a one-dimensional (1D) system is obtained in a similar way. The 1D density, *i. e.* the number of carriers per unit length is given by

$$n^{1D} = \int_{E_C}^{\infty} \frac{1}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_C)}} e^{-(E - E_F)/kT} dE, \quad (12.37)$$

$$n^{1D} = N_c^{1D} e^{-(E_C - E_F)/kT}. \quad (12.38)$$

The one-dimensional effective density of states is obtained as

$$N_c^{1D} = \sqrt{\frac{m^* kT}{2 \pi \hbar^2}} \quad (12.39)$$

The evaluation of a zero-dimensional density of states does not yield a simplification of the carrier-density calculation, since the zero-dimensional density of states is δ -function like. Table 12.1 summarizes the dispersion relation, the density of states, and the effective density of states of semiconductors with various degrees of freedom.

Degrees of freedom	Dispersion (kinetic energy)	Density of states	Effective density of states
3 (bulk)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2)$	$\rho_{\text{DOS}}^{3\text{D}} = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{E - E_C}$	$N_c^{3\text{D}} = \frac{1}{\sqrt{2}} \left(\frac{m^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}}$
2 (slab)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$	$\rho_{\text{DOS}}^{2\text{D}} = \frac{m^*}{\pi \hbar^2} \sigma(E - E_C)$	$N_c^{2\text{D}} = \frac{m^*}{\pi \hbar^2} kT$
1 (wire)	$E = \frac{\hbar^2}{2m^*}(k_x^2)$	$\rho_{\text{DOS}}^{1\text{D}} = \frac{m^*}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_C)}}$	$N_c^{1\text{D}} = \sqrt{\frac{m^* kT}{2\pi \hbar^2}}$
0 (box)	–	$\rho_{\text{DOS}}^{0\text{D}} = 2\delta(E - E_C)$	$N_c^{0\text{D}} = 2$

Table 12.1 Density of states for semiconductor with 3, 2, 1, and 0 degrees of freedom for propagation of electrons. The dispersion relations are assumed to be parabolic. The formulas can be applied to anisotropic semiconductors if the effective mass m^* is replaced by the density-of-states effective mass m_{DOS}^* . If the semiconductor has a number of M_c equivalent minima, the corresponding density of states must be multiplied by M_c . The bottom of the band is denoted as E_C and $\sigma(E)$ is the step-function.