Final Exam - Solution, Spring Semester 2007

ECSE-6961, Light-emitting diodes and solid-state lighting, Prof. Schubert

Note: In this solution, some of the intermediate steps are left out for brevity.

1. Consider a white-light source consisting of a blue LED (λ = 460 nm) and a yellow phosphor (λ = 560 nm). Assume that for every blue photon emitted by the white-light emitter, one yellow photon is being emitted, i.e. 50% of the blue photons are converted to yellow photons.
   (a) Identify a fundamental energy loss mechanism in this device.
   (b) What is the maximum attainable power efficiency ($\eta_{\text{power}} = P_{\text{optical}} / P_{\text{electrical}}$) of the device?
   (c) What is the maximum attainable luminous efficiency of the device?
   (d) Assume that the blue LED and the yellow phosphor have the following quantum efficiencies: $\eta_{\text{internal}} = \eta_{\text{external}} = 1.0$. Assume further that the device is injected with 1 mA. How many yellow photons are being emitted per second?

(a) Wavelength-conversion loss (also called Stokes-shift loss)

(b) Maximum attainable power efficiency, $\eta_{\text{power}} = \dfrac{\dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2}}{2\,\dfrac{hc}{\lambda_1}} = \dfrac{1}{2}\left(\dfrac{\lambda_1 + \lambda_2}{\lambda_2}\right) = 0.91 = 91\%$

where λ_1 = 460 nm and λ_2 = 560 nm.

(c) Maximum attainable luminous efficiency, η_{lum} =

$$683 \ \frac{\text{lm}}{\text{W}} \left(\frac{P(\lambda_1)\, V(\lambda_1) + P(\lambda_2)\, V(\lambda_2)}{I\ V} \right)$$

We assume that energy loss occurs only in the phosphor due to the Stokes-shift and no energy is lost inside the semiconductor when converting the input electrical power ($I\,V$) into blue photons ($h\,c / \lambda_1$). Therefore,

$$\eta_{\text{lum}} = 683 \frac{\text{lm}}{\text{W}} \left(\frac{\frac{1}{2}\frac{hc}{\lambda_1} V(\lambda_1) + \frac{1}{2}\frac{hc}{\lambda_2} V(\lambda_2)}{\frac{hc}{\lambda_1}} \right) = 683 \frac{\text{lm}}{\text{W}} \frac{1}{2} \left(V(\lambda_1) + \frac{\lambda_1}{\lambda_2} V(\lambda_2) \right)$$

$$= 683 \frac{\text{lm}}{\text{W}} \frac{1}{2} \left(0.06 + \frac{460}{560} \times 1 \right) = 301 \text{ lm/W}$$

(d) Number of yellow photons emitted per second $= \dfrac{1}{2} \times \dfrac{I}{e} = \dfrac{1}{2} \times \dfrac{1 \times 10^{-3}\,\text{A}}{1.6 \times 10^{-19}\,\text{C}} =$

$= 3.125 \times 10^{15}\ \text{s}^{-1}$

2. Consider a white-light source consisting of a UV LED ($\lambda = 400$ nm) and a blue, a green, and a red phosphor emitting at $\lambda = 450$ nm, 550 nm, and 650 nm, respectively. Assume that for every 3 UV photons emitted by the LED, one blue, one green, and one red photon is being emitted.
   (a) What is the maximum attainable power efficiency of the device?
   (b) What is the maximum attainable luminous efficiency of the device?
   (c) Compare the following approaches:
      (i) White LED made from blue LED and yellow phosphor.
      (ii) White LED made from UV LED and a blue, green, and red phosphor combination.
      Which of the two has the higher power efficiency? Explain the reason in words.
   (d) Which of the two has the higher luminous efficiency? Explain the reason in words.

(a) Maximum attainable power efficiency, $\eta_{\text{power}} = \dfrac{\dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2} + \dfrac{hc}{\lambda_3}}{3\,\dfrac{hc}{\lambda}} = \dfrac{1}{3}\lambda\left(\dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2} + \dfrac{1}{\lambda_3}\right)$

$$= \frac{400}{3}\left(\frac{1}{450} + \frac{1}{550} + \frac{1}{650}\right) = 74.4\%$$

where $\lambda = 400$ nm, $\lambda_1 = 450$ nm, $\lambda_2 = 550$ nm and $\lambda_3 = 650$ nm.

(b) Maximum attainable luminous efficiency, $\eta_{\text{lum}} =$

$$= 683\,\frac{\text{lm}}{\text{W}}\left(\frac{P(\lambda_1)\,V(\lambda_1) + P(\lambda_2)\,V(\lambda_2) + P(\lambda_3)\,V(\lambda_3)}{I\,V}\right)$$

We assume that energy loss occurs only in the phosphor due to the Stokes-shift and no energy is lost inside the semiconductor when converting the input electrical power (IV) into UV photons (hc/λ). Therefore,

$$\eta_{\text{lum}} = 683\,\frac{\text{lm}}{\text{W}}\left(\frac{\frac{1}{3}\frac{hc}{\lambda_1}V(\lambda_1) + \frac{1}{3}\frac{hc}{\lambda_2}V(\lambda_2) + \frac{1}{3}\frac{hc}{\lambda_3}V(\lambda_3)}{\frac{hc}{\lambda}}\right) =$$

$$= 683\,\frac{\text{lm}}{\text{W}}\,\frac{\lambda}{3}\left(\frac{V(\lambda_1)}{\lambda_1} + \frac{V(\lambda_2)}{\lambda_2} + \frac{V(\lambda_3)}{\lambda_3}\right) = 683\,\frac{\text{lm}}{\text{W}} \times \frac{400}{3}\left(\frac{0.05}{450} + \frac{1}{550} + \frac{0.09}{650}\right) =$$

$= 188.3$ lm/W

(c) White LED made from blue LED and yellow phosphor has higher power efficiency, because wavelength conversion loss from UV to red is significantly higher thereby reducing the power efficiency.

(d) White LED made from blue LED and yellow phosphor has higher luminous efficiency, because human eye is significantly less sensitive in the red light region.

3. Consider a 10 km long multimode step-index fiber with a core refractive index of $n = 1.50$ and a cladding refractive index of $n = 1.45$. Assume that the fiber input comes from an LED emitting at 850 nm.
   (a) Calculate the numerical aperture of the fiber.
   (b) What is the solid angle of a light source that can be coupled into the fiber?
   (c) Assume that the LED emits 1 mW of optical power isotropically into a hemisphere (solid angle 2π). What is the optical power that can be coupled into the fiber?
   (d) What optical power exits the fiber?

(a) Numerical Aperture $NA = n_1 \sin \theta_c$

$$\theta_c = \arccos\left(\frac{n_{\text{clad}}}{n_{\text{core}}}\right) = \arccos\left(\frac{1.45}{1.5}\right) = 14.8°$$

$NA = 1.5 \sin 14.8° = 0.38$

(b) Solid Angle $= 2\pi\,[1 - \cos(\arcsin(\text{NA}))\,] = 0.47$ steradian

(c) Power coupled $= 1\,\text{mW} \times \dfrac{0.47}{2\pi} = 74.8\ \mu\text{W}$

(d) The attenuation loss is about 2 db/km for a silica fiber at 850 nm. Therefore total loss for 10 km will be $2\ \text{db}/\text{km} \times 10\ \text{km} = 20\,\text{db}$

Loss $= 10 \log\,(P_{\text{out}}/P_{\text{in}})$ or $-20 = 10 \log \dfrac{P_{\text{out}}}{74.8\ \mu\text{W}}$

Solving this equation yields $P_{\text{out}} = 0.748\ \mu\text{W}$

4. This question concerns the design a Distributed Bragg Reflector optimized for a wavelength of 300 nm.
   (a) Select two materials from the table above which would form the DBR with highest reflectivity and least number of periods for 300 nm.
   (b) Calculate the layer thickness of each material.
   (c) Which material layer should form the top layer if the ambient is air?

(a) To maximize the refractive-index contrast and also have optical transparency at 300 nm, we select CaF_2 and Si_3N_4 as the two DBR materials.

(b) CaF_2 layer thickness $= \dfrac{\lambda}{4 \times \bar{n}_{1,h}} = \dfrac{300}{4 \times 1.43} = 52.4$ nm

Si_3N_4 layer thickness $= \dfrac{\lambda}{4 \times \bar{n}_{1,h}} = \dfrac{300}{4 \times 2} = 37.5$ nm

(c) To maximize reflection, material with higher refractive-index contrast with air should be used for the top layer. Therefore, Si_3N_4 should form the top layer.

5. This question concerns colorimetry.
   (a) Deduce the temperature of a black body whose Planckian spectrum has a maximum intensity at 443 nm.
   (b) Schematically draw a chromaticity diagram and mark the location of the above Planckian black-body radiator. (Hint: Appendix 18.1 on page 312)
   (c) Can we obtain a light source with the same location on chromaticity diagram as the above Planckian black-body radiator by mixing the blue and yellow sources marked on the chromaticity diagram below?
   (d) Can we obtain a light source with the same location on chromaticity diagram as the above Planckian black-body radiator by mixing the Blue, Green and Red sources marked on the chromaticity diagram below?
   (e) Which of the following properties will be the same for the three light sources obtained in (b), (c) and (d)? (i) Spectrum (ii) CRI (iii) *x*, *y* color coordinates (iv) *u*, *v* color coordinates (v) luminous efficacy.

(a) $T = \dfrac{2880\ \mu\text{m K}}{\lambda} = \dfrac{2880\ \mu\text{m K}}{443 \times 10^{-3}\ \mu\text{m}} = 6500\ \text{K}$

(b) $x = 0.315$, $y = 0.327$

(c) Yes, because the chromaticity point of the planckian radiator is located on the straight line that can be drawn between the chromaticity locations of the blue and yellow source.

(d) Yes, because the chromaticity point of the planckian radiator is within the color gamut opened up by the three sources.

(e) Only the x, y and u, v coordinates will be the same for the light sources.