Midterm Exam – Solutions, Spring 2007

ECSE-6961, Light-Emitting Diodes and Solid-State Lighting

1. Consider a light-emitting diode, through which a constant electrical current flows for a time of 10 s so that the total electrical charge flowing through the diode is 1C. A photon counter counts 10^{17} photons being emitted from the LED per second. The LED emits at a wavelength of 650 nm and has a forward voltage of 1.91 V.
   (a) What is the external quantum efficiency of the device?
   (b) What is the injection current of the device?
   (c) What is the electrical input power of the device?
   (d) What is the optical output power of the device?
   (e) What is the power efficiency of the device?
   (f) What is the series resistance of the device?
   (g) What is the total power converted to heat in the device?
   (h) What is the internal quantum efficiency of the device assuming that the light-extraction efficiency is 100%?
   (i) What is the energy consumed, after the 1 C charge has flown through the device?

a) External quantum efficiency, $\eta_{\text{ext}} = \dfrac{\text{Number of photons emitted per second}}{\text{Number of electrons injected per second}} =$

$$\frac{\text{\# of photons emitted per second}}{\left(\dfrac{\text{total charge}}{\text{charge of electron}}\right)\times\dfrac{1}{\text{total time}}} = \frac{10^{17}\,\text{s}^{-1}}{\left(\dfrac{1\,\text{C}}{1.602\times10^{-19}\,\text{C}}\right)\times\dfrac{1}{10\,\text{s}}} = 0.1602 = 16.0\%$$

b) Injection current, I_{inj} = Injected charge per unit time = 1 C per 10 seconds = 100 mA

c) Electrical input power, $P_{\text{in}} = I_{\text{inj}} \times V_{\text{F}}$ = 100 mA × 1.91 V = 191 mW

d) Optical output power, P_{out} = # of photons emitted per sec. × hc/λ = $10^{17}\ \text{s}^{-1}$ ×

$$\left(\frac{6.63\times10^{-34}\ \text{Js}\times3\times10^{8}\ \text{m/s}}{650\times10^{-9}\ \text{m}}\right) = 30.6\ \text{mW}$$

e) Power efficiency, $\eta_{\text{power}} = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{30.6\,\text{mW}}{191\,\text{mW}} = 16.0\%$

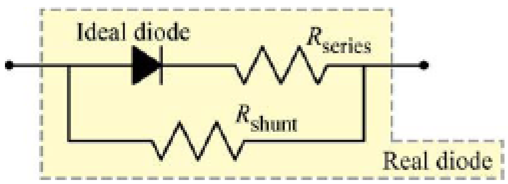
f) Series resistance, $R_{\text{S}} = \dfrac{V_{\text{F}} - V_{\text{th}}}{I_{\text{inj}}} = \dfrac{V_{\text{F}} - E_{\text{g}}/e}{I_{\text{inj}}} = \dfrac{V_{\text{F}} - \left(\dfrac{hc}{\lambda\times e}\right)}{I_{\text{inj}}} = \dfrac{1.91\,\text{V} - 1.91\,\text{V}}{100\times10^{-3}\ \text{A}} = 0\ \Omega$

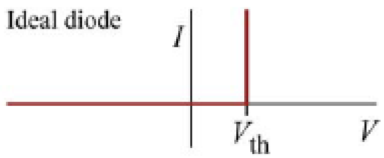
g) Power converted to heat, $P_{\text{heat}} = P_{\text{in}} - P_{\text{out}}$ = 191 mW – 30.6 mW = 160.4 mW

h) Internal quantum efficiency, $\eta_{\text{int}} = \dfrac{\eta_{\text{ext}}}{\eta_{\text{extraction}}}$ = 16.0 %

i) Energy consumed, E = charge × electric potential = 1 C × 1.91 V = 1.91 J
   Or, alternatively, $E = P\,t = I\,V\,t$ = 100 mA × 1.91 V × 10 s = 1.91 J

2. A 460 nm blue LED is tested in a manufacturing environment by measuring the V_{f1} = 4.5 V and V_{f2} = 2.0 V, measured at 100 mA and 1 μA, respectively. Assume that the device can be modeled by the equivalent circuit shown below. Also assume that the "ideal diode" shown in the circuit has negligibly small reverse saturation current.
(a) What is V_{th} of the "ideal diode"?
(b) What is the series resistance of the device?
(c) What is the shunt resistance of the device?
(d) What would be the value of V_{f1} if the series resistance would be zero?
(e) What would be the value of V_{f2} if the shunt resistance would be infinitely high?
(f) Consider the choice between 4 diodes with relative V_{f1} and V_{f2} values of: (V_{f1}, V_{f2}) = (high, high)$_{\text{for Diode 1}}$, (high, low)$_{\text{for Diode 2}}$, (low, high)$_{\text{for Diode 3}}$, and (low, low)$_{\text{for Diode 4}}$. Which of the diodes is the best and why?





a) V_{th} for the "ideal diode":

$$V_{th} = \frac{E_g}{e} = \frac{h\,c}{\lambda\,e} = \frac{6.63\times10^{-34}\ \text{Js}\times3\times10^{8}\ \text{m/s}}{650\times10^{-9}\ \text{m}\times1.602\times10^{-19}\ \text{C}} = 2.7\ \text{V}$$

b) Series resistance of the device, $R_S = \dfrac{V_{f1}-V_{th}}{I_{f1}} = \dfrac{4.5\ \text{V}-2.7\ \text{V}}{100\times10^{-3}\ \text{A}} = 18\ \Omega$

c) Shunt resistance of the device, $R_{Shunt} = \dfrac{V_{f2}}{I_{f2}} = \dfrac{2.0\ \text{V}}{1\times10^{-6}\ \text{A}} = 2\ \text{M}\Omega$

d) $V_{f1,Rs=0} = V_{th} = 2.7$ V

e) $V_{f2,Rshunt=\infty} = V_{th} = 2.7$ V

f) Diode 3 is the best, because it has the lowest series resistance (lowest V_{f1}) and the largest shunt resistance (highest V_{f2}).

3. The temperature dependence of the optical output power of an LED can be described in terms of characteristic temperature T_1. A red-emitting diode has $T_1 = 200$ K and it emits an optical power of 100 mW at 300 K.
   (a) Give the formula describing the temperature dependence of the output power of the LED.
   (b) What is the physical meaning of T_1?
   (c) What is the optical output power at 400 K and 500 K?
   (d) What is the main physical cause of the temperature dependence of the output power?
   (e) How could the inherent temperature dependence of the device be decreased?
   (f) Assume that the temperature dependence of the device could be decreased. Would T_1 then increase or decrease?
   (g) Show a circuit that can compensate for the temperature dependence of the LED optical emission power, so that the LED emits a power that is independent of temperature.
   (h) Explain the circuit.

a) Temperature dependence of output power of an LED is given as,

   $I = I_{300K} \exp[-(T - 300\text{ K}) / T_1]$

b) Physically, T_1 is the temperature at which the intensity of the output optical power decreases by a factor of e = 2.718.

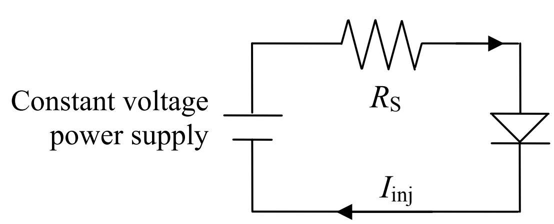
c) $I_{300K} = 100$ mW
   $I_{400K} = I_{300K} \exp -((400\text{ K} - 300\text{ K}) / 200\text{ K}) = 60.65$ mW
   $I_{500K} = I_{300K} \exp -((500\text{ K} - 300\text{ K}) / 200\text{ K}) = 36.79$ mW

d) The main physical cause of temperature dependence of the output power is non-radiative recombination via deep levels.

e) The inherent temperature dependence of the device can be decreased by improving the material quality, proper design of the device geometry and by optimized band-gap engineering.

f) If the temperature dependence of the device could be decreased, the T_1 would increase.

g) Below is the circuit that can compensate for the temperature dependence of the LED optical emission power.



h) As the temperature increases, the threshold voltage, V_{th}, of the diode decreases, increasing the voltage across the series resistance, R_S, which results in increased current, I_{inj}, through the diode. With proper selection of R_S, the increase in current with temperature can offset the decrease in the output optical power, thereby maintaining a constant power output.