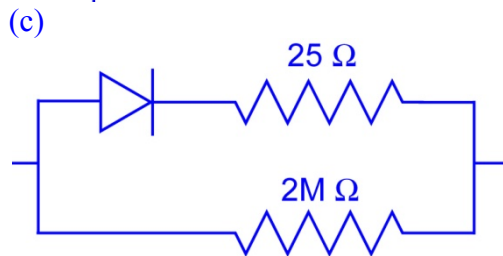
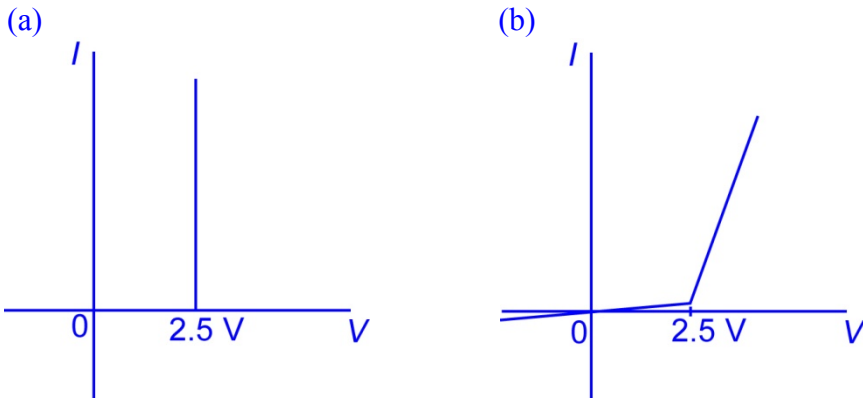


Final Exam - Solution, Spring 2009
 ECSE-6962, Light-Emitting Diodes and Solid-State Lighting

Note: Put your name on paper. Show your work. Underline results. Use appropriate approximations. Show units. All materials are allowed (books, manuscripts, excerpts, calculators, etc.). Use ball-point pen, no pencil.

1. Consider a circuit consisting of an ideal diode ($V_{th} = 2.5 \text{ V}$, abrupt turn on, no series resistance, capable of carrying an infinite current for $V > V_{th}$), a series resistance, and a parallel resistance (bypassing the diode and the series resistance). Assume that the series resistance has a value of 25Ω and the parallel resistance has a value of $2 \text{ M}\Omega$.
 - (a) Accurately draw the I-V characteristic of the ideal diode.
 - (b) Sketch the I-V characteristic of the diode including the series and parallel resistance.
 - (c) What is the value of V_{f1} , measured at a total diode current of 100 mA ?
 - (d) What is the value of V_{f2} , measured at a total diode current of $100 \mu\text{A}$?
 - (e) What is the value of V_{f3} , measured at a total diode current of $1 \mu\text{A}$?
 - (f) Assume that one could choose between one diode that has $V_{f3} = 1.5 \text{ V}$ and another one with $V_{f3} = 2.0 \text{ V}$ (all other factors are the same). Which diode would you choose, and why?

Solutions:



The total diode current can be expressed as:

When $V_f < 2.5 \text{ V}$ (or $I < \frac{2.5 \text{ V}}{2 \times 10^6 \Omega} = 1.25 \mu\text{A}$):

$$I = \frac{V_f}{2 \times 10^6 \Omega}$$

When $V_f > 2.5 \text{ V}$ (or $I > \frac{2.5 \text{ V}}{2 \times 10^6 \Omega} = 1.25 \mu\text{A}$):

$$I = \frac{V_f - 2.5 \text{ V}}{25 \Omega} + \frac{V_f}{2 \times 10^6 \Omega} = V_f \left(\frac{1}{25 \Omega} + \frac{1}{2 \times 10^6 \Omega} \right) - 0.1 \text{ A}$$

$I_1 = 100 \text{ mA} > 1.25 \mu\text{A}$, so

$$V_{f1} = (I + 0.1 \text{ A}) \times 25 \Omega = (0.1 \text{ A} + 0.1 \text{ A}) \times 25 \Omega = 5.0 \text{ V}$$

(d) $I_2 = 100 \mu\text{A} > 1.25 \mu\text{A}$, so

$$V_{f2} = (I + 0.1 \text{ A}) \times 25 \Omega = (0.0001 \text{ A} + 0.1 \text{ A}) \times 25 \Omega = 2.5 \text{ V}$$

(e) $I_3 = 1 \mu\text{A} < 1.25 \mu\text{A}$, so

$$V_{f3} = I \times 2 \times 10^6 \Omega = 1 \mu\text{A} \times 2 \times 10^6 \Omega = 2 \text{ V}$$

- (f) $V_{f3} = 2.0 \text{ V}$ is preferred, because a larger V_{f3} means a larger parallel resistance and therefore a smaller leakage current.

2. This question is about the design of a white light source that has a white point located at the “equal-energy-point” ($x = y = 0.333$). Given are two blue LEDs one emitting at 460 nm and the other one emitting at 492.5 nm.
- Using the chromaticity diagram, what are the complementary wavelengths of an additional two emitters (two LEDs) that allow one to get white light? (Assume that the complementary emitter is also located on the perimeter of the chromaticity diagram)
 - Assume that the intensity needed from each of the emitter is inversely proportional to the distance of the emitter’s chromaticity point from the desired white point. Given this, what is the intensity ratio for the two white sources?
 - Assuming that the total optical power of each of the white light sources is 1 W, calculate the individual powers of the 4 LEDs.
 - Using the eye sensitivity curve $V(\lambda)$, what are the luminous efficacies of radiation for the 4 LEDs.
 - Calculate the luminous efficacy of radiation for the 2 white sources.
 - One of the two white-light sources probably has a higher efficacy of radiation than the other one. Give an intuitive explanation as to why one of the light sources has a higher efficacy.

Solutions:

- The complementary wavelength for 460 nm is 572 nm, and the complementary wavelength for 492.5 nm is 630 nm (no need to be very accurate, for example, 620-640 nm is fine).
- Intensity ratio: 0.73 and 1.27.
- Individual powers of the 4 LEDs:

460 nm	572 nm	492.5 nm	630 nm
0.42 W	0.58 W	0.56 W	0.44 W
- Luminous efficacy of radiation for the 4 LEDs:

460 nm	572 nm	492.5 nm	630 nm
40 lm/W	600 lm/W	150 lm/W	160 lm/W (no need to be very accurate)
- Luminous efficacy of radiation for
White source 1:
$$0.42 \times 40 \frac{\text{lm}}{\text{W}} + 0.58 \times 600 \frac{\text{lm}}{\text{W}} = 364 \text{ lm/W}$$
White source 2:
$$0.56 \times 150 \frac{\text{lm}}{\text{W}} + 0.44 \times 160 \frac{\text{lm}}{\text{W}} = 154 \text{ lm/W}$$
- White source 1 has a higher efficacy because the 572 nm LED is very close to the highest human-eye sensitivity point (555 nm).

3. In an LED display, three different light-emitters are used, emitting monochromatically in the red (620 nm), green (520 nm), and blue (450 nm). Each of them consists of a UV LED (400 nm) and a phosphor. Assume that the external quantum efficiency for each of the three light-emitters (UV LED + phosphor) is 20%.
- Under a forward current of 500 mA (LEDs are in a series circuit), what is the number of photons emitted per second from each of the three light-emitters (UV LED + phosphor)?
 - What is the optical power emitted from each of the three light-emitters?
 - Given the above discussion, what could be the energy loss mechanisms in these light-emitters?
 - Is it possible to reach the “equal-energy-point” in the chromaticity diagram with the three light-emitters?
 - What is the entire range of colors that can be created from combinations of the three light-emitters (draw the region in the chromaticity diagram)?
 - Light-emitters usually have a spectral broadening. Consider the locations of the three light-emitters in the chromaticity diagram, which chromaticity location (red, green, or blue?) will move more towards the center of the diagram upon spectral broadening, and why?
 - Which chromaticity location (red, green, or blue?) will be affected least upon spectral broadening, and why?
 - What is the effect of spectral broadening of the light-emitters to the range of colors (area of color gamut) that can be created from combinations of the three light-emitters?
 - Give an example as to what kind of color is lost (no longer displayable) because of the spectral broadening of the three light-emitters?

Solutions:

- (a) Number of photons emitted per second from each of the three light-emitters:

$$\frac{I}{e} \times \eta_{\text{ext}} = \frac{0.5 \text{ C/s}}{1.60 \times 10^{-19} \text{ C}} \times 20\% = 6.25 \times 10^{17} \text{ s}^{-1}$$

- (b) Optical power emitted from each of the three light-emitters:

$$\text{Blue LED: } 6.25 \times 10^{17} \text{ s}^{-1} \times \frac{6.63 \times 10^{-34} \text{ Js} \times 3.0 \times 10^8 \text{ ms}^{-1}}{450 \text{ nm}} = 0.28 \text{ W}$$

$$\text{Green LED: } 6.25 \times 10^{17} \text{ s}^{-1} \times \frac{6.63 \times 10^{-34} \text{ Js} \times 3.0 \times 10^8 \text{ ms}^{-1}}{520 \text{ nm}} = 0.24 \text{ W}$$

$$\text{Red LED: } 6.25 \times 10^{17} \text{ s}^{-1} \times \frac{6.63 \times 10^{-34} \text{ Js} \times 3.0 \times 10^8 \text{ ms}^{-1}}{620 \text{ nm}} = 0.20 \text{ W}$$

- (c) Possible energy loss mechanisms: Internal quantum efficiency of the UV LED, conversion efficiency of phosphor, wavelength conversion loss (Stokes-shift loss), and total internal reflection.
- (d) Yes.
- (e) The area within the triangle in Fig. 1.
- (f) Green is affected most, because the perimeter of the chromaticity diagram is strongly curved in the green region.
- (g) Red is affected least, because the perimeter is approximately a straight line in the red spectral region.
- (h) Due to the spectral broadening, the locations of the three LEDs in the chromaticity diagram move away from the perimeter and towards the center of the chromaticity diagram, therefore the number of colors which can be produced are reduced.
- (i) For example, saturated green (i.e., green that is very close to the upper perimeter of the chromaticity diagram). Other answers, like pure colors, pure green, pure blue and pure red etc., are also correct (“pure” here means “saturated” or “monochromatic”).

Figure 1: CIE 1931 x, y chromaticity diagram

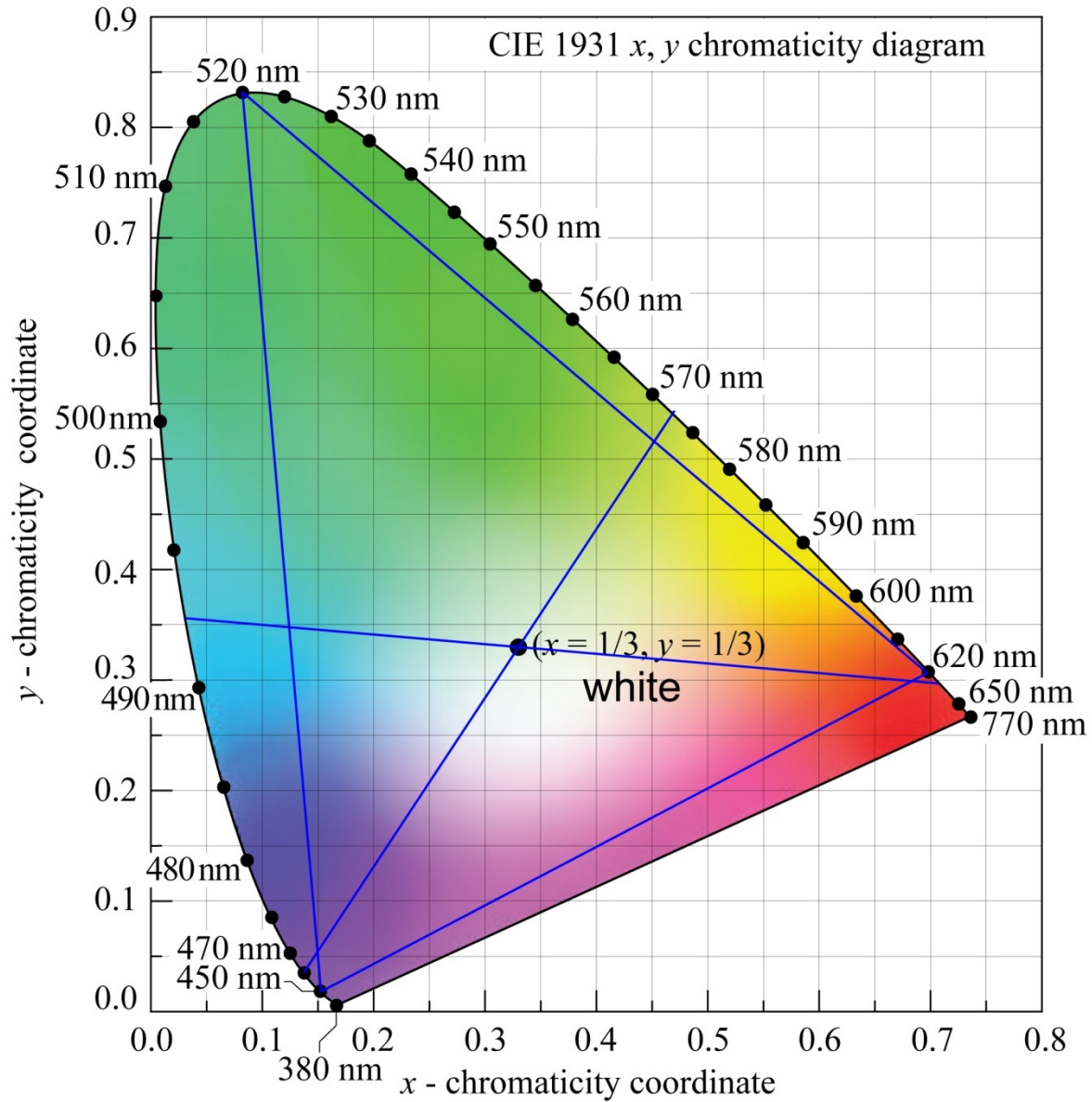


Figure 2: CIE 1978 Photopic vision

