

# Quantum Mechanics Applied to Semiconductor Devices

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## Introduction

Quantum mechanics plays an essential role in modern semiconductor heterostructure devices. The spatial dimensions of such devices are frequently on the scale of just Angstroms. In the domain of microscopic structures with dimensions comparable to the electron de Broglie wavelength, size quantization occurs. Classical and semi-classical physics no longer gives a correct description of many physical processes. The inclusion of quantum mechanical principles becomes mandatory and provides a most useful description of many physical processes in electronic and photonic heterostructure devices.

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