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(1925)

18

Tunneling structures

18.1 Tunneling in ohmic contact structures

Attempt rate of carriers in a semiconductor
Calculation of the success rate using the WKB approximation
Calculation of the current-vs.-voltage characteristic
Contact resistance

18.2 Tunneling in metal-oxide-semiconductor structures

Same principle as above but
At thin oxide thicknesses such as 10 Å, the tunneling current is 10 A / cm^2 – check with Ellis at Lucent or Don Monroe at Lucent

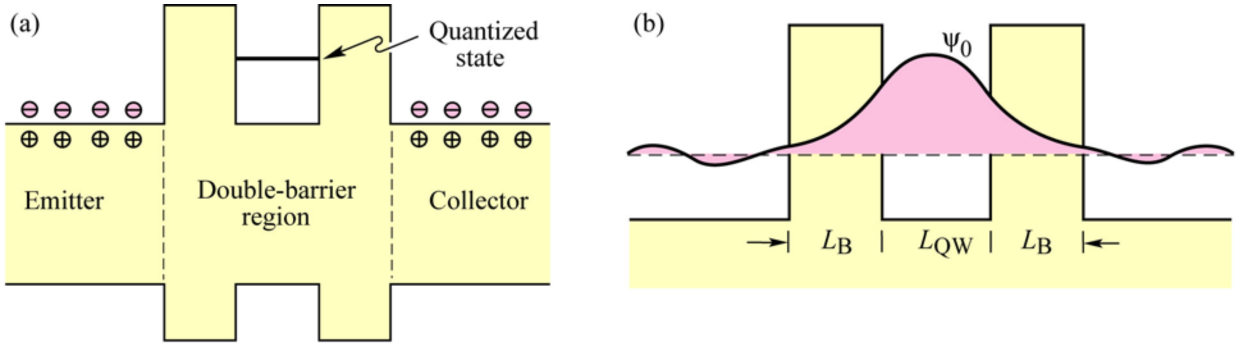

Fig. 18.1. (a) Band diagram of an n-type resonant-tuneling structure and (b) ground-state wave function in the well.

18.3 Resonant-tunneling structures

Resonant tunneling structures (RTS) consist of two tunneling barriers and a quantum well layer between the barriers. Current flows from the one electrode, denoted as *emitter*, through the double barrier structure to the receiving electrode, denoted as *collector*, as shown in ***Fig***. 18.1. Emitter and collector are doped to provide charge carriers for transport. RT structures are usually n-type due to the larger quantum energies attainable with the lighter electrons as compared to the heavier holes.

The quantum well layer is sufficiently thin so that just one or two quantized states occur in the well. The tunneling barriers are sufficiently this so that the tunneling probability through one of the barriers is sufficiently large to carry a current density of reasonable magnitude, *e. g.* in the A / cm^2 to kA / cm^2 range. As a bias is applied to the RTS, the energy of quantum state in the center well will change, and, at some bias, the quantum state will be in resonance with electrons injected from the emitter.

Let us first assume that the RTS is biased in such a way that the emitter is in resonance with the quantum state in the quantum well. In this case, carriers need to tunnel only through the first barrier to reach an allowed state. The tunneling probability can be conveniently calculated by the WKB approximation. Recall that the tunneling probability is given by

$$T \quad = \quad \mathrm{e}^{-\int_{x=0}^{L_B} 2\hbar^{-1}\sqrt{2m^*\left[U(x)-E\right]}\,\mathrm{d}x} \tag{18.1}$$

where L_B is the thickness of the tunnel barrier. *All* carriers that tunnel through the first barrier and reach the well, will eventually escape from the well and tunnel through the second barrier to the lower-energy states of the collector.

Let us next consider the case that the RTS is biased in such a way that the emitter is *not* in resonance with the quantum state in the quantum well. In this case, carriers need to tunnel through the first barrier, the well, and the second barrier to reach an allowed state in the collector. Again, the tunneling probability can be conveniently calculated by the WKB approximation. However, the carriers need to tunnel much farther.

$$T \quad = \quad e^{-\int_{x=0}^{L_B+L_W+L_B} 2\hbar^{-1}\sqrt{2m^*\left[U(x)-E\right]}\,\mathrm{d}x} \tag{18.2}$$

Comparison of the tunneling probabilities for the on-resonance case (Eq. 18.50) with the off-resonance case (Eq. 18.2) yields that the tunneling probability is different by many orders of magnitude. There is a distinct peak in the current-voltage characteristic is expected when the resonance condition is satisfied.

The current-voltage (I-V) curve of a resonant tunneling structure is shown in ***Fig***. 18.2 for different bias conditions. The I-V curve exhibits a clear peak and decreases again at higher bias voltages. If an RTS has several levels in the well, several peaks in the I-V characteristic might be observed. However, if the well has several levels the energy spacing between them is reduced and any broadening mechanism will obscure the manifestation of distinct peaks.

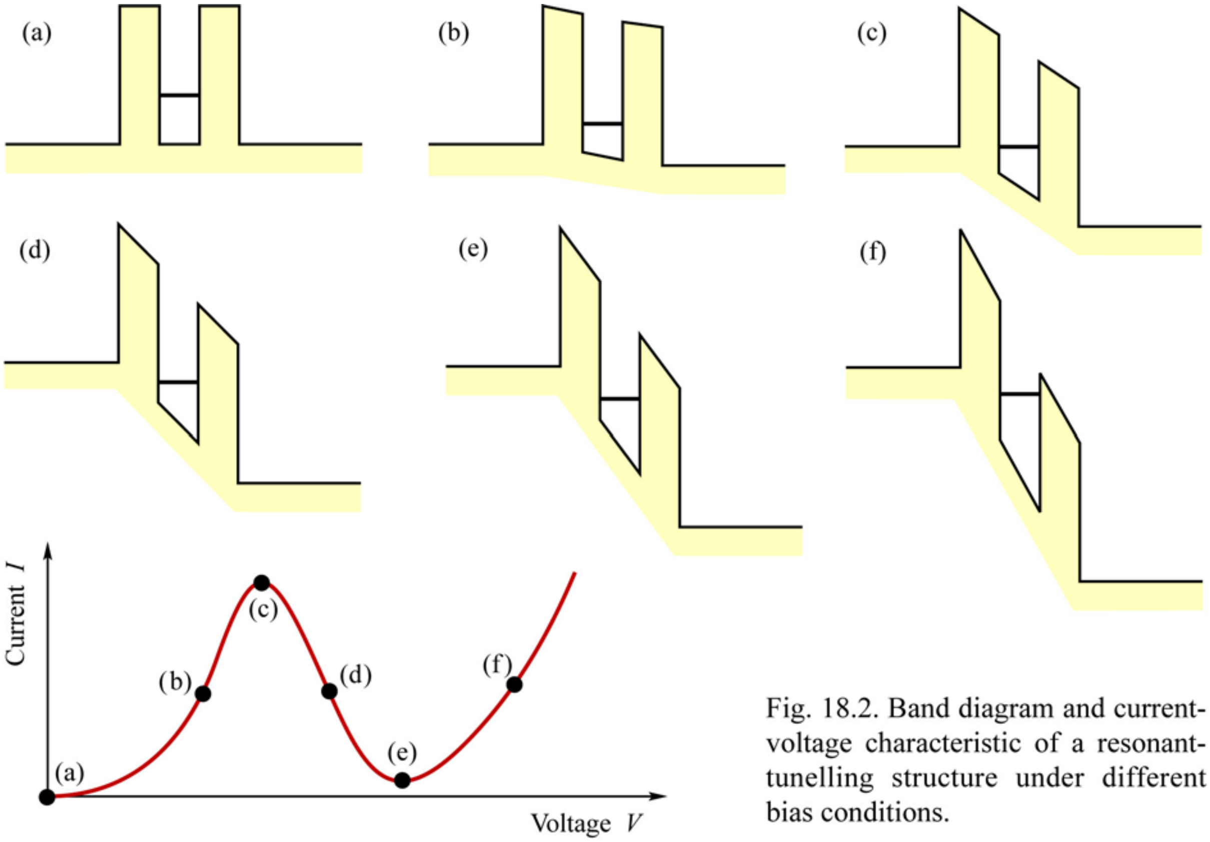

Fig. 18.2. Band diagram and current-voltage characteristic of a resonant-tunelling structure under different bias conditions.

Potential drops in a resonant tunneling structure

There are different contributions to the potential drop in a resonant tunneling structure. Assume that the RTS is under bias and a potential drop occurs at the resistive region, *i. e*. the double barrier region. The different potential drops are shown in ***Fig***. 18.2.

As the bias is applied to the structure an accumulation layer forms at the emitter of the RTS. Band bending is induced by free carriers accumulating in front of the first barrier. At the collector side, the band bending is induced by a depletion region, *i. e*. by fixed donor charges. In the center well region, a small stored charge will be stored if a current flows across the double barrier structure. At all boundaries, the electrostatic boundary condition must be fulfilled, namely that the normal component of the electric displacement $D = \varepsilon$ E be continuous.

The total potential drop is a sum of the potential drops in the accumulation layer, the barrier and well region, and the depletion region. We assume that the electric field in the first barrier is given by E. We next calculate the different potential drops in the RTS.

The electric field at the emitter-barrier layer interface in the emitter layer is given by the boundary condition $\varepsilon_{\mathrm{Emit}} \mathcal{E}_{\mathrm{Emit}} = \varepsilon_{\mathrm{B}} \mathcal{E}$ due to the boundary condition. For simplicity, we assume that the semiconductors forming the RTS have the same dielectric constant so that $\varepsilon_{\mathrm{Emit}} = \varepsilon_{\mathrm{B}}$ and thus $\mathcal{E}_{\mathrm{Emit}} = \mathcal{E}_{\mathrm{B}}$, *i. e*. the electric field is continuous at the boundaries. The potential in the accumulation layer of the emitter is shown in ***Fig***. 18.3. The band bending in the accumulation layer is caused by the electrons in the accumulation layer. Thus the total charge in the accumulation ayer is given by Gauss' law

$$\mathcal{E} \;=\; \frac{e\,n_{\text{acc}}^{\text{2D}}}{\varepsilon} \qquad (18.5\text{XX})$$

where $n_{\text{acc}}{}^{\text{2D}}$ is the density of electrons per unit area in the accumulation layer.

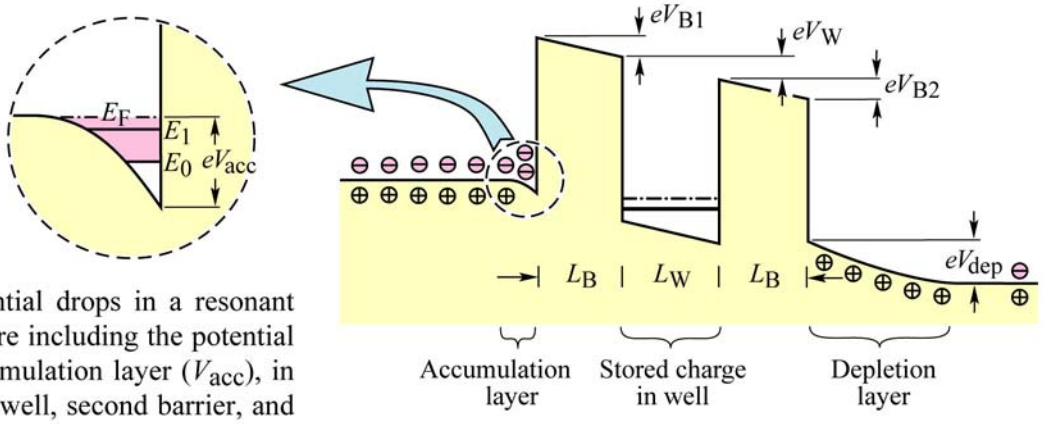

Fig. 18.3 Potential drops in a resonant tunelling structure including the potential drop in the accumulation layer (V_{acc}), in the first barrier, well, second barrier, and the depletion region (V_{dep}).

Due to the narrow size of the triangular accumulation layer potential, size quantization occurs. A suitable wave function for electrons in the accumulation layer is the Fang-Howard wave function (which was discussed earlier). The ground-state energy level is given by

$$E_0 \;=\; \frac{3}{2}\left(\frac{3}{2}\,\frac{e\,\mathcal{E}\,\hbar}{\sqrt{m^*}}\right)^{2/3} \qquad (18.5\text{XX})$$

Assuming that the quantum well has only one quantized state, the Fermi level can be inferred from the electron concentration according to

$$n_{\text{acc}}^{\text{2D}} \;=\; \int_{E_0}^{\infty} \rho^{\text{2D}}(E)\; f_{\text{FD}}(E)\,\text{d}E \qquad (18.5\text{XX})$$

where ρ^{2D} is the two-dimensional density of states. In the high-density approximation, this simplifies to

$$n_{\text{acc}}^{\text{2D}} \;=\; \rho^{\text{2D}}\,(E_{\text{F}} - E_0)\ . \qquad (18.5\text{XX})$$

Thus the accumulation potential is given by

$$eV_{\text{acc}} \;=\; E_0 + (E_{\text{F}} - E_0) \;=\; \frac{3}{2}\left(\frac{3}{2}\,\frac{e\mathcal{E}\hbar}{\sqrt{m^*}}\right)^{\frac{2}{3}} + \frac{n_{\text{acc}}^{\text{2D}}}{\rho^{\text{2D}}} \;=\; \frac{3}{2}\left(\frac{3}{2}\,\frac{e\mathcal{E}\hbar}{\sqrt{m^*}}\right)^{\frac{2}{3}} + \frac{\varepsilon\mathcal{E}}{e\rho^{\text{2D}}} \qquad (18.5\text{XX})$$

We hae this expressed the voltage drop in the accumulation layer as a function of the electric field.

Next we determine the potential drop in the center regions of the RTS. We first assume, that the charge stored in the well is negligibly small. Then, the potential drops in the first barrier, the well, and the second barrier are simply given by

$$V_{B1} = \mathcal{E} L_{B1} \ , \qquad V_W = \mathcal{E} L_W \ , \text{ and } \qquad V_{B2} = \mathcal{E} L_{B2} \ . \tag{18.5XX}$$

If the charge in the well is taken into account, the electric field in the second barrier is further increased by that charge. The increase can be calculated from Gauss' law. The charge in the center well will be discussed in detail below.

The potential drop in the collector layer is caused by the charged depletion layer. If the collector is doped with a donor concentration N_D, and the electric field in the collector at the barrier-collector boundary is given by E, then Gauss' law yields the thickness of the depletion layer

$$W_D = \varepsilon \mathcal{E} / N_D \tag{18.5XX}$$

Using Poisson's equation, one obtains the potential drop in the depletion layer

$$V_{dep} = \frac{e}{2\,\varepsilon} N_D W_D^2 \ . \tag{18.5XX}$$

The total voltage applied to the device is the sum of the different potentials discussed above, *i. e.*

$$V = V_{acc} + V_{B1} + V_W + V_{B2} + V_{dep} \ . \tag{18.5XX}$$

We thus can determine the relationship between the electric field in the RTDs and the applied voltage.

The attempt to escape model

Assume that an electron has tunneled through the emitter barrier and is confined by the quantum well. The electron confined by the well has a kinetic energy and thus bounces back and forth between the two barriers. Each time the electron impinges on the barrier, it attempts to escape from the well. However, due to the low tunneling probability, the electron is unlikely to escape from the well at its first attempt. After sufficiently many attempts, the electron will eventually escape from the well. It will escape through the second barrier into the collector due to the availability of unoccupied states in the collector. The escape to the emitter is unlikely because electrons in the accumulation layer occupy states with the same energy as the electrons in the well.

The rate of attempts to escape from the quantum well can be derived from the kinetic energy of electrons in the well. Assume that electrons in the well have a state energy E_0 with respect to the bottom of the well. Using the infinite well-approximation, the energy and the k value of the electron are related by

$$E_0 = \frac{\hbar^2}{2\,m^*}\left(\frac{\pi}{L_{QW}}\right)^2 = \frac{\hbar^2 k^2}{2\,m^*} = \frac{p^2}{2\,m^*} \ . \tag{18.5X}$$

Thus the electron velocity is give by

$$v \;=\; \frac{p}{m^*} \;=\; \sqrt{2\,E_0\,/\,m^*}\;\;. \qquad (18.5X)$$

The attempt rate of the electron to escape through the exit barrier is then given by

$$A \;=\; \frac{v}{2\,L_{\mathrm{QW}}} \;=\; \frac{\sqrt{2\,E_0\,/\,m^*}}{2\,L_{\mathrm{QW}}} \;=\; \frac{\sqrt{2\,E_0\,/\,m^*}}{2\,\pi\,\hbar\,/\sqrt{2\,E_0\;m^*}} \;=\; \frac{2\,E_0}{\pi\,\hbar}\;\;. \qquad (18.5X)$$

where we have used the infinite well approximation to express L_{QW} in terms of E_0. Each attempt has a success probability of T, the tunneling probability. Thus the rate of successful attempts is given by

$$AT \;=\; \frac{2\,E_0}{\pi\,\hbar}\,T\;\;. \qquad (18.5X)$$

The inverse of the rate of successful attempts is the lifetime of the electron in the well, that is,

$$\Delta\tau \;=\; \left(A\,T\right)^{-1} \;=\; \frac{\pi\,\hbar}{2\,E_0\,T}\;\;. \qquad (18.5X)$$

The finite lifetime of electrons in the well leads, according to the uncertainty principle, to a broadening of the quantum state in the well. Since the uncertainty principle is given by $\Delta E\;\Delta\tau \approx \hbar$, the quantum state has the width

$$\Delta E \;=\; \frac{\hbar}{\Delta\tau} \;=\; \frac{2\,E_0\,T}{\pi}\;\;. \qquad (18.5X)$$

The broadening of the energy level leads to a broadening of the voltage resonance. The width of the voltage resonance is then given by $\Delta V = \Delta E\,/\,e$. Calculating the width of the energy resonance for a typical resonant tunneling structure reveals that the calculated energy resonance broadening is much smaller than the linewidth of the voltage peak of a typical resonant tunneling structure. This is because other broadening mechanisms cause additional broadening of the current peak in the I-V characteristic. The additional broadening mechanisms include the thermal energy distribution of carriers in the emitter, any random fluctuations of the quantum barriers or the quantum well width, or any compositional fluctuations of the barrier and well materials.

Exercise. ***Linewidth of the current resonance in an RTS***. Calculate the linewidth of the current peak in a typical $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ / GaAs resonant tunneling structure with x = 30 %, L_{B} = 40 Å, and L_{W} = 100 Å. Compare your result to typical experimental linewidths in RTS which are several tens of meV at low temperatures and several hundreds meV at room temperature.

The charge stored in the well

As a current flows through the resonant tunneling structure, a small amount of charge is stored in the well layer. Assume that a current I is flowing through the resonant tunneling structure. The charge in the well will remain a time τ within the well before escaping from the well by tunneling. Thus the charge per unit area in the well is given by

$$Q_{\mathrm{W}}^{\mathrm{2D}} = J\,\tau \qquad (18.\mathrm{XXX})$$

where J is the current density. The charge in the well isusually very small and can be neglected, in particular in the off-resonance state. However, under on-resonance conditions, the current increases and therefore the charge in the well increases.

The charge in the well can be measured directly by a capacitance measurement. Fig. XXX shows the capacitance vs. voltage curve ...

The charge in the well can give rise to a hysteresis in the I-V characteristic of the RTS. Goldmann has shown ...

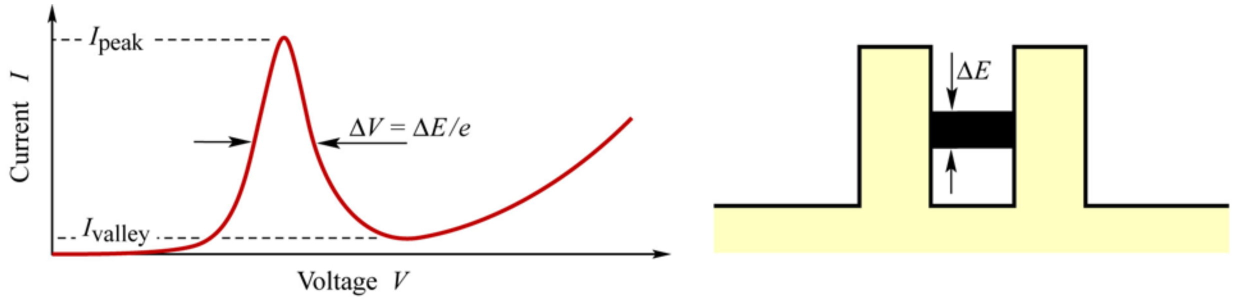

Fig. 18.4 Linewidth of current resonance peak. Also shown are the peak and valley current.

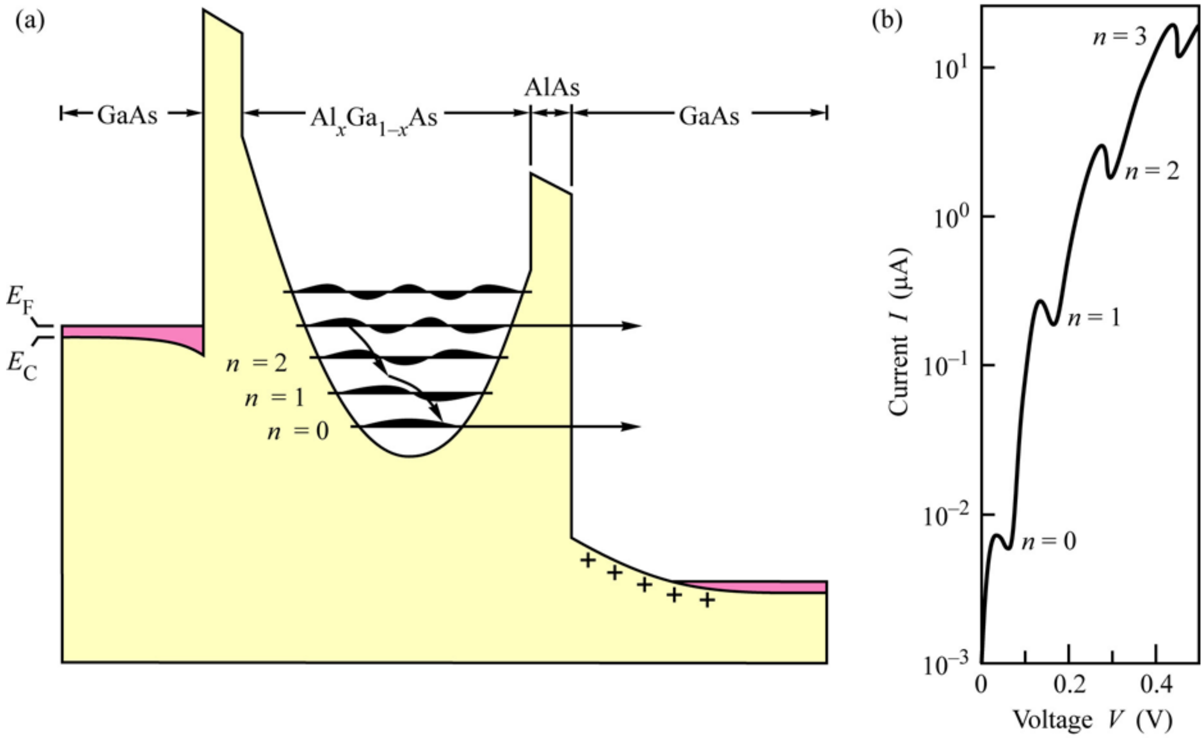

Fig. 18.5. Illustration of the conduction band diagram of a resonant-tunneling structure with a parabolic well. The two tunneling processes, elastic (energy conserved) and inelastic (electron undergoes energy relaxation in the well), are indicated by arrows. (b) Current-voltage characteristic of an Al$_x$Ga$_{1-x}$As / GaAs parabolic well resonant tunneling structure at T = 4.2 K. Four resonances, n = 0 to n = 3, are observed.

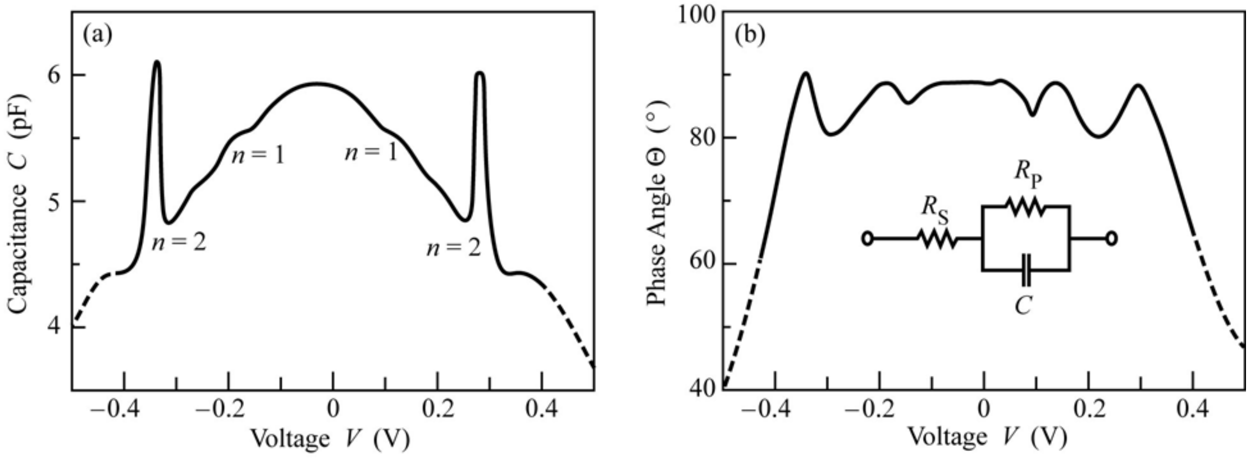

Fig. 18.6. (a) Capacitance-voltage characteristic of a resonant tunneling structure with parabolic well at T = 4.2 K. A shoulder and a peak are observed at the n = 1 and n = 2 resonances, respectively. (b) Phase angle between current and voltage measured at a frequency of 10 MHz.

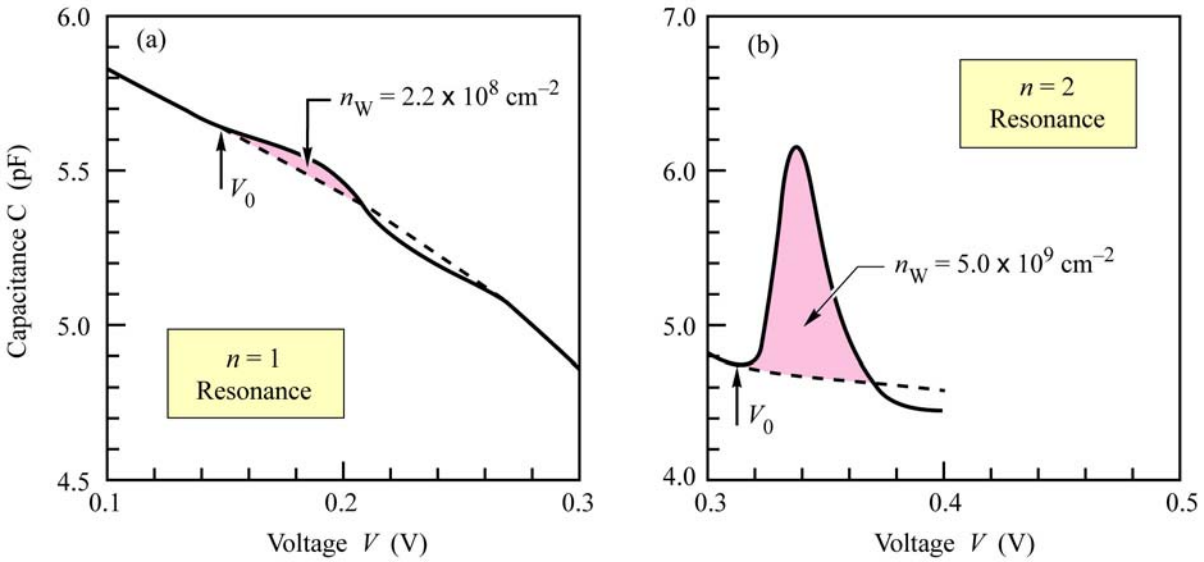

Fig. 18.7. Capacitance-voltage curve at 4.2 K in the vicinity of the (a) $n = 1$ and (b) $n = 2$ resonance. The maximum charge densities are 2.2×10^8 cm^{-2} and 5.0×10^9 cm^{-2} for the $n = 1$ and $n = 2$ resonance, respectively.

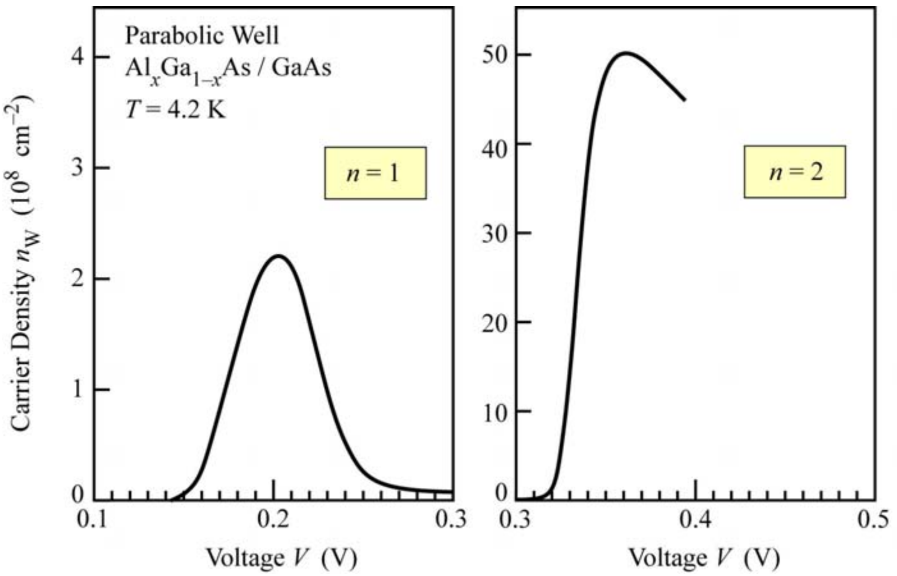

Fig. 18.8. Evolution of the charge density in the well with voltage for the $n = 1$ and $n = 2$ resonance.

Exercise. ***Resonant tunneling structures***. Suggest some ways to improve the peak-to-valley current ratio of resonant tunneling structures. Explain the dependence of the peak-to-valley ratio on the emitter doping concentration and the measurement temperature.

Applications of resonant tunneling structures

High-frequency oscillators

Bistable device