

The length of a photon

- A photon is a difficult entity. It frequently behaves as a particle. It also behaves as an electromagnetic wave. What is the amplitude of the wave? What is the duration of the wave? What is the length of the wave?
- The above questions may appear simple but they are in fact quite difficult. Albert Einstein said about photons: “Every fool thinks he knows what the photon is, but he is mistaken”
- Consider a photon emitted due to an electron-hole recombination in a semiconductor. Assume that the lifetime of the recombination process is τ .
- The emission of the photon occurs within the time τ . The (monomolecular) rate equation for electron-hole recombination is given by

$$\frac{dn}{dt} = -\alpha n$$

where α is a proportionality constant. The equation can also be written as

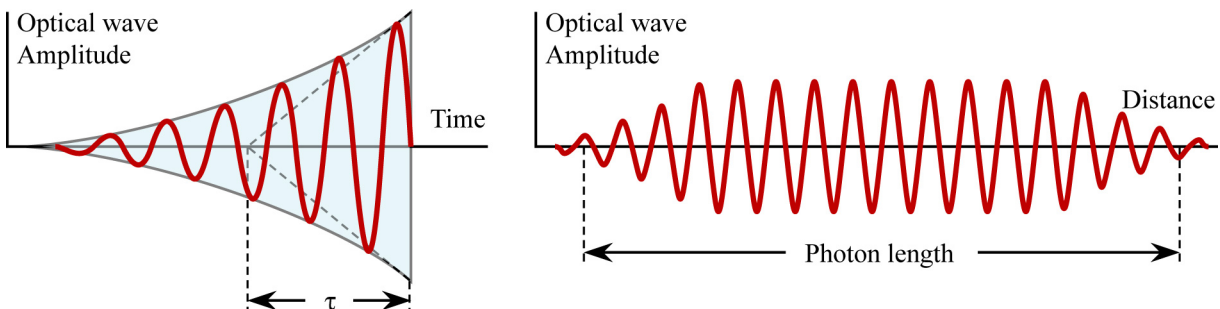
$$\frac{dn}{dt} = -n/\tau \quad \text{where} \quad \tau = 1/\alpha.$$

The solution of this differential equation is given by

$$n(t) = n_0 e^{-t/\tau}.$$

The solution shows that the carrier concentration decreases exponentially. The solution also shows that the lifetime τ is the *average* time it takes for an electron-hole recombination.

- Let us assume that the intensity of a photon emission has the same time dependence. That is, the photon is emitted within the time τ and the average oscillation time of a photon (time during which an electromagnetic radiation is emitted) is also τ . The illustration shows that the electric field amplitude of the photon decays within the time τ (left-hand side figure).



The illustration also shows the electric field of a photon based on intuition (right-hand side of illustration). Although we do not know for certain if a photon indeed looks like this, it will surely look something like that.

- In free space, the photon propagates with the velocity of light. Thus, the physical length of the photon is given by

$$L = c \tau$$

where c is the velocity of light.

- *Example:* The spontaneous emission time in a semiconductor is $\tau_{\text{spont}} = 5$ ns. What is the length of photons emitted by the semiconductor?

Solution: The length of the photon is given by $L = c \tau_{\text{spont}} = 1.5$ m.

- **Student exercise:** Discuss the propagation of a photon in a Fabry-Perot resonator. Discuss the occurrence of optical modes. What is the role of the cavity length and the photon length? Can we distinguish different regimes, *e.g.* cavity length \gg or \ll photon length?
- Recall that in the case of a semiconductor, the energy of the photon is approximately given by the energy gap, *i. e.*

$$h\nu \approx E_g \quad (\text{Planck's equation})$$

That is, the photon frequency depends only on the photon energy.

- Also recall that the photon momentum is very small compared to typical momenta of electrons and holes in semiconductors

$$p = h/\lambda \quad (\text{de Broglie relation})$$