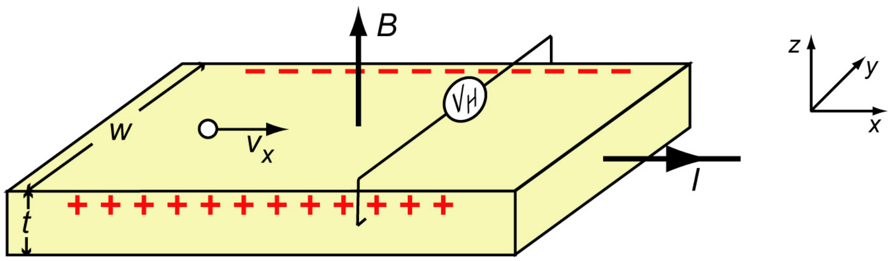
Hall Effect

The Hall Effect was discovered by E. H. Hall in 1879 when he investigated the nature of the force acting on a conductor carrying a current in a magnetic field. He found that the magnetic field exerts a transverse force (Lorentz force) on the moving charge carriers and the force tends to push the carriers to one side of the conductor, as illustrated in the figure below. A buildup of charges at the sides of the conductors will create an electric field that balances the Lorenz force, thereby producing a measurable voltage in the transverse direction.



Assume a p-type semiconductor with carrier concentration p, a current I flowing in the x-direction, and a magnetic field B applied in the z-direction. The electric field occurring in the y-direction produces the Hall voltage V_H given by

$$V_H = \int_0^w \varepsilon_y \, dy = \int_0^w \frac{B\,I}{q\,w\,t\,p} \, dy = \frac{B\,I}{q\,t\,p}$$

The Hall coefficient R_H (given in units of m^3/C) is defined as

$$R_H = \frac{t\,V_H}{B\,I}$$

giving

$$p_H = \frac{1}{q\,R_H}$$

Similarly, the carrier concentration of n-type semiconductor can be determined by

$$n_H = -\frac{1}{q\,R_H} \,.$$

The Hall mobility μ_H is defined by

$$\mu_H = \frac{|R_H|}{\rho} = |R_H|\,\sigma \,.$$

Hall Effect measurement

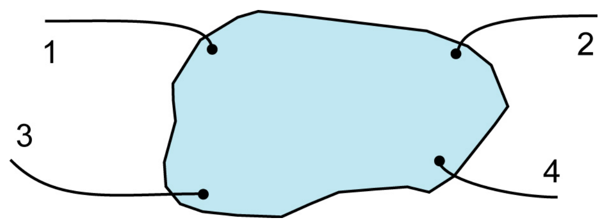
For a flat sample of irregular shape, the van-der-Pauw model can be applied to determine the resistivity, carrier concentration, and mobility. The following conditions need to be met.

(1) the contacts are at the circumference of the sample
(2) the contact are sufficiently small
(3) the sample is uniformly thick

For the sample illustrated in the figure below, the resistivity is given by

$$\rho \;=\; \frac{\pi\, t}{\ln 2}\;\frac{R_{12,34} \;+\; R_{23,41}}{2}\;F$$

where $R_{12,34} = V_{34} / I$. The current I goes from Contact 1 to Contact 2. $V_{34} = V_4 - V_3$ is the voltage between Contact 3 and 4. For symmetric samples, like squares or circles, $F = 1$ (F indicates the symmetry of the sample and contacts).



The Hall coefficient is given by

$$R_{\mathrm{H}} \;=\; \frac{t\,\Delta V_{34}}{2\,B\,I}$$

where $\Delta V_{34} = V_{34}$ (for $+B$) – V_{34} (for $-B$) and the Hall mobility can be calculated by

$$\mu_{\mathrm{H}} = \frac{|R_{\mathrm{H}}|}{\rho}\;.$$

Let us assume that we measure a sample that has two distinct layers. For such a simple two-layer structure with thickness t_1, t_2 and conductivity σ_1, σ_2 for Layer 1 and Layer 2, respectively, as shown in the figure below, the Hall coefficient is given by

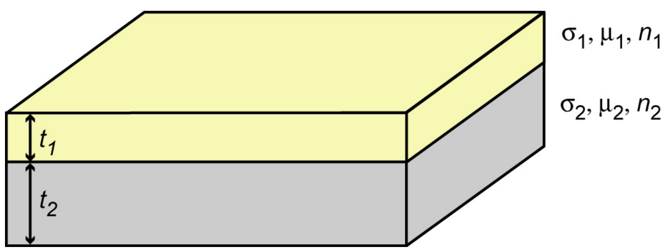
$$R_H = \frac{(t_1+t_2)\left[\left(R_{H1}\,\sigma_1^2\,t_1 + R_{H2}\,\sigma_2^2\,t_2\right) + R_{H1}\,R_{H2}\,\sigma_1^2\,\sigma_2^2\,(R_{H1}\,t_2 + R_{H2}\,t_1)\,B^2\right]}{(\sigma_1\,t_1 + \sigma_2\,t_2)^2 + \sigma_1^2\,\sigma_2^2\,(R_{H1}\,t_2 + R_{H2}\,t_1)\,B^2}$$

where R_{H1} and R_{H2} is the Hall coefficient for layer 1 and layer 2, respectively. In low magnetic field limit, it becomes

$$R_H = \frac{(t_1+t_2)\left(R_{H1}\,\sigma_1^2\,t_1 + R_{H2}\,\sigma_2^2\,t_2\right)}{(\sigma_1\,t_1 + \sigma_2\,t_2)^2} = R_{H1}\frac{t_1}{t_1+t_2}\left(\frac{\sigma_1}{\sigma}\right)^2 + R_{H2}\frac{t_2}{t_1+t_2}\left(\frac{\sigma_2}{\sigma}\right)^2$$

where

$$\sigma = \frac{t_1}{t_1+t_2}\sigma_1 + \frac{t_2}{t_1+t_2}\sigma_2 \,.$$



A detailed analysis shows that the *measured* carrier concentration is given by

$$n_H = \frac{n_1\,n_2\,\sigma^2\,(t_1+t_2)}{n_2\,t_1\,\sigma_1^2 + n_1\,t_2\,\sigma_2^2} = \frac{(n_1\,\mu_1\,t_1 + n_2\,\mu_2\,t_2)^2}{\left(n_1\,\mu_1^2\,t_1 + n_2\,\mu_2^2\,t_2\right)(t_1+t_2)}$$

and the *measured* mobility is given by

$$\mu_H = \mu_1\frac{t_1}{t_1+t_2}\frac{\sigma_1}{\sigma} + \mu_2\frac{t_2}{t_1+t_2}\frac{\sigma_2}{\sigma} = \frac{n_1\,\mu_1^2\,t_1 + n_2\,\mu_2^2\,t_2}{n_1\,\mu_1\,t_1 + n_2\,\mu_2\,t_2}\,.$$