Fresnel reflection in multi-layer structures

As electromagnetic waves and photons pass through space they inevitably reach materials with different electrical and magnetic properties. In semiconductors, we are concerned with extracting photons generated in a material with a high index of refraction (active layer) to a low index of refraction (air). As the photons are incident upon a boundary between materials of different refractive indices, a reflection occurs, known as the **Fresnel reflection**.

For near-normal incidence angles at the boundary, ***Fresnel Power Reflection Coefficient*** can be expressed as

$$R_f = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

or in terms of transmittance it may be written as the ***Fresnel Power Transmission Coefficient***

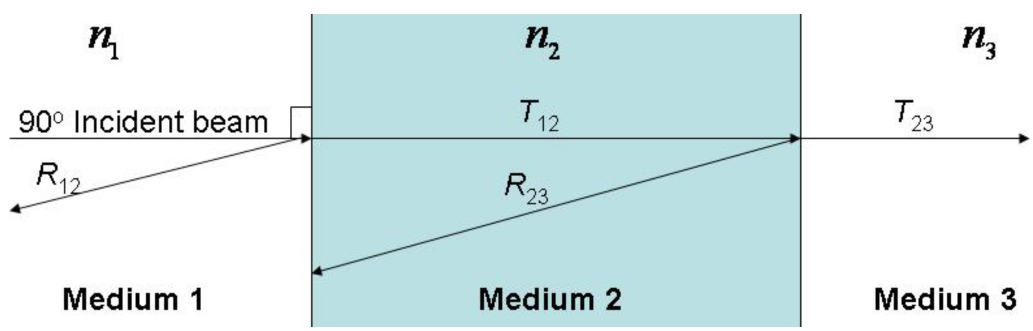
$$T_f = 1 - R_f = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

where n_1 and n_2 are the refractive indices of the two media.

To determine the transmittance across single or multiple boundaries, we will consider different cases.

Calculation of transmittance without taking into account multiple reflections

We assume that the materials will not exhibit any optical absorption loss and multiple reflections will not be taken into account as shown in the figure below.



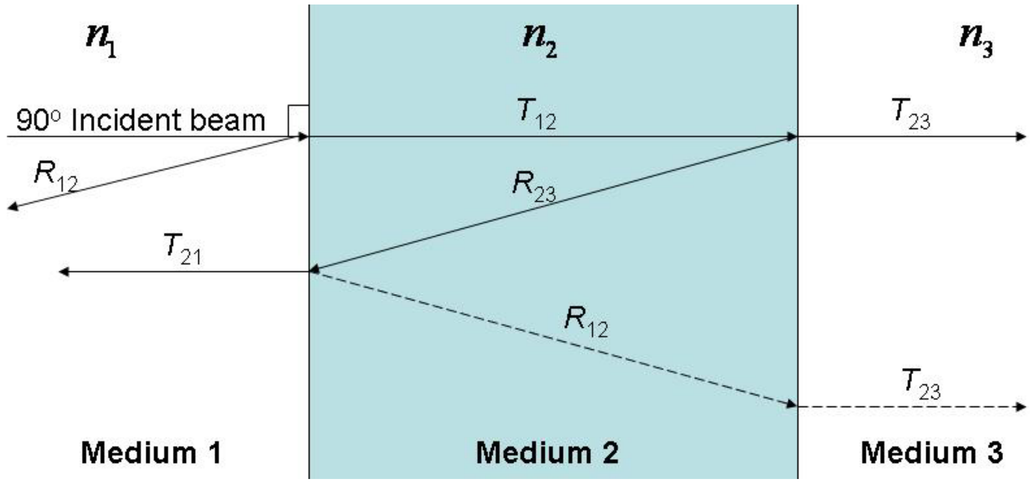
We have derived the following formula to describe the transmission across *m* number of boundaries based on Fresnel Reflection

$$T(n,m) = \frac{4^m n_1 \left(\prod_{j=2}^{m} n_j^{\,2} \right) n_{m+1}}{\prod_{j=1}^{m} (n_j + n_{j+1})^2}$$

where n_1 represents the first medium and n_j represents the subsequent mediums.

Calculation of transmittance by taking into account multiple reflections

The figure below shows a 3 layer structure, which is the simplest of all the multiple reflection cases.



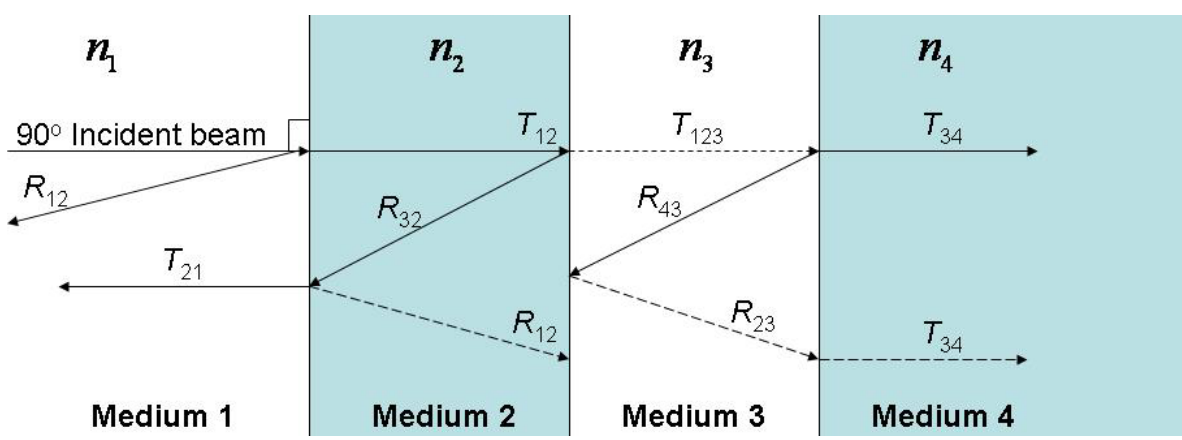
The total transmittance becomes

$$T = T_{12}T_{23} + T_{12}R_{32}R_{12}T_{23} + T_{12}(R_{32}R_{12})^2 T_{23} + T_{12}(R_{32}R_{12})^3 T_{23} + \dots$$

which converges to

$$T = \frac{T_{12}T_{23}}{1 - R_{32}R_{12}} .$$

For a more general case, we consider more layers. Using the previous equation, we assume that the first 2 boundaries have a total transmittance of T_{123} as shown in the following figure.



The total transmittance becomes

$$T = T_{123}T_{34} + T_{123}R_{43}R_{23}T_{34} + T_{123}(R_{43}R_{23})^2 T_{34} + T_{123}(R_{43}R_{23})^3 T_{34} + \ldots$$

which converges to

$$T = \frac{T_{123}T_{34}}{1 - R_{43}R_{23}}.$$

The 4 layer transmittance equation has the same form as the 3 layer equation. We, therefore, derive the following equation to calculate the total transmittance for L number of layers

$$T_{tot} = T_{12} \prod_{j=2}^{L-1} \frac{T_{j,j+1}}{(1 - R_{j.j-1}\ R_{j,j+1})}$$

which expands to

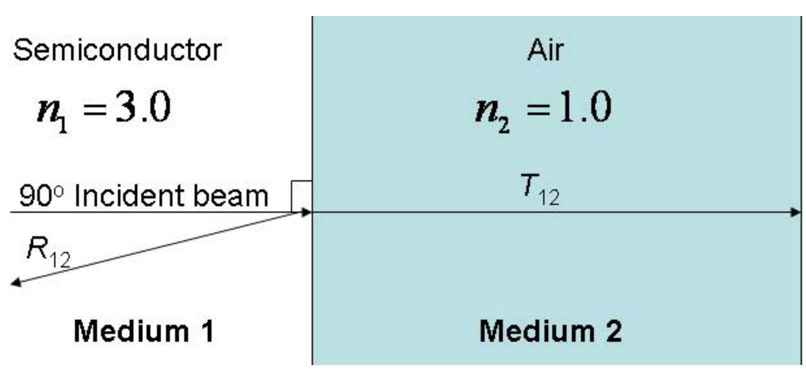
$$T_{tot} = 4\frac{n_1 n_2}{(n_1 + n_2)^2} \prod_{j=2}^{L-1} \frac{4 \cdot \frac{n_j n_{j+1}}{(n_j + n_{j+1})^2}}{(1 - \frac{(n_j - n_{j-1})^2}{(n_j + n_{j-1})^2} \frac{(n_j - n_{j+1})^2}{(n_j + n_{j+1})^2})}$$

Note: This equation does not take into account light transmission from Medium 3 back to Medium 2 and the alike to keep the calculation simple.

Example of transmittance calculation through an index graded material

Next we consider an example demonstrating the differences between an abrupt and gradual index step using the two cases considered above.

In the abrupt step case, there is only one semiconductor-air interface as shown below. Therefore we do not need to take into account multiple reflections and will use the original Fresnel transmission coefficient formula. We assume that the semiconductor refractive index is 3.0 and the refractive index of air is 1.0.

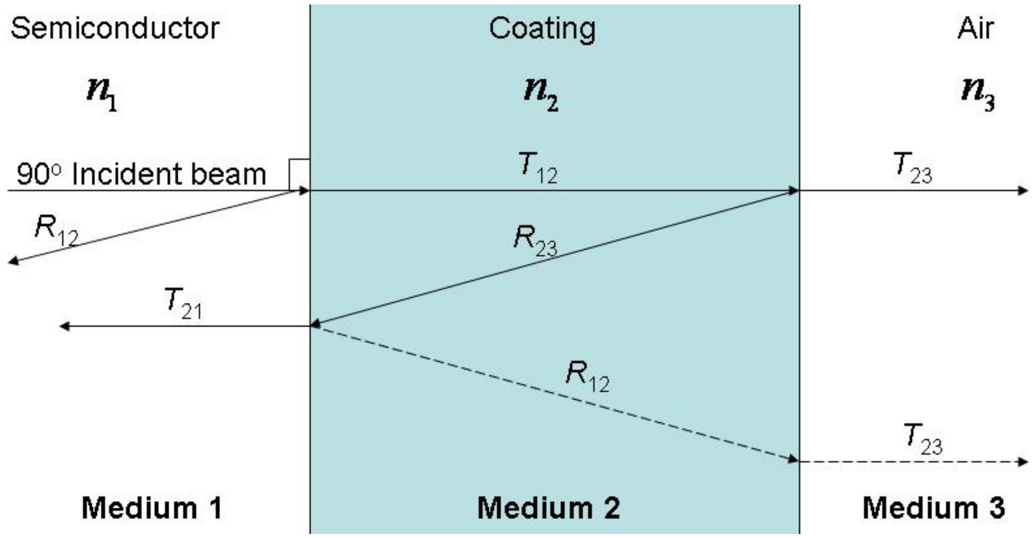


The transmittance value is

$$T_{tot} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

$$T_{tot} = 0.75$$

Next we consider the graded step case with one intermediate layer with an index of 2.0.



The transmittance values are as follows:

Without the multiple reflection case

$$T_{tot} = 0.85333$$

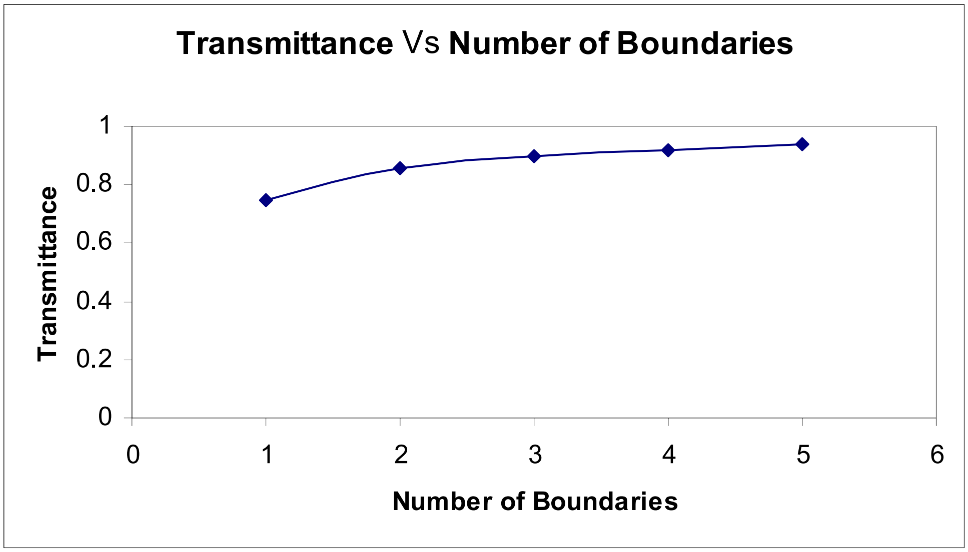
With the multiple reflection case

$$T_{tot} = 0.857$$

The graph below shows the transmittance as a function of the number of boundaries. The graph demonstrates how graded steps of refraction index affect the Fresnel transmission coefficient. In the graph, the index steps are equally spaced with the first and last index of refraction being 1.0 and 3.0, respectively.

The previous examples demonstrate the usefulness of adding graded-index steps between layers with different refractive indices to help increase the Fresnel transmission coefficient, thus improving overall photon extraction.

Note: The above calculations do not take into account optical interference effects. Therefore, the thickness of the individual layers does not enter the calculations.



Note: The graph above does not take into account multiple reflections.

Definitions

Below are a few useful definitions used when describing reflected and transmitted light rays.

Qualitative Terms

- **Reflection:** The act of a light wave (or any wave) that originates in a medium, having a boundary, reaching the boundary then returning back in the medium.
- **Transmission:** The act of a light wave (or any wave) that originates in a medium, having a boundary, reaching the boundary then transmitting through the boundary.

Quantitative Terms

- **Power Reflectance:** The ratio of the power reflected by a boundary to the total power incident on the boundary.
- **Fresnel Power Reflection Coefficient:** Describes the light power that is reflected due to Fresnel Reflection.
- **Power Transmittance:** The ratio of the total power of the transmitted wave to the total power of the incident wave.
- **Fresnel Power Transmission Coefficient:** Describes the light power that is transmitted due to Fresnel Reflection.