Black-body radiation

In physics, a black body is an object that absorbs all electromagnetic radiation that falls onto it. No radiation passes through it and none is reflected, yet it theoretically radiates every possible wavelength of energy.

The black-body radiation spectrum is used to evaluate the spectral intensity of radiation from a black body at temperature T. The black-body spectrum is characterized by only one parameter T, the temperature of the body. The black-body ***spectral irradiance*** was first derived by Max Planck (1900) and is given by:

$$I(\lambda) = \frac{2hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]}$$

The unit of $I(\lambda)$ is W m^{-2} nm^{-1} and thus is the power emitted per unit surface area (per m^2) of the black body per unit wavelength (per nanometer). The plot of $I(\lambda)$ at different black-body temperatures is shown in Figure 1.

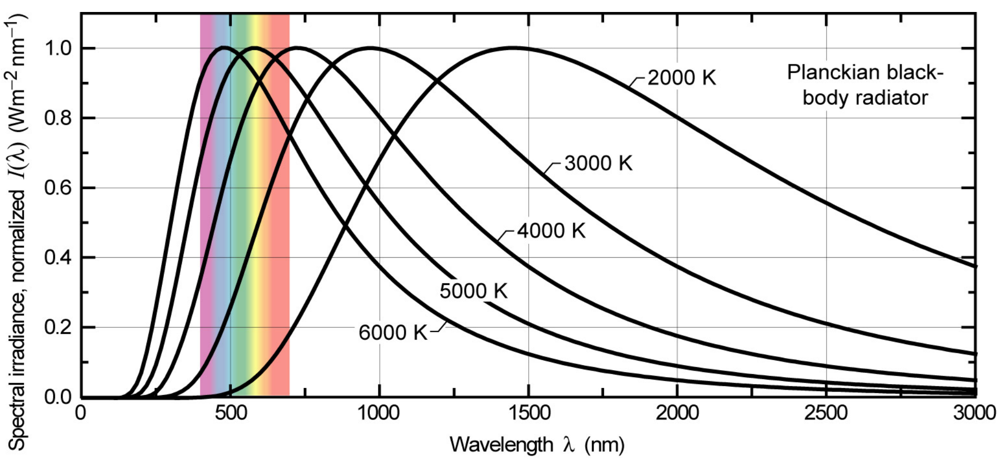

Figure 1: Spectral irradiance, $I(\lambda)$, versus wavelength.

The integral of $I(\lambda)$ over λ gives the ***irradiance***, i.e. the optical power emitted per unit surface area of the black body. The integral is given by:

$$\int_\lambda I(\lambda)\,\mathrm{d}\lambda = \frac{P_{\text{optical}}}{Area}$$

Another important parameter is $I(E)$, the ***spectral irradiance*** per unit area ***per unit energy***. The unit of $I(E)$ is W m^{-2} eV^{-1} and thus is the power emitted per unit surface area (per m^2) of the black body per unit energy (per eV). In order to calculate $I(E)$, we use:

$$\int_\lambda I(\lambda)\,\mathrm{d}\lambda = \int_E I(E)\,\mathrm{d}E = \frac{P_{\text{optical}}}{Area}$$

or

$$I(\lambda)\,\mathrm{d}\lambda = I(E)\,\mathrm{d}E$$

This equation allows us to calculate $I(E)$:

$$I(E) = I(\lambda)\frac{\mathrm{d}\lambda}{\mathrm{d}E}$$

Using the relation $E = h\nu = hc/\lambda$ and $\mathrm{d}E = -hc/\lambda^2\mathrm{d}\lambda$, and neglecting the minus sign ("–"), we obtain:

$$I(E) = I(\lambda)\frac{\lambda^2}{hc} = \frac{2hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda kT}\right)-1\right]}\frac{\lambda^2}{hc} = \frac{2c}{\lambda^3\left[\exp\left(\frac{hc}{\lambda kT}\right)-1\right]}$$

$$= \frac{2c}{\left(\frac{hc}{E}\right)^3\left[\exp\left(\frac{E}{kT}\right)-1\right]} = \frac{2E^3}{h^3c^2\left[\exp\left(\frac{E}{kT}\right)-1\right]}$$

$$\boxed{I(E) = \frac{2E^3}{h^3c^2\left[\exp\left(\frac{E}{kT}\right)-1\right]}}$$

The plot of $I(E)$ versus E for different blck-body temperatures is shown in Figure 2.

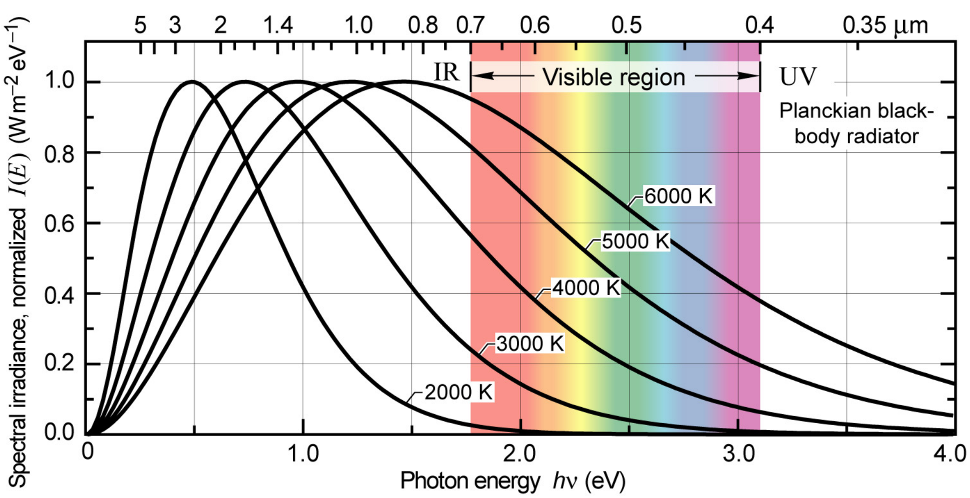

Figure 2: Spectral irradiance, $I(E)$, versus energy.

References

Light-Emitting Diodes, 2nd edition, E. F. Schubert, Cambridge University Press, UK (2006)

Planck M. "On the theory of the law on energy distribution in the normal spectrum (translated from German)" *Verhandlungen der Deutschen Physikalischen Gesellschaft* **2**, 237 (1900)

http://en.wikipedia.org/wiki/Black_body