Absorption Coefficient – Measurement and Calculations

An ***ellipsometer*** can not estimate the *refractive index* and the *absorption coefficient* of a material that is much thicker than the wavelength of light. Instead, the transmittance and reflectance of a 200 µm thick GaN wafer is measured using a ***JASCO spectrophotometer*** and the absorption coefficient is calculated from the measured values.

Theory

The intensity of light, traveling in a medium, reduces according to the equation

$$I(z) = I_0 e^{-\alpha z} \qquad (1)$$

where α is the absorption coefficient and z is the distance along the propagation direction of light. The reflectance and transmittance at each interface is given by,

$$R = \left(\frac{n_1 - n_2}{n_1 - n_2} \right)^2 \qquad (2a)$$

$$T = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \qquad (2b)$$

where n_1 and n_2 are the refractive indices of the two media.

The wafer has two interfaces with air. ***Figure*** 1 depicts a model describing the transmittance and reflectance. The thickness of the GaN wafer, *d,* is 200 µm. Because the following calculation is limited to very think films, $d >> \lambda$, we do not take into account the phase of the wave and thus do not take into account any optical interference effects.

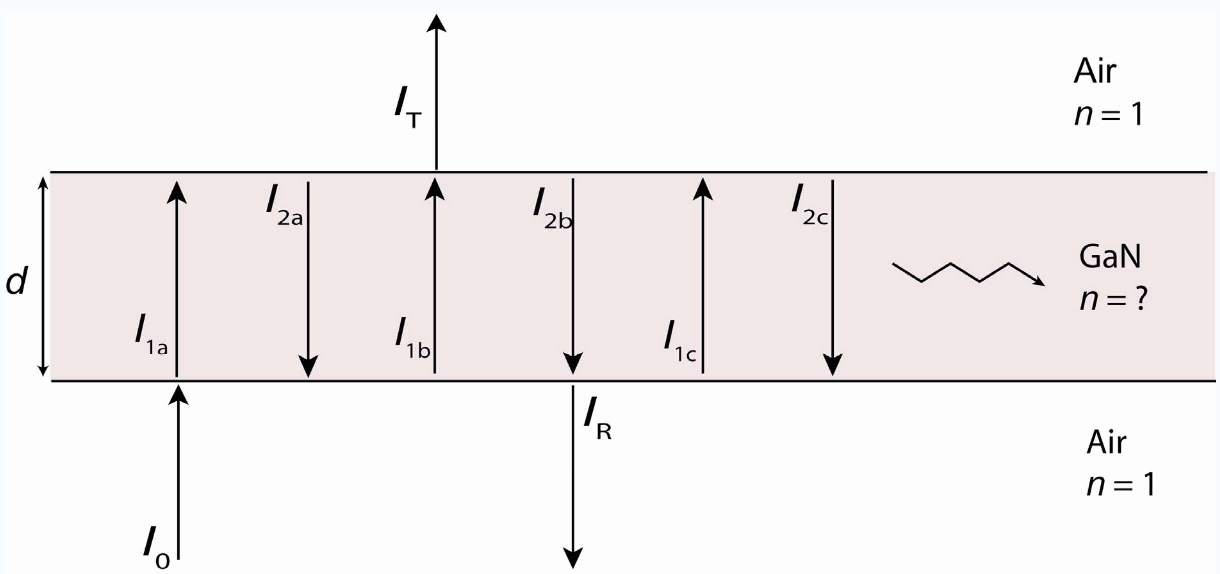

Figure 1: The reflection and transmission at each interface of a GaN wafer

The equations describing I_{1a}, I_{1b}, I_{1c} etc. and I_{2a}, I_{2b}, I_{2c} etc. are

$$I_{1a} = I_0\,T \qquad (3a) \qquad\qquad I_{2a} = I_{1a}\,e^{-\alpha d} R \qquad (3b)$$

$$I_{1b} = I_0\,T\,e^{-2\alpha d} R^2 \qquad (4a) \qquad\qquad I_{2b} = I_{1b}\,e^{-\alpha d} R \qquad (4b)$$

$$I_{1c} = I_0\,T\,e^{-4\alpha d} R^4 \qquad (5a) \qquad\qquad I_{2c} = I_{1c}\,e^{-\alpha d} R \qquad (5b)$$

and so on …

Summation of I_{1a}, I_{1b}, I_{1c} etc. is defined as I_1 and the summation of I_{2a}, I_{2b}, I_{2c} etc. is defined as I_2.

$$I_1 = I_{1a} + I_{1b} + I_{1c} + ... \qquad (6a) \qquad\qquad I_2 = I_{2a} + I_{2b} + I_{2c} + ... \qquad (6b)$$

Substituting equations (3b), (4b), (5b) etc. into equation (6b) yields,

$$\begin{aligned} I_2 &= I_{1a}\,e^{-\alpha d} R + I_{1b}\,e^{-\alpha d} R + I_{1c}\,e^{-\alpha d} R + ... \\ &= (I_{1a} + I_{1b} + I_{1c} + ...)e^{-\alpha d} R \quad = \quad I_1 e^{-\alpha d} R \qquad (7) \end{aligned}$$

Substituting equations (3a), (4a), (5a) etc. into equation (6a) yields,

$$\begin{aligned} I_1 &= I_0\,T + I_0\,T\,e^{-2\alpha d} R^2 + I_0\,T\,e^{-4\alpha d} R^4 + ... \\ &= I_0\,T\,(1 + e^{-2\alpha d} R^2 + e^{-4\alpha d} R^4 + ...) \quad = \quad I_0\,T \big/ (1 - e^{-2\alpha d} R^2) \end{aligned}$$

as $1 + x + x^2 + x^3 ... = 1 \big/ 1 - x$ for $x < 1$

(Geometric series)

$$\begin{aligned} I_1 - I_1 e^{-2\alpha d} R^2 &= I_0 T \\ I_1 &= I_0 T + I_1 e^{-2\alpha d} R^2 \\ &= I_0 T + I_2 e^{-\alpha d} R \qquad (8) \end{aligned}$$

The transmitted light, I_T and the reflected light, I_R is

$$\begin{aligned} I_T &= I_{1a}\,e^{-\alpha d} T + I_{1b}\,e^{-\alpha d} T + I_{1c}\,e^{-\alpha d} T + ... \\ &= (I_{1a} + I_{1b} + I_{1c} + ...)e^{-\alpha d} T \quad = \quad I_1 e^{-\alpha d} T \qquad (9) \end{aligned}$$

$$\begin{aligned} I_R &= I_0 R + I_{2a}\,e^{-\alpha d} T + I_{2b}\,e^{-\alpha d} T + I_{2c}\,e^{-\alpha d} T + ... \\ &= I_0 R + (I_{2a} + I_{2b} + I_{2c} + ...)\,e^{-\alpha d} T \quad = \quad I_0 R + I_2 e^{-\alpha d} T \qquad (10) \end{aligned}$$

Rearranging equations (7) and (9) and using equations (2a) and (2b),

$$I_1 = I_T e^{\alpha d} \frac{(n+1)^2}{4n} \tag{11}$$

$$I_2 = I_T \frac{(n-1)^2}{4n} \tag{12}$$

By simplifying equations (8) and (10) using equations (11) and (12),

$$I_T \left(e^{\alpha d}(n+1)^4 - e^{-\alpha d}(n-1)^4 \right) = I_0 \, (4n)^2 \tag{13}$$

$$I_R = \left(I_0 + I_T e^{-\alpha d} \right) \left(\frac{n-1}{n+1} \right)^2 \tag{14}$$

Calculating Absorption Coefficient

The values of reflectance, I_R and transmittance, I_T were measured using the *JASCO* spectrometer. The graphs in ***Figure*** 2 show the measured values. The reflectance values for wavelength lower than 400 nm is incorrect as it does not fall in the range of the DBR used as a reference in the *JASCO* setup.

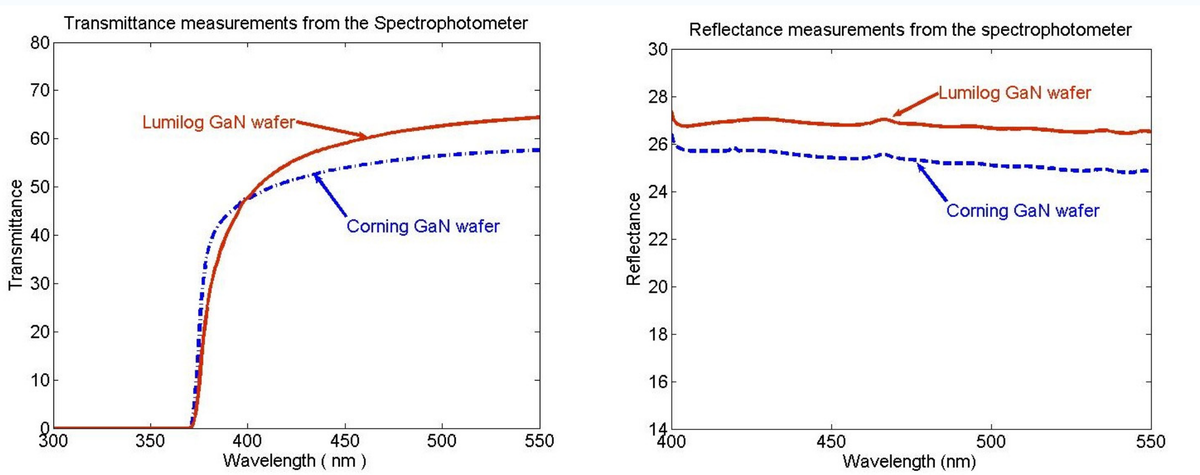

Figure 2: Transmittance and reflectance of a 200 µm thick GaN wafer measured using JASCO spectrophotometer

Using the equations (13) and (14), we can calculate the refractive index and the absorption coefficient as a function of wavelength.

To solve these two equations we use a simple iterative technique. The algorithm is as follows:

Iterative loop: wavelength goes from 400 nm to 550nm in steps of 1 nm

Iterative loop: *n* goes from 1.5 to 3 in steps of 0.01

Calculate the value of α using equation (14) using the measured value of I_R and I_T.

Calculate the value of I_T using equation (13) using the above calculated α.

Calculate the difference between the actual value of I_T and the calculated value of I_T and save the difference value.

Repeat for another value of *n*.

Find the value of α, which gives the minimum error in calculating I_T.

Find the corresponding value of *n*.

Repeat for the next value of wavelength.

The graphs in ***Figure*** 3 show the values of *n* and *α* determined by this method.

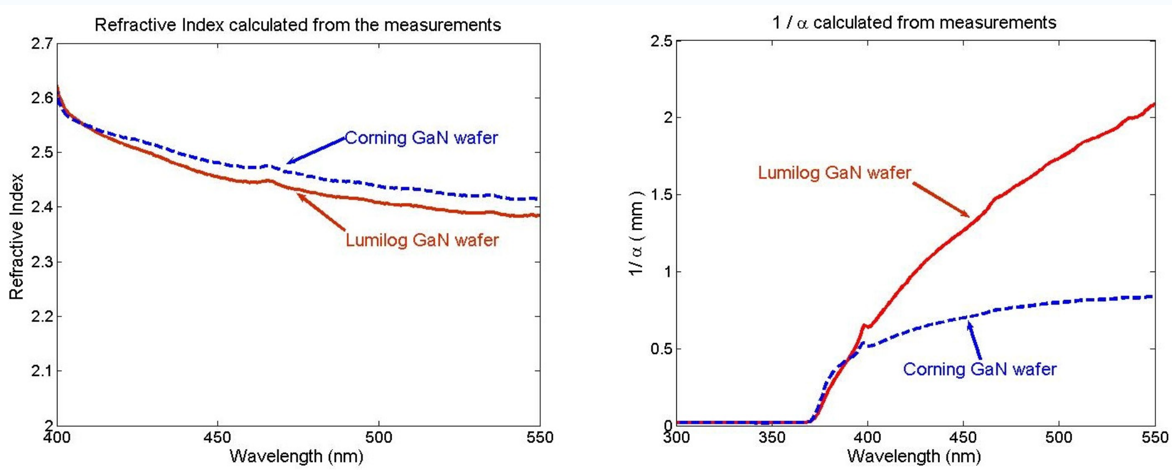

Figure 3: Calculated refractive index and inverse of absorption coefficient

Scattering

The measured value of absorption coefficient does not take into account the light lost due to optical scattering. The Corning wafer has a rougher surface compared to Lumilog wafer. Thus the scattering at the surface is higher for the Corning wafer as compared to the Lumilog. To discount the effect of scattering in our measurements, we need to either use a polished GaN or setup a laser source with an integrating sphere placed very close to the wafer.

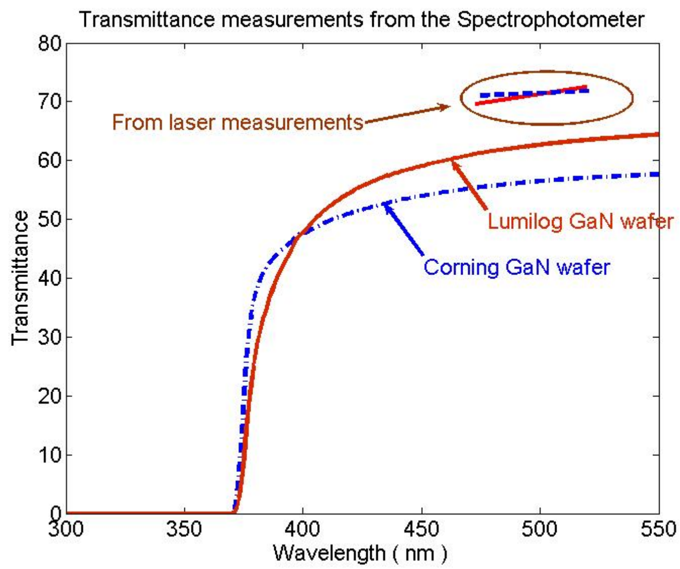

Figure 4: Circled values are the measured using Argon laser

Result

At λ = 500 nm, ***Figure*** 3 shows the following data.

	$1/\alpha$ mm	$1/\alpha$ cm	α cm $^{-1}$
Corning wafer (rougher surface)	0.8	0.08	12.5
Lumilog wafer	1.7	0.17	5.9