Hopping conduction

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1. General introduction

Hopping conduction is defined as electric conduction in which the carrier transport is via electrons hopping from one localized state to another. Electron transport through localized states (shallow-level states or deep-level states) within the bandgap of a semiconductor includes (as shown in Fig. 1):

1. Electron hops from a state to another state that has a higher energy. A thermal energy is required for this move. Let us denote the energy difference as E_{hop}. This process is thermally assisted tunneling. It depends on temperature.
2. Electron hops from a state to another state that has equal energy. This transport is tunneling process. It does not depend on temperature.
3. Electron hops from a state to another state that has a lower energy. This transport is a tunneling process with the emission of a phonon(s). It does not depend on temperature.

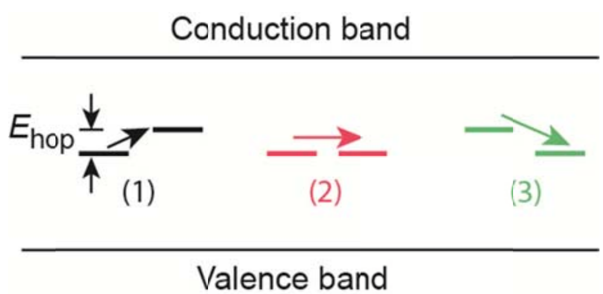

Figure 1 Electron transport between localized states: (1) thermally-assisted tunneling; (2) tunneling; and (3) tunneling with the emission of a phonon(s).

The necessary conditions for the occurrence of hopping are:

1. Wave functions of the two localized states must overlap.
2. Occupied and empty states must be present for the hopping to occur. This condition requires the hopping should happen between states close to the Fermi level.
3. An energy is necessary for the electron hopping from a localized state to another localized state with a higher energy level.

2. Nearest neighbor hopping conduction (Mott, 1974)

Nearest neighbor hopping (NNH) conduction is the hopping conduction in which an electron in a localized state obtains energy and hops to a nearest localized state with an energy E_{hop} above the former state. NNH conduction is rate limited by the thermal energy of electrons.

When the temperature is low, electrons in the localized states cannot be thermally activated to the conduction band. The energy difference of two neighbor states usually is smaller than the activation energy for the electron from the localized state to the conduction band. When one electron hops from one state to another, there could be a phonon(s) absorbed or emitted. An activation energy E_{hop} is associated with the process of absorption of a phonon(s). The average distance d_{nn} of the two states depends on the concentration of the localized states N_{LS} ($d_{\text{nn}} = [3/(4\pi N_{\text{LS}})]^{1/3}$). The analysis using percolation approach for NNH conduction yields the conductivity:

$$\sigma_{\text{nn}} = C_{\text{nn}} \exp\left(-\frac{\alpha d_{\text{nn}}}{a_{\text{d}}}\right) \exp\left(-\frac{E_{\text{hop}}}{kT}\right) \tag{1}$$

where C_{nn} is a constant independent of temperature, a_{d} is the spatial extent of the wave function,

d_{nn} is the average distance between neighbors, α is a constant and is about 2, E_{hop} is the thermal activation energy, k is the Boltzmann constant and T is temperature. In Eq. (1), the first exponential term is determined by the wavefunction overlap and the second exponential term is determined by the thermal activation energy.

3. Variable range hopping conduction (Shklovskii and Efros, 1984; Mott, 1974)

When the temperature is very low, the probability of the electron thermal activation between states that are close in space but far in energy becomes smaller than that of electron hopping between some more remote states whose energy levels happen to be very close to each other. In this case, the characteristic hopping length d_{vr} increases with decreasing temperature. Therefore, this kind of hopping is called variable range hopping (VRH). The temperature dependence of the conductivity is described by Mott's law:

$$\sigma_{vr} = C_{vr}\exp\left[-\left(\frac{T_0}{T}\right)^{1/4}\right], \quad (2)$$

where C_{vr} is a constant independent of temperature, T is temperature and T_0 is the characteristic temperature. In the Appendix, we derive Mott's law following Mott's original derivation, which is only a qualitative derivation. But it elucidates the physical essence of VRH conduction.

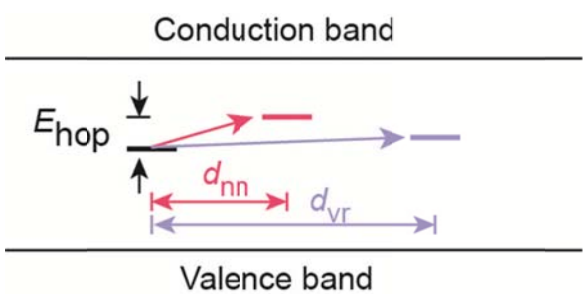

Figure 2 Nearest neighbor hopping and variable range hopping

From Appendix, we find that the average hopping length d_{vr} is a function of temperature and follows:

$$d_{vr} \approx a_d \left(\frac{T_0}{T}\right)^{1/4} \quad (3)$$

Thus, as temperature decreases, the average hopping length grows as $T^{-1/4}$. In NNH, the average hopping length is on the order of the average distance between localized states, and does not vary with temperature.

The derivative – d (ln σ) / d (k T)$^{-1}$ at a given temperature may be called the activation energy. This derivative is proportional to the absolute value of the slope in the Arrhenius plot of ln σ vs. 1 / T. From Mott's law, it follows that the activation energy monotonically decreases as $T^{3/4}$ with decreasing temperature. For this reason, conductivity obeying Mott's law is sometimes referred to the conductivity with decreasing activation energy when temperature decreases. In other words, the absolute value of the slope in the Arrhenius plot decreases with decreasing temperature.

4. Hopping conduction in the depletion region of a *pn* junction and Schottky diode

The depletion region of a *pn* junction and a Schottky diode can be considered as an insulating layer. There are no mobile electrons or holes in this region. In a *pn* junction, under reverse bias, electrons tunnel from the valence band of the p-type region to the localized states in the bandgap and then hop through the localized states and finally tunnel to the conduction band of the n-type region. The differences of the hopping conduction in a *pn* junction and the bulk material are:

1. The electric field in the depletion region can be much larger than that in the bulk material.
2. With increasing external reverse bias, the depletion width increases while in the bulk material, the material thickness does not change with the bias.

The electric field in the depletion region enhances hopping between localized states, since it reduces the thermal activation energy between states. If we consider the field dependence and non-uniform density of states near the Fermi level, the current of the field-assisted VRH hopping follows (Hill, 1971):

$$j \propto \exp\{-C(\frac{T_0}{T})^{1/4}[1-\frac{C'F^2}{T^{1/2}}]\} \quad (4)$$

where C and C' are constants independent of temperature and the electric field, F is the electric field, T is temperature, T_0 is the characteristic temperature. The temperature dependence of the field term is slight if the temperature range is small. Therefore, at a constant voltage, the temperature dependence of the reverse current follows Mott's $T^{-1/4}$ law. Kuksenkov et al. (Kuksenkov et al., 1998) claim that the reverse current of a GaN *pn* junction with $V_R < 7$ V can be well fitted by the theoretical current-field equation for hopping conduction under a moderate electric field (temperature ranges from 25°C to 250°C):

$$j = j(0)\exp[C\frac{eFa}{2kT}(\frac{T_0}{T})^{1/4}] \quad (5)$$

where $j(0)$ is the low field current density, C is a constant on the order of unity, e is the electron charge, a is Bohr radius of the localized states, F is the electric field, k is the Boltzmann constant, T is temperature, T_0 is the characteristic temperature. The characteristic temperature T_0 is obtained from the low field conductivity (($\sigma(F) = j(F) / F$, with $F \rightarrow 0$)) using Mott's $T^{-1/4}$ law and $T_0 \approx 10^7$ K. Equation (5) was originally derived for the bulk material and is valid when the electric field satisfies $e F a / (2 k T) < 1$. If we assume $a = 10$ Å and $T = 300$ K, the electric field should be smaller than 5×10^5 V / cm. The electric field is estimated by $F = (V_R + V_i) / w$, where, V_R is the absolute reverse bias, V_i is the build-in junction voltage and estimated at about 1 V, w is the depletion width. At $V_R = 7$ V, $F \approx 2.2 \times 10^5$ V / cm.

A 1-D VRH was proposed as one possible mechanism for the reverse leakage current of GaN Schottky diode (temperature ranges from 250 K to 400 K) (Miller et al., 2004). Miller et al. proposed that when the Schottky diode is under the reverse bias, an electron in the metal would fall into a state associated with a threading dislocation very near or below the Fermi level in the metal with subsequent transport described by a 1D-VRH conduction model. The conductivity is then given by:

$$\sigma = \sigma_0 \exp\left[-\left(\frac{T_0}{T}\right)^{1/2}\right] \quad (6)$$

In our experiments, we plot the current of a GaN LED at a fixed reverse voltage as a function of temperature. The experimental results are fitted by using Eqs. (1), (2), and (4) to (6). We find that the temperature dependence of the current at a fixed voltage in the temperature range 80 K to 250 K follows Mott's $T^{-1/4}$ law and can be fitted well with Eq. (2) and Eq. (4).

Appendix: Mott's law of variable range hopping conduction – a qualitative derivation (Mott, 1974)

We consider a system with localized states near the Fermi level. From Eq. (1), at very low temperature, only hopping with small E_{hop} contributes to conduction ($E_{\text{hop}} \in (E_F - \varepsilon_0, E_F + \varepsilon_0)$). The localized states in this energy range form a band (let us denote it as the "$2\varepsilon_0$ band"). Because of the narrow width of the $2\varepsilon_0$ band, its constituent states are far away from each other. At very low temperature, the density of state (DOS) near the Fermi level can be considered as constant. Therefore, the concentration of states in the band is given by:

$$N(\varepsilon_0) = 2g(E_{\text{F}})\varepsilon_0 \quad \text{(A1)}$$

The average distance d_{vr} between localized states from Eq. (1) is then written as:

$$d_{\text{vr}} = \left(\frac{3}{4\pi}\frac{1}{N(\varepsilon_0)}\right)^{1/3} \quad \text{(A2)}$$

It should be noted that $N(\varepsilon_0)$ depends on ε_0 and accordingly, temperature. Thus d_{vr} depends on temperature. Replacing E_{hop} by ε_0 and d_{nn} by d_{vr}, Eq. (1) becomes:

$$\sigma_{\text{vr}} = C\exp\left(-\frac{\alpha}{a_{\text{d}}[g(E_{\text{F}})\varepsilon_0]^{1/3}}\right)\exp\left(-\frac{\varepsilon_0}{kT}\right) \quad \text{(A3)}$$

For large ε_0, the second exponential function is the dominant limitation to the conductivity. Consequently, hopping conductivity increases when ε_0 decreases because the thermal activation energy decreases. When ε_0 is smaller, both terms are equally important. A further decrease in ε_0 leads to a decrease in σ_{vr}. Because smaller ε_0 indicates narrower band around the Fermi level, fewer sites take part in the hopping conduction and fewer wavefunctions overlap. Due to the competition of wavefunction overlap and the thermal activation, the conductivity has a maximum value at:

$$\varepsilon_0(T) = \frac{(\alpha kT)^{3/4}}{\left[g(E_{\text{F}})a_{\text{d}}^3\right]^{1/4}} \quad \text{(A4)}$$

One may call the $2\varepsilon_0$ band around the Fermi level as the optimal band. By inserting Eq. (A4) into Eq. (A3), we obtain Mott's law:

$$\sigma_{\text{vr}} = C_{\text{vr}}\exp\left(-\frac{T_0}{T}\right)^{1/4} \quad \text{(A5)}$$

References

Shklovskii B. I. and A. L. Efros, *Electronic properties of doped semiconductors,* Springer (1984)

Hill R. M., "Hopping conduction in amorphous solids", **Phil. Mag. 24**, 1307 (1971)

Kuksenkov D. V., Temkin H., Osinsky A., Gaska R., and Khan M. A., "Low-frequency noise and performance of GaN *p-n* photodetectors", **Appl. Phys. Lett. 72**, 1366 (1998)

Miller E. J., Yu E. T., Waltereit P., and Speck J. S., "Analysis of reverse-bias leakage current mechanisms in GaN grown by molecular-beam epitaxy", **Appl. Phys. Lett. 84**, 535 (2004)

Mott N. F., *Metal-insulator transitions,* Taylor& Francis (1974)