# A complementary matching technique to reduce the variance of optical and electrical properties of light-emitting diodes

Guan-Bo Lin Martin F. Schubert Jaehee Cho (SID Member) E. Fred Schubert Hyungkun Kim **Abstract** — Light-emitting diodes (LEDs) using phosphor conversion inherently have a wide variation of multiple parameters, including correlated color temperature (CCT), light output power (LOP), and forward voltage  $V_{\rm F}$ . A method, based on the formation of LED pairs with complementary characteristics, is presented to produce LED-based light sources with narrow CCT, LOP, and  $V_{\rm F}$  distributions. A weighted matching algorithm was developed to select LED pairs under a multiple selection criteria. Based on the weighted matching algorithm, 96 LEDs were combined to form 48 LED pairs, and it was experimentally demonstrated that the variance of LED CCT and LOP distributions decreased by 93% and 71%, respectively.

**Keywords** — LED backlighting, color uniformity, complementary LEDs.

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#### 1 Introduction

Solid-state light emitters are becoming the dominant light source for applications ranging from low-power indicators to high-power illumination sources. Especially, liquid-crystal-display (LCD) backlighting units (BLUs), automotive headlights, and interior and exterior lighting have received extensive attention. The core components for high-power solid-state lighting sources are (i) a III-nitride-based light-emitting diode (LED) that uses GaInN multiple quantum wells and (ii) a phosphor, such as a cerium-doped YAG (YAG:Ce), which is optically excited by the LED chip. As a result of this optical excitation, the phosphor emits yellow light that, when mixed with the blue light of the LED chip, results in white light.  $^2$ 

The color of a light source can be specified by its chromaticity coordinates; for example, the CIE 1931 xy coordinates.<sup>3</sup> Light emitted by an LED may be regarded as white if its chromaticity coordinates are suitably close to the Planckian locus. For a light source with chromaticity coordinates close to the Planckian locus, one can define the correlated color temperature (CCT), which is the temperature at which a blackbody radiator best matches the color of the light source. Although all points on the Planckian locus are considered white, there is visible variation in the color of white light. On a large scale, a lower CCT yields a yellowish appearance and a higher CCT yields a bluish appearance. On a small scale, chromaticity points with an equal color difference from a defined chromaticity point describe an ellipse in the chromaticity diagram. The sizes of these ellipses were recorded by MacAdam and are called MacAdam ellipses. The radius of the MacAdam ellipses are known as the standard deviation of color matching (SDCM).<sup>4</sup> Although a chromaticity spread of up to seven SDCM is generally acceptable for a daylight illumination application, a chromaticity difference of larger than two SDCM is distinguishable to the human eye.  $^5$ 

The variance of properties (CCT, light output power, peak wavelength, and forward voltage) is much greater for LEDs than for conventional light sources. This fact is one of the great challenges of solid-state-lighting (SSL) technology. Because of the fundamentally much greater variance in optical and electrical properties, binning (sorting of LED devices into a number of bins according to their characteristics) is needed to ensure that an application appears uniform in color (i.e., within acceptable color differences), output power, and forward voltage. The origins of color variance for white LEDs include the variation of chip wavelength, chip position inside the package, and phosphor quantity. Therefore, it is necessary to reduce the variation of parameters influencing the color distribution. Prior-art technologies developed by Steranka et al. 7 and Braune et al. 8 alter phosphor dispensing method to have a conformal coating. The phosphor concentration and thickness is more precisely controlled compared to non-conformal phosphor dispensing, such that the wavelength shift and the intensity of yellow phosphorescence is confined into a smaller range. Another method includes the fabrication of phosphor plates that have an intentional variation of chromaticity. 9 By matching specific blue LEDs with specific yellow phosphor plates, the variance in chromaticity of the resulting whitelight source can be reduced. In this paper, we discuss a method that reduces the variance of white LEDs by about one order of magnitude. We first present the theory and then experimentally demonstrate the concept.

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## 2 Design and theoretical modeling

Consider that we have 2N LEDs; after pairwise combination we will form N LED pairs. There are  $(2N)!/N!(2!)^N$  possible configurations of N LED pairs. Each configuration has an arrangement of LED pairs in a particular combination. The optimal configuration of N LED pairs would be the configuration that creates N LED pairs that are characterized by the least possible variance of their properties. Therefore, the question arises, what is the optimal configuration and how can we find it? The question could be solved by comparing the variance of each configuration and the configuration with the smallest variance is the optimal configuration. However, the number of possible N-pair configurations  $(2N)!/N!(2!)^N$  is too large for practical computation, particularly when N is large. For example, for N = 50, the number of possible configurations is  $2.73 \times 10^{78}$ . In order to solve the above question, the weight function  $w_{ii}$  for the pair consisting of the *i*th and *j*th LEDs is defined as

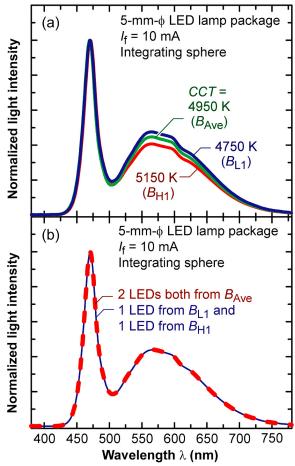
$$w_{ij}(p_1, p_2, ..., p_n) = \sqrt{\sum_{k=1}^{n} (p_k - p_{\text{target},k})^2},$$

where  $p_k$  and  $p_{\mathrm{target},k}$  is the kth property value and the kth target property value (e.g., CCT, light output power LOP, forward voltage  $V_{\mathrm{F}}$ , and forward current  $I_{\mathrm{F}}$ ), and n is the number of selection criteria. When considering three properties, e.g. CCT, LOP, and  $V_{\mathrm{F}}$ , then the number of selection criteria is n=3. Here the property  $p_k$  of the pair of ith and jth LEDs is calculated from the superposition of experimental data of each LED. It is notable that because an LED pair can have a serial connection or a parallel connection between two LEDs, the property of either  $V_{\mathrm{F}}$  or  $I_{\mathrm{F}}$  can be selected at once. For example, the property of  $I_{\mathrm{F}}$  under an assumption of a constant voltage can be calculated for the parallel connection of the LED pair, while the property of  $V_{\mathrm{F}}$  under an assumption of a constant current can be calculated for the serial connection of the LED pair.

The weight function represents the deviation of an LED pair from the target values. After having defined the weight function, we find the optimal configuration by identifying the one that minimizes the sum of weights. In other words, we are interested in the  $x_{ij}$  which minimize the sum of weights  $\sum_{i,j} w_{ij} x_{ij}$ . Here,  $x_{ij}$  represents an LED pair of the *i*th and *j*th LED;  $x_{ij} = 1$  if the *ij* pair is formed and  $x_{ij} = 0$  if the pair is not formed. Because  $x_{ii}$ , which means a self-combination, is not allowed to be formed, we force  $w_{ii} \to \infty$ . Besides,  $\sum_{i=1}^{2N} x_{ij} = 1$  for j = 1, 2, ... 2N to guarantee that one LED can be selected into an LED pair only once. By formulating this problem, we avoid to identify the optimal configuration from the huge number of possible configurations  $(2N)!/N!(2!)^N$ , which approximately equal to  $\sqrt{2}(2N/e)^N$ (from Stirling's approximation); instead, the number of possible pairs is N(2N-1), which approximately equal to  $2N^2$ and is much smaller than the number of possible configurations. The above problem is known as weighted matching in the field of computer science and can be solved efficiently by Edmond's algorithm. <sup>10</sup> Briefly speaking, this problem can be seen as a linear programming problem of finding the extreme value of a linear combination of weights and variables with certain constrains. In our case, the variable is  $x_{ij}$  representing the matching of two single LEDs and the constraint is that an LED can be selected only once into a pair. This algorithm gives the optimal configuration of N LED pairs which has the narrowest distribution closest to the chosen target value. For our case, it takes less than 30 sec of computation time to match 96 LEDs for two selection criteria. As another example, it takes less than 5 minutes of computation time to get the optimal matching result for 1000 LEDs on the Intel Core 2 Duo 2.4-GHz computer equipped with a 2-GB memory.

#### 3 Results and discussions

Although the algorithm can be used in either parallel or serial arrangement of an LED pair, we assume that the LED pair has a serial arrangement of two LEDs in this section to simplify proof-of-concept. For the experimental part of this work, we measure the spectral power distribution (SPD) of each LED in the integrating sphere at a DC current of 10 mA. Figure 1(a) shows the measured SPD of three indi-

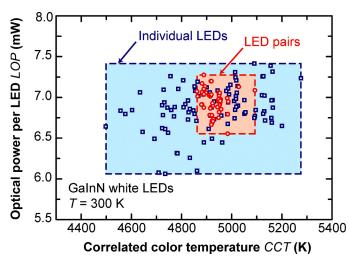


**FIGURE 1** — (a) Spectral power distributions of typical individual LEDs from the  $B_{H1}$ ,  $B_{ave}$ , and  $B_{L1}$  bins. (b) Spectral power distributions of two LED pairs from  $B_{ave} - B_{ave}$  and  $B_{H1} - B_{L1}$ .

vidual LEDs from different CCT bins, where  $B_{L1}$  ranges from 4700 to 4900 K,  $B_{ave}$  ranges from 4900 to 5100 K, and  $B_{H1}$  ranges from 5100 to 5300 K. From the measured SPDs of the LEDs, we can superpose and calculate the SPD of any LED pair. Figure 1(b) illustrates that a combined LED pair with complementary characteristics from  $B_{L1}$  and  $B_{H1}$  could have a very similar SPD of the LED from  $B_{ave}$ . Further, from the SPD of the LED pair we can determine the CCT and LOP. To minimize the variance of LED pairs with respect to correlated color temperature CCT and light output power LOP, we define a weight function  $w_{ij}$  of the pair of the ith and jth LED as follows:

$$w_{ij}(CCT, LOP) = \sqrt{\left(CCT - CCT_{\rm target}\right)^2 + \left(LOP - LOP_{\rm target}\right)^2}\,,$$

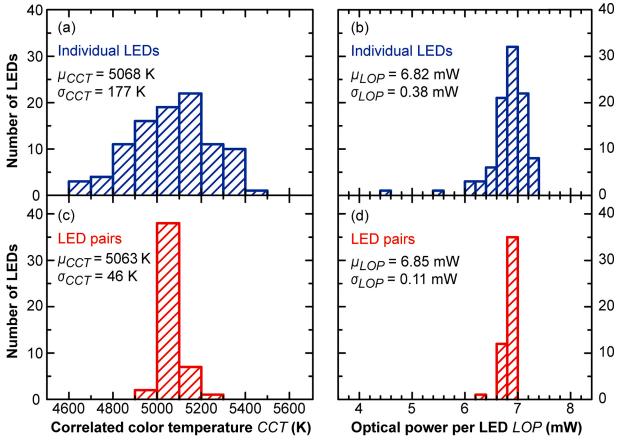
where  $CCT_{target}$  and  $LOP_{target}$  is the target value of CCT and LOP. After implementing the weighted matching method for 96 LEDs, we can obtain the optimal configuration. Distributions of the CCTs and the optical powers from the original individual LEDs are shown in the first row of Figs. 2(a) and 2(b). For the targeted values ( $CCT_{target}$ ,  $LOP_{target}$ ) = (5000 K, 7 mW), the calculated distributions of the CCTs and optical powers of the LED pairs are shown in the second row of Figs. 2(c) and 2(d), respectively. The average  $\mu$  and standard deviation  $\sigma$  are also shown in Fig. 2. By using 2N LEDs to form N LED pairs, the CCT standard deviation and optical power standard deviation changes from 177 to



**FIGURE 3** — Measured two-dimensional distribution chart of the CCT and the LOP for the individual LEDs (blue box) and the LED pairs (red box).

46 K and from 0.38 to 0.11 mW, respectively. It means that the variance of CCT and LOP is narrowed down to 7% and 8% of its original value, respectively. Therefore, by the weighted-matching algorithm, the distributions can be simultaneously narrowed by a very large amount.

Based on the weighted matching method, we combine 96 individual LEDs to form 48 LED pairs and measure the CCT and the LOP at the same current, 10 mA, used in the single-LED measurements. Figure 3 shows a two-dimen-



**FIGURE 2** — (a) The CCT and (b) the calculated LOP distributions from the individual LEDs. (c) The calculated CCT and (d) the LOP distributions for the LED pairs after optimal combination.

sional distribution chart for the CCT and LOP before and after the LED pair formation. The measured CCT and LOP standard deviations decrease from 165 to 43 K and 0.28 to 0.15 mW, respectively. In other words, the variances of measured CCT and LOP decrease by 93% and 71%, respectively.

## 4 Summary

In summary, the consistency in CCT, LOP, and V<sub>F</sub> is of particular importance for many LED applications. However, phosphor-based white LEDs have inherently a much wider variation of these properties compared to that for conventional white-light sources. An LED-matching method is presented to produce LEDs with narrow CCT and LOP distributions. This method is based on the combination of complementary LEDs, and a weighted matching algorithm is adopted to determine the optimal LED pairs that have smallest variance among all possible LED-pair configurations. The advantage of the method is to increase the number of usable LEDs restricted by certain criteria. We demonstrate experimentally that the CCT and LOP distributions of LEDs are decreased simultaneously by about one order of magnitude (93% and 71%) after formation of complementary LED pairs. The method presented here overcomes one of the greatest obstacles of solid-state lighting.

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