An Adaptive V-Grid Algorithm for Diffuse Optical Tomography

Murat Guven, Birsen Yazici
Drexel University
Electrical and Computer Engineering

Xavier Intes, Britton Chance
University of Pennsylvania
Department of Biophysics and Biochemistry

***Abstract*—In this work, we investigate an adaptive multigrid approach to improve the computational efficiency and the quantitative accuracy of DOT image reconstruction. The key idea is based on locally refined grid structure for region of interest (ROI). A Least Squares (LS) solution is formulated for the inverse problem. A Fast Adaptive Composite (FAC) V-Grid algorithm is employed to solve the inverse problem. Same problem is also solved using a fixed fine grid and FAC 2-Grid acheme for a 2-level locally refined grid. Our numerical studies demonstrate that the proposed FAC based adaptive V-Grid approach provides better image quality and up to 90% reduction in computational requirements as compared to the fixed grid and at least 10% reduction as compared to FAC 2-grid algorithms.**

I. Introduction

The propagation of light in the tissue is not restricted to a plane. This makes the diffuse optical image reconstruction a challenging 3D inverse problem with a relatively large number of unknowns and a limited number of measurements. Furthermore, the diffuse nature of light photons, the spatial resolution of DOT is poor. In this study, we propose to balance these two opposing issues resolution/accuracy versus computational complexity within an adaptive multigrid framework.

Standard multigrid methods have been applied to DOT image reconstruction problem recently [4], where the problem has been solved on a hierarchy of globally coarsened grids. However, one can prespecify a Region of Interest (ROI) that may contain diagnostically pertinent information by using geometric prior information. ROI may require a higher resolution level with respect to background. Adaptive multigrid differs from standard multigrid in the sense that a hierarchy of local and global grids is considered from the finest level to the coarsest level, which sequentially reduces the dimension of the problem during relaxation. Eventually the solution is determined on the nonuniform composite grid (Fig. 1), use of which allows us to effectively treat the resolution requirements of the ROI while improving the quantitative accuracy with reduced computations.

In this work, we focus to improve the computational efficiency and the quantitative accuracy of DOT by formulating a Fast Adaptive Composite (FAC) V-Grid algorithm, which is an extension of the previously formulated FAC 2-grid scheme [1]. An adaptively refined multilevel composite grid, which is based on the ROI information and corresponding spatial resolution requirements, is generated to provide higher resolution for ROI and sufficiently high resolution for the background (BG). A least squares solution is formulated for the 2D inverse problem and a conjugate gradient method is employed at the relaxation stage of the multigrid solver. The reconstruction results are presented to compare the quantitative accuracy and computational advantages of the proposed V-grid algorithm over fixed-grid and FAC 2-Grid algorithms.

II. Forward Model

Photon propagation in soft tissue can be adequately modeled by the diffusion equation. In this work, we focus on the reconstruction of absorption coefficients. Assuming a spatially constant diffusion coefficient D, we make use of the following frequency domain diffusion equation as the forward model:

$$-\frac{i\omega}{c}\phi(r) - D\,\nabla^2\,\phi(r) + \mu_a(r)\phi(r) = A\delta(r_s) \qquad (1)$$

$\phi(r)$ represents the spatially varying total field due to the point source $A\delta(r_s)$ located at $r = r_s$. ω denotes the frequency, c is the speed of light and $i = \sqrt{-1}$. D is the spatially invariant diffusion coefficient and $\mu_a(r)$ stands for the spatially varying absorption coefficient.

In this work, we have applied Rytov approach [3] using a first order approximation to express the forward problem in terms of system of linear equations in the form:

$$y = W \times x \qquad (2)$$

where y is the measurement vector holding the perturbative Rytov phase for each source-detector pair and W denotes the linear forward model which relates the differential absorption coefficient distribution x to the measurement vector y.

III. Inverse Problem

In this work we have formulated a least squares (LS) solution for the inverse problem:

$$\hat{x}_{LS} = arg\min_{x} \|y - Wx\|_2^2 \qquad (3)$$

where $\hat{x}_{LS}$ is the estimate of the unknown image x and $\|.\|_2$ denotes the $L_2 - norm$.

We have employed a Fast Adaptive Composite (FAC) V-grid algorithm to solve the resulting problem. A hierarchy of grids Ω^{4h}, Ω^{2h} and Ω^{h}_{ROI} is considered where Ω^{4h} and Ω^{2h} are the global coarse grids on the whole image with grid size $4h$ and $2h$ and Ω^{h}_{ROI} is the local fine grid on the ROI, which is a subset of the global fine grid Ω^h, with grid size h, combination of which leads to the 3-level composite grid Ω^c (Fig. 1). The LS formulation on the fine grid Ω^h can be

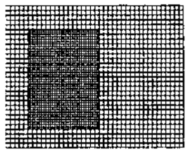

Fig. 1. The composite grid Ω^c designed for the inverse problem solution

equivalently expressed for the error on the coarse grid e^{2h} as

$$\begin{aligned} \|y - W^h x^h\|_2^2 &= \|y - W^h(\hat{x}^h + e^h)\|_2^2 \\ &= \|r - W^{2h} e^{2h}\|_2^2 \end{aligned} \quad (4)$$

where the residual $r = y - W^h \hat{x}^h$ and $W^{2h} = W^h I_{2h}^h$ and I_{2h}^h is the interpolation matrix. We have selected $I_{2h}^h = 4(I_h^{2h})^T$ where I_h^{2h} is the full weighting operator in 2D [2,5].

We have extended previously formulated FAC 2-grid scheme[1] for this multi-level problem by applying the 2-grid correction scheme as we move to the coarser grids, which leads to a V-grid structure [5].

Case 1: For this case, a solution is iteratively obtained on the fixed grid Ω^h. Fig. 3. shows the image reconstruction after 1200 iterations and the convergence rate.

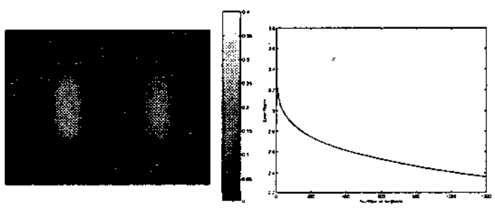

Fig. 2. The image reconstruction and the corresponding ROI error norm vs. iteration number plot for the fixed grid method.

Case 2: FAC 2-Grid algorithm is applied. Fig. 4 shows the significant enhancement in the ROI image quality and the corresponding convergence rate.

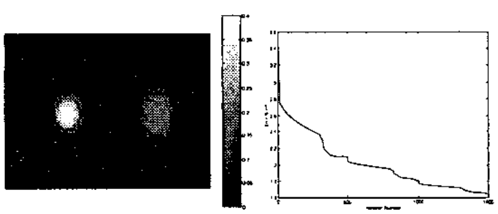

Fig. 3. The image reconstruction and the corresponding ROI error norm vs. iteration number plot for the FAC 2-Grid algorithm after 3 2-grid cycles.

Case 3: FAC V-Grid algorithm is applied where 3 resolution levels are considered. Fig. 4 shows the corresponding image reconstruction and the error norm vs. iteration number curve.

Finally in Fig. 5, the ROI error-norm vs. multiplication

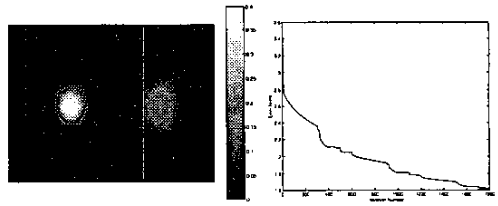

Fig. 4. The image reconstruction and the corresponding ROI error norm vs. iteration number plot for the proposed FAC V-Grid algorithm after 3 cycles.

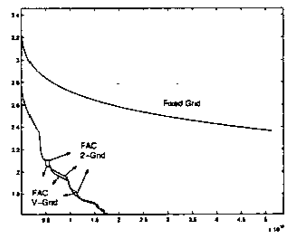

Fig. 5. Error norm vs. multiplication number plots for Case 1,2 and the proposed FAC 2-Grid algorithm.

number plots are presented for all algorithms. The plot clearly shows that the computational efficiency is greatly improved by the FAC algorithms. FAC V-grid has slightly improved performance over 2-grid.

IV. Conclusion

The proposed FAC based adaptive V-Grid approach provides significant reduction in computational requirements as compared to the FAC 2-Grid method while providing better image quality. Increasing the number of levels will eventually reduce the computational costs further and increase the image quality for both ROI and BG.

Acknowledgment

Xavier Intes and Britton Chance acknowledge partial support from NIH CA 87046.

References

[1] M. Guven, B. Yazici, X. Intes, B. Chance, "An Adaptive Multigrid Algorithm for Region of Interest Diffuse Optical Tomography," submitted to ICIP 2003.

[2] U. Trottenberg, C. Oosterlee, A. Schüller, "Multigrid," Academic Press, New York, 2001.

[3] M. A. O'Leary, D. A. Boas, B. Chance, and A. G. Yodh, "Experimental images of heterogeneous turbid media by frequency-domain diffusing-photon tomography," *Opt. Lett.* **20**, pp.426-428, 1995.

TABLE I

THE NUMBER OF MULTIPLICATIONS REQUIRED TO ACHIEVE THE SPECIFIED ERROR NORM FOR EACH ALGORITHM:

Error norm	2.6	2.1	1.62
Fixed grid soln.	18.5×10^9	Not achieved	Not achieved
FAC 2-Grid soln.	1×10^9	5.7×10^9	Not achieved
FAC V-Grid soln.	1×10^9	4.9×10^9	1.8×10^{10}