RECURSIVE LEAST SQUARES ALGORITHM FOR OPTICAL DIFFUSION TOMOGRAPHY

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***Abstract*- Algebraic reconstruction techniques (ART) is a family of practical algorithms which sets algebraic equations for the unknowns in terms of the measured data and solves these equations iteratively. It is typical that the system of linear equations obtained in Diffuse Optical Tomography (DOT) is underdetermined and/or ill-conditioned. ART is one of the most popular image reconstruction techniques used in DOT to solve this kind of system of linear equations. There is, however, no natural way of including *a priori* information about the image in ART algorithm. Moreover ART requires a large number of iterations to reconstruct the image and hence convergence to the solution is slow. In this paper, for the inverse problem in DOT, we apply a Recursive Least Squares Algorithm (RLS) that converges in only one iteration and enables the use of *a priori* information such as image smoothness.We present comparison between the images reconstructed by ART and RLS.**

Keywords- **Tomography; Inverse problem; Photon migration; Medical Imaging; Algebraic techniques.**

I. INTRODUCTION

Diffuse Optical Tomography (DOT) uses the low-energy of near infra-red (NIR) light to probe highly scattering media in order to derive qualitative or quantitative images of their optical properties. NIR tomography takes advantage of a 'therapeutic window' between 600 and 1000 nm in which tissues exhibit low absorbance, but high scattering characteristics. In this spectral range, the propagation of light is accurately modeled by the diffusion equation.[1]

DOT, similar to the other tomographic schemes, is divided into two parts: the forward problem and the inverse problem.[2] The solution of the forward problem, which is the diffusion equation, predicts the photon field expected at boundaries of the measured system. The inverse problem uses the appropriate forward solution to construct an operator that is applied to the measured data from an unknown medium to yield the internal optical composition of this medium.

It is typical that the system of equations obtained in DOT is underdetermined and/or ill-conditioned. ART, among the linear inversion techniques, provides a convenient way to solve the inverse problem in DOT. This method is best suited for projections that are sparse, noisy or non-uniformly distributed[3] and furthermore, it allows efficient processing of large inversion problems since it has minimum storage requirements and can be easily implemented with constraints such as object shape or non-negativity.

However, there are some disadvantages of applying ART for the inverse problem in DOT. One of those disadvantages is that ART does not allow a natural addition of *a priori* knowledge to the algorithm such as image positivity and smoothness. Furthermore a large number of iterations is required for the convergence to the solution.

In this paper for the inverse problem in Optical Tomography, we apply a RLS algorithm that converges to the minimum-norm solution in only one iteration and enables the use of *a priori* information such as image smoothness. We present comparison between the images reconstructed by ART and RLS algorithm respectively and discuss the pertinence of the results in a clinical context.

II. METHODS

A. Forward Model

In this paper we have employed analytical solutions of the heterogeneous diffusion equation using the first order Rytov approximation.[4] The Rytov approach writes the heterogeneous field as:

$$U(\vec{r},\vec{r}_s) = U_0(\vec{r},\vec{r}_s)e^{\Phi_{sc}(\vec{r},\vec{r}_s)} \tag{1}$$

where $U_0(\vec{r},\vec{r}_s)$ is the solution of the homogeneous equation and $\Phi_{sc}(\vec{r},\vec{r}_s)$ the diffuse Rytov phase.

In the case of DOT, multiple source-detector pairs are used. The medium of interest is sampled in voxels and the forward problem is expressed in terms of a system of linear equations:

$$\begin{bmatrix} \Phi_{sc}(r_{s1}, r_{d1}) \\ \vdots \\ \Phi_{sc}(r_{sm}, r_{dm}) \end{bmatrix} = \begin{bmatrix} W_{11} & \cdots & W_{1n} \\ \vdots & \ddots & \vdots \\ W_{m1} & \cdots & W_{mn} \end{bmatrix} \begin{bmatrix} \delta\mu_a(r_1) \\ \vdots \\ \delta\mu_a(r_n) \end{bmatrix} \tag{2}$$

where $\Phi_{sc}(r_{si}, r_{di})$ is the diffuse perturbative phase for the i^{th} source-detector pair; W_{ij} is the weight for the j^{th} voxel and the i^{th} source-detector pair and $\delta\mu_a(r_j)$ is the differential absorption coefficient of the j^{th} voxel. The relation above can be expressed equivalently as follows:

$$P = W \cdot F \qquad (3)$$

where P is a vector holding the measurements for each source-detector pair, W is the matrix of the forward model (weight matrix), and F is the vector of unknowns (object function).

B. Inversion using ART

ART solves a system of linear equations by projecting the *(i-1)*th solution estimate onto the i^{th} hyperplane defined by the i^{th} row of the weight matrix in order to obtain the i^{th} solution estimate for the next projection. The process can be mathematically described by:

$$\vec{f}_i = \vec{f}_{i-1} - \lambda \frac{(\vec{f}_{i-1} \cdot \vec{w}_i - p_i)}{\vec{w}_i \cdot \vec{w}_i} \vec{w} \qquad (4)$$

where p_i is the i^{th} measurement, $\vec{f}_i$ is the i^{th} solution estimate, $w_i = (w_{i1}, w_{i2}, \ldots, w_{in})$ and $\vec{w}_i \cdot \vec{w}_i$ is the dot product of w_i with itself. λ is the relaxation parameter and it is set to $\lambda = 0.1$ for this study.

C. Inversion using Least Squares Algorithm

The same set of linear equations (1) can be solved using a RLS algorithm.[5] This algorithm describes the iterative approach mathematically as follows:

$$f_i = f_{i-1} + \lambda_i (p_i - w_i f_{i-1}) P_{i-1} w \qquad (5)$$

where

$$P_i = P_{i-1} - \lambda_i P_{i-1} w_i^T w_i P_{i-1} \qquad (6)$$

and

$$\lambda_i = \frac{1}{w_i P_{i-1} w_i^T + \sigma_i^2} \qquad (7)$$

where p_i is the i^{th} measurement, f_i is the i^{th} solution estimate, w_i is the i^{th} row of the weight matrix and σ_i is the standard deviation of the noise at the i^{th} measurement.

Choosing P_i and λ_i according to equation (5) enables the solution f_i, given by (4), to minimize the quadratic cost function for every i:

$$C_i(f) = \sum_{m=1}^{i} \frac{1}{\sigma_m^2} (w_m f - p_m)^2 + (f - f_0) P_0^{-1} (f - f_0) \qquad (8)$$

where f_0 and P_0 are the a priori image mean and covariance and σ_i^2 is the noise variance at the i^{th} data point.

III. Results and Discussion

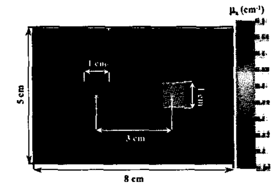

Figure 1. Model used for the simulations

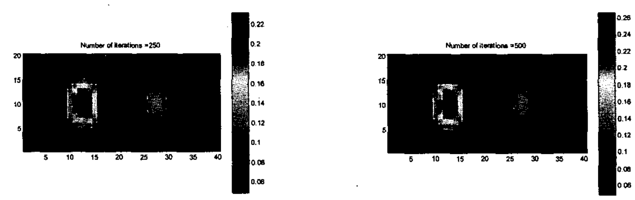

Figure 2. Reconstruction results by ART

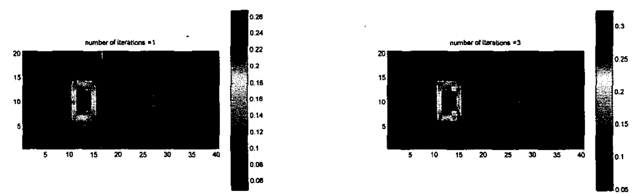

Figure 3. Reconstruction results by RLS Algorithm

We give reconstruction results obtained by ART and RLS algorithms respectively. The measurements in this study were obtained by solving the frequency-domain diffusion equation with a finite difference approach. The simulations are restricted to 2D geometry for computational efficiency.

The configuration used in this study is shown in Figure 1. Figure 2 shows the reconstructed images using ART after 250 and 500 iterations respectively. Figure 3 shows the reconstructed images using RLS Algorithm after 1 and 3 iterations respectively.

We observe that the RLS algorithm achieves better reconstruction with much fewer iterations.

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