COMBINED L_2-L_1-NORM REGULARIZATION IN FLUORESCENCE DIFFUSE OPTICAL TOMOGRAPHY

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ABSTRACT

The inverse problem of fluorescence diffusse optical tomography (FDOT) is often highly ill-posed, which needs regularization techniques. In this paper, we propose a combined l_1-l_2-norm regularization method to address the ill-posed FDOT inverse problem. Compared with the traditional Tikhonov regularization, the proposed method is able to effectively remove the noise in the reconstructed image without much over-smoothness. The performance of the proposed method is demonstrated in 3D numerical simulation.

Index Terms— optical imaging, reconstruction

1. INTRODUCTION

The inverse problem of fluorescence diffusse optical tomography (FDOT) requires reconstruction of 2D or 3D fluorophore map inside the imaging domain using boundary measurements obtained at the emission and excitation wavelengths [1]. However, the number of measurements available is usually insufficient. Furthermore, the solution is not stable, i.e., a small perturbation in the measurement data can result in large deviations in the solution [1]. Thus, regularization techniques are usually necessary.

In the FDOT, the fluorophore yield, which is proportional to the fluorophore concentration, often has large value in a small foreground region, and is close to zero in the background [2]. The most widely used Tikhonov regularization with a quadratic regularization function imposes heavy penalty on large argument values, which often leads to over-smoothing in the foreground region [3]. A regularization function that increases less rapidly for large argument values than quadratic function works better to preserve the foreground region [2, 4]. However, such penalty function might be insufficient in smoothing out large spikes of noise, which is possible to exit in the reconstructed image due to the ill-posedness of the forward matrix and the measurement noise. In this work, we propose a new regularization function that combines both the quadratic regularization and the l_1-norm regularization, which we refer to as the combined l_2-l_1-norm regularization in the following discussion. The proposed method is capable of preserving large argument values in the foreground region while effectively removing noise in the reconstructed image. We demonstrate the performance of the proposed method by 3D numerical simulation.

2. FDOT IMAGING PROBLEM

The FDOT inverse problem involves reconstructing unknown fluorophore yield, which is proportional to the fluorophore concentration, from the boundary measurements. Given N_s sources and N_d detectors, the measurement $\Gamma_{i,j}$ of the jth detector due to ith source is given as follows [1],

$$\Gamma_{i,j} = \int_\Omega g_j^*(\boldsymbol{r})\phi_i(\boldsymbol{r})\mu(\boldsymbol{r})d\boldsymbol{r}, \qquad (1)$$

where Ω is the imaging domain, $\phi_i(\boldsymbol{r})$ is the excitation light field due to the ith source, $g_j^*(\boldsymbol{r})$ is the Greens function of the jth detector, and $\mu(\boldsymbol{r})$ is the fluorophore yield. We discretize the domain into N voxels, and obtain the discretized form,

$$\boldsymbol{\Gamma} = \boldsymbol{A}\boldsymbol{\mu}, \qquad (2)$$

where $\boldsymbol{y} \in \mathbb{R}^M$ ($M = N_s \times N_d$) is the measurement vector, $\boldsymbol{A} \in \mathbb{R}^{M\times N}$ is the vector-valued forward operator, and $\boldsymbol{\mu} \in \mathbb{R}^N$ is the discretized fluorophore yield.

Regularization techniques are often applied to solve for $\boldsymbol{\mu}$, which trades off the quadratic error of the measurement with a regularization function $R(\boldsymbol{\mu})$,

$$\hat{\boldsymbol{\mu}} = \arg\min_{\boldsymbol{\mu}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{\mu}\|^2 + \lambda R(\boldsymbol{\mu}), \qquad (3)$$

where λ is the regularization parameter. If $R(\boldsymbol{\mu})$ is continuous and differentiable, (3) can be easily solved by gradient based method, such as the nonlinear conjugate gradient method [5].

3. THE COMBINED L_2-L_1-NORM REGULARIZATION

In this section, we propose a combined l_2-l_1-norm regularization. Assume that the amplitude of $\boldsymbol{\mu}$ in the foreground region are approximately in a known range $[\alpha,\ \beta]$, which can be determined from empirical values. The combined l_2-l_1-norm regularization imposes small penalty when $\alpha \leq \mu_i \leq \beta$, and

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large penalty when $\mu_i \notin [\alpha,\ \beta]$. The regularization function $R(\boldsymbol{\mu})$ is the summation of the cost functions of each voxel,

$$R(\boldsymbol{\mu}) = \sum_{i=1}^{N} r(\mu_k), \tag{4}$$

where $r(\mu_k)$ is the cost function of the μ_k. $r(\mu_k)$ has the following properties: (i) $r(\mu_k)$ is continuous and differentiable, $\forall \mu_k \in \mathbb{R}$, (ii) $r(\mu_k)$ has large quadratic penalty when $\mu_k \notin [\alpha, \beta]$, (iii) $r(\mu_k)$ has l_1-norm penalty when $\mu_k \in [\alpha, \beta]$. Based on the properties listed above, $r(\mu_k)$ is given by

$$r(\mu_k) = \begin{cases} \frac{a\mu_k^2}{2\epsilon}, & |\mu_k| \le \epsilon; \\ a|\mu_k| - \frac{a\epsilon}{2}, & \epsilon \le |\mu_k| \le \epsilon'; \\ c_2\mu_k^2 + c_1|\mu_k| + c_0, & |\mu_k| > \epsilon', \end{cases} \tag{5}$$

where ϵ and ϵ' are two constants satisfying $0 < \epsilon < \alpha < \epsilon' < \beta$. a, c_0, c_1 and c_2 are constant parameters chosen such that $R(\boldsymbol{\mu})$ is continuous and differentiable,

$$a = (2\lambda\alpha^2)/(2\alpha - \epsilon), \tag{6}$$

$$c_2 = (\lambda\beta^2 - a\beta + \frac{a\epsilon}{2})/(\beta - \epsilon')^2, \tag{7}$$

$$c_1 = a - 2\epsilon' c_2, \tag{8}$$

$$c_0 = c_2\epsilon'^2 - \frac{a\epsilon}{2}. \tag{9}$$

4. NUMERICAL SIMULATION RESULTS

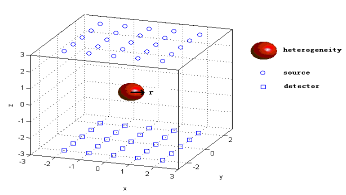
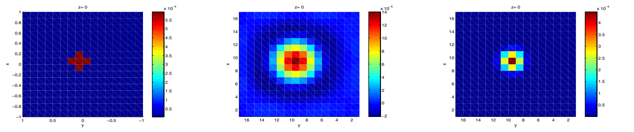


Fig. 1. Phantom configuration.

In this section, we present 3D numerical simulation results. The simulated imaging domain is a $6\times\ 6\ \times\ 6$ cm^3 cubic region, with sources and detectors uniformly distributed, as shown in Fig.1. The measurements are corrupted by i.i.d Gaussian noise with $30dB$ *signal to noise ration* (SNR). The fluorophore is concentrated in the center of radius 0.5cm.

Fig.2(a) shows the cross section of the original phantom. Reconstruction results of the Tikhonov regularization is shown in Fig.2(b) with severer over-smoothing. Fig.2(c) shows the reconstruction result of the proposed combined l_1-l_2-norm regularization. Fig.2(c) has much less over-smoothness and clear foreground region.

The *mean square error* (MSE) between the reconstructed fluorophore map and the original phantom is shown in Fig.3. The combined l_2-l_1-norm regularization has a much lower MSE compared to the Tikhonov regularization at different SNR levels. The MSE analysis matches visual results.



(a) Phantom (b) Tikhonov (c) Combined l_2-l_1-norm

Fig. 2. Cross section of the original phantom and the reconstruction results.

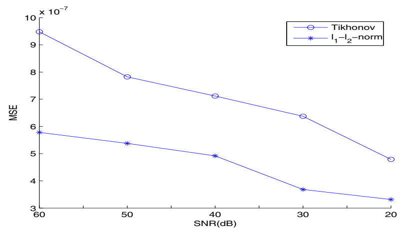


Fig. 3. MSE-SNR plot of the reconstruction results.

5. CONCLUSION

In this work, we propose a combine l_1-l_2-norm regularization technique to address the ill-posed FDOT inverse problem. Numerical results show that the proposed method is able to preserve the fluorophore region, and effectively remove large spike of noise without much over-smoothing.

6. REFERENCES

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