Passive Imaging of A Non-Cooperative Moving RF Emitter using Sparsely Distributed Receivers

Ling Wang

College of Electronic and Information Engineering
Nanjing University of Aeronautics and Astronautics
Nanjing 210016, China
Email: wanglrpi@gmail.com

Birsen Yazıcı**

Department of Electrical, Computer and Systems Engineering,
Rensselaer Polytechnic Institute,
110 8th Street, Troy, NY 12180 USA
Email: yazici@ecse.rpi.edu

Abstract—The localization and tracking of radio frequency (RF) emitters are important in both civilian and military applications, such as in search and rescue operations and in tracking mobile threats employed by dismounts. In this paper, we consider passive imaging of a non-cooperative moving RF emitter in a multiple-scattering environment, such as an urban area, using an array of sparsely distributed receivers. We develop a method that exploits multiple-scattering to estimate the location and velocity of the emitter. We present numerical simulations to demonstrate the performance of our method.

I. INTRODUCTION

The RF emitter imaging problem [1]–[3] falls into the well-studied area of source localization with a wide range of applications in geophysics, acoustics, wireless communication networks or cognitive radio systems. Some of the recent work in the area can be found in [4]–[9].

Majority of these work with the exception of [1]–[4], address the localization of stationary emitters in free-space environment. In this paper, we present a novel method for passive imaging of a moving RF emitter in multiple-scattering environments using sparsely distributed receivers. To the best of our knowledge, our method is the first in the literature that exploits multiple-scattering for imaging of moving RF emitters.

We use the framework that we introduced in [10], [11] and derive a measurement model which relates the Doppler as well as delay information measured at a receiver location to the delay and Doppler information measured at other receiver locations. We formulate the imaging problem as a Generalized Likelihood Ratio Test (GLRT) with unknown emitter location and unknown velocity. The GLRT is a particularly suitable framework for the sparse arrays due to the limited aperture provided by such arrays. We present numerical simulations demonstrating the performance of our method.

While we mainly focus on the radar application, the results presented in our paper are also applicable to other source localization problems, such as those that arise in wireless systems, acoustics, geophysics and seismology.

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** Corresponding author

II. MEASUREMENT MODEL

We reserve $\mathbf{x}$ to denote location in 3D Euclidean space and $\mathbf{x}$ to denote location in 2D space. We denote operators ($\mathcal{G}, \mathcal{R}$, etc.) with calligraphic letters. For a function f , $\hat{f}$ denotes its Fourier transform and f^* denotes its complex conjugate. Bold fonts denote vector quantities. Non-bold italic fonts denote scalar quantities.

The propagation of electromagnetic waves due to an arbitrary source distribution $s(\mathbf{x}, t)$ in a medium can be described using the scalar wave equation:

$$[\nabla^2 - c^{-2}(\mathbf{x}, t)\partial_t^2]E(\mathbf{x}, t) = s(\mathbf{x}, t) \quad (1)$$

where c is the wave speed in the medium and E is the electric field. The propagation medium is characterized by the *Green's function* satisfying

$$[\nabla^2 - c_0^{-2}\partial_t^2]g(\mathbf{x}, \mathbf{z}, t) = \delta(\mathbf{x} - \mathbf{z})\delta(t) \quad (2)$$

where c_0 is the speed of light in free space.

Let E^{in} denote the incident field due to the source distribution $s(\mathbf{x}, t)$. Using (1) and (2), the incident field is modeled as

$$E^{\text{in}}(\mathbf{y}, t) = \int g(\mathbf{y}, \mathbf{z}, t - \tau)s(\mathbf{z}, \tau)d\tau d\mathbf{z}. \quad (3)$$

We assume that there is a single RF emitter with isotropic antenna present in the scene of interest. Thus, we express the source distribution as

$$s(\mathbf{x}, t) = \delta(\mathbf{x} - \mathbf{z} - \mathbf{v}t)p(t) \quad (4)$$

where $\mathbf{z}$ denotes the location of the emitter at time $t = 0$, which moves with velocity $\mathbf{v}$ and transmits the waveform $p(t)$. Then, (3) can be expressed as

$$E^{\text{in}}(\mathbf{y}, t) = \int g(\mathbf{y}, \mathbf{z} + \mathbf{v}\tau, t - \tau)p(\tau)d\tau. \quad (5)$$

We consider a sparse distribution of N receivers located at $\mathbf{x}_i, i = 1, \dots, N$. Considering the field only due to the emitter (referred to as “direct signals” in passive detection and imaging), the measurement at the i^{th} receiver can be modeled as

$$m_i(t) = E^{\text{in}}(\mathbf{x}_i, t) + n(t) \quad (6)$$

$$= \int g(\mathbf{x}_i, \mathbf{z} + \mathbf{v}\tau, t - \tau)p(\tau)d\tau + n(t), \quad (7)$$

where $n(t)$ is the receiver thermal noise.

The experimental radar studies conducted in urban environments have shown that the multiple scattering responses are primarily due to reflections from the specular region in the background environment, such as a wall or a light pole, etc., [12]. In this case, the Green's function of the background can be approximated by the shoot-and-bounce model [13], which is considered to be a sufficient model for the multiple-scattering in urban areas [14], [15]:

$$\hat{g}(\mathbf{x}, \mathbf{y}, \omega) = \frac{e^{-ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} + \sum_{l=1}^L a_l \frac{e^{-ik|\mathbf{x}^l-\mathbf{y}|}}{4\pi|\mathbf{x}^l-\mathbf{y}|} \quad (8)$$

$$= \sum_{l=0}^L a_l \frac{e^{-ik|\mathbf{x}^l-\mathbf{y}|}}{4\pi|\mathbf{x}^l-\mathbf{y}|}, \quad a_0 = 1, \mathbf{x}^0 = \mathbf{x} \quad (9)$$

where $k = \omega/c_0$, $\mathbf{x}^l$ is reflection of $\mathbf{x}$ about the mirror plane, $\mathcal{A}_l$ with respect to the l th, ($l = 1, \dots, L$) multipath bounce and a_l are the corresponding attenuation coefficients. We define the receivers located at $\mathbf{x}^l$, ($l = 1, \dots, L$) as *mirror receivers*. (9) can be further expressed as

$$\hat{g}(\mathbf{x}, \mathbf{y}, \omega) = \sum_{l=0}^L a_l \hat{g}_0(\mathbf{x}^l, \mathbf{y}, \omega) \quad (10)$$

where $\hat{g}_0$ denotes the Fourier transform of the free-space Green's function given by

$$\hat{g}_0(\mathbf{x}^l, \mathbf{y}, \omega) = \frac{e^{-ik|\mathbf{x}^l-\mathbf{y}|}}{4\pi|\mathbf{x}^l-\mathbf{y}|}. \quad (11)$$

Using (10), the measurement in Fourier domain is given as follows:

$$\hat{m}_i(\omega) = \sum_{l=0}^L a_l \hat{g}_0(\mathbf{x}_i^l, \mathbf{z}, \omega) \hat{p}(\mu_{\mathbf{z}, \mathbf{v}, \mathbf{x}_i^l} \omega) + \hat{n}_i(\omega) \quad (12)$$

where the scale factor $\mu_{\mathbf{z}, \mathbf{v}, \mathbf{x}_i^l}$ is given by

$$\mu_{\mathbf{z}, \mathbf{v}, \mathbf{x}_i^l} = 1 - \frac{\widehat{\mathbf{x}_i^l - \mathbf{z} \cdot \mathbf{v}}}{c_0}, \quad (13)$$

which denotes the Doppler-scale-factor observed at location $\mathbf{x}_i$ due to a moving emitter with velocity $\mathbf{v}$ at location $\mathbf{z}$ associated with the l th multipath propagation.

We are interested in determining the position and velocity of the transmitting source using the measurements, $m_i(t)$, $i = 1, \dots, N$. We assume that the emitter related information including the transmitted waveform and its center frequency, is completely unknown. We develop a passive measurement model that does not explicitly depend on the emitter related terms. The model is based on the fact that the measurements at all receivers are due to the same source distribution. The model involves back propagating the measurement at a receiver location to a hypothetical emitter location moving at a hypothetical velocity and then forward propagating the resulting signal, i.e., the source signal, to another receiver location.

Using (12), we define the forward-propagation operator, $\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}$, with respect to the i th receiver as follows:

$$\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}[\hat{p}](\omega) = \sum_{l=0}^L a_l \mathcal{G}_{\mathbf{z}, i}^l \mathcal{S}_{\mathbf{v}, i}^l[\hat{p}](\omega) \quad (14)$$

where $\mathcal{S}_{\mathbf{v}, i}^l$ is the scaling operator associated with the l th, $l = 1, \dots, L$ multipath bounce and the direct-path corresponding to $l = 0$ defined as

$$\mathcal{S}_{\mathbf{v}, i}^l[\hat{p}](\omega) = \hat{p}(\mu_{\mathbf{z}, \mathbf{v}, \mathbf{x}_i^l} \omega) \quad (15)$$

and $\mathcal{G}_{\mathbf{z}, i}^l$ is the operator that accounts for the wave propagation in the stationary background from the emitter to the i th receiver via the l th, $l = 1, \dots, L$ multipath bounce and the direct-path corresponding to $l = 0$ defined as

$$\mathcal{G}_{\mathbf{z}, i}^l[\hat{p}](\omega) = \hat{g}_0(\mathbf{x}_i^l, \mathbf{z}, \omega) \hat{p}(\omega). \quad (16)$$

where $\hat{g}_0$ is the Fourier transform of the free-space Green's function as described in (11).

We define the back-propagation operator as the adjoint of $\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}$ and denote it with $\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}^\dagger$. Using (14), $\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}^\dagger$ is expressed as

$$\mathcal{P}_{\mathbf{z}, \mathbf{v}, i}^\dagger = \sum_{l=0}^L a_l^* \mathcal{S}_{\mathbf{v}, i}^{l, \dagger} \mathcal{G}_{\mathbf{z}, i}^{l, \dagger}, \quad (17)$$

where $\mathcal{S}_{\mathbf{v}, i}^{l, \dagger}$ and $\mathcal{G}_{\mathbf{z}, i}^{l, \dagger}$ are the adjoint of $\mathcal{S}_{\mathbf{v}, i}^l$ and $\mathcal{G}_{\mathbf{z}, i}^l$, respectively.

We can now express the measurement, $\hat{m}_i$, at the i th receiver in terms of the measurement, $\hat{m}_j$ at the j th receiver using the following model:

$$\hat{m}_i(\omega) = \mathcal{P}_{\mathbf{z}, \mathbf{v}, i} \mathcal{P}_{\mathbf{z}, \mathbf{v}, j}^\dagger \hat{m}_j(\omega) + \hat{n}_i(\omega). \quad (18)$$

Note that in the back-propagation operation, the noise at the j th receiver is back-propagated along with the noise-free measurement.

Assuming the j th receiver as a reference, we form the vectorized passive measurement model for N receivers as follows:

$$\mathbf{m}(\omega) = \mathbf{P}_{\mathbf{z}, \mathbf{v}} \mathbf{m}_r(\omega) + \mathbf{n}(\omega) \quad (19)$$

where $\mathbf{m} = [\hat{m}_1 \ \hat{m}_2 \ \dots \ \hat{m}_N]^T$, $\mathbf{m}_r = [\hat{m}_j \ \hat{m}_j \ \dots \ \hat{m}_j]^T$, $\mathbf{n} = [\hat{n}_1 \ \hat{n}_2 \ \dots \ \hat{n}_N]^T$, and $\mathbf{P}_{\mathbf{z}, \mathbf{v}} = \text{diag}[\mathcal{P}_{\mathbf{z}, \mathbf{v}, 1} \mathcal{P}_{\mathbf{z}, \mathbf{v}, j}^\dagger \mathcal{P}_{\mathbf{z}, \mathbf{v}, 2} \mathcal{P}_{\mathbf{z}, \mathbf{v}, j}^\dagger \dots \mathcal{P}_{\mathbf{z}, \mathbf{v}, N} \mathcal{P}_{\mathbf{z}, \mathbf{v}, j}^\dagger]$ with $i = 1, \dots, N$ and $i \neq j$. Note that $\mathbf{P}_{\mathbf{z}, \mathbf{v}}$ is $(N-1) \times (N-1)$ and all the operations in (19) are understood to be elementwise.

III. IMAGE FORMATION

Considering the sparsity of the aperture formed by the distributed stationary receivers, we formulate the image formation problem as a binary hypothesis testing problem in the context of Generalized Likelihood Ratio Testing (GLRT) with unknown emitter location and velocity [16]. The GLRT formulation generates a test statistic for each possible target location and velocity and we use this test statistic as the pixel value to form an image in position and velocity spaces.

We define the position- and velocity-resolved binary hypothesis test as follows:

$$\begin{aligned}\mathcal{H}_0: & \mathbf{m} = \mathbf{n} \\ \mathcal{H}_1: & \mathbf{m} = \mathbf{P}_{\mathbf{z}, \mathbf{v}} \mathbf{m}_r + \mathbf{n}\end{aligned}\quad (20)$$

where $\mathbf{P}_{\mathbf{z}, \mathbf{v}}$, $\mathbf{m}_r$, $\mathbf{m}$ and $\mathbf{n}$ are as defined above. The null hypothesis states that the measurement is due to noise, whereas the alternative hypothesis states that the measurement is due to an RF emitter located at $\mathbf{z}$ moving with velocity $\mathbf{v}$. Note that hypothesis $\mathcal{H}_1$ in (20) is for a particular location and velocity.

We design a linear discriminant functional to solve the binary hypothesis testing problem:

$$\lambda = \langle \mathbf{m}, \mathbf{w} \rangle := \int \mathbf{w}^H \mathbf{m} d\omega = \sum_{i \neq j} w_i^* (\omega) \hat{m}_i(\omega) d\omega \quad (21)$$

where λ denote the the output of the discriminant functional, which we call the *test-statistic* and $\mathbf{w}$ is a template given by $\mathbf{w} = [w_1 \ w_2 \ \dots \ w_N]^T$.

We determine the test-statistic by maximizing the Signal-to-Noise Ratio (SNR) of the test-statistic. The image is then formed in $(\mathbf{z}, \mathbf{v})$ domain by the position- and velocity-resolved test-statistic where the location and the velocity can be identified by thresholding the resulting image.

IV. NUMERICAL SIMULATIONS

A. Settings

We considered a scene of size $[0, 3\text{e}3] \times [0, 3\text{e}3] \text{ m}^2$ with flat topography. We discretized the scene into 201×201 pixels where $[0, 0, 0] \text{ m}$ and $[3\text{e}3, 3\text{e}3, 0] \text{ m}$ correspond to the pixels $(1, 1)$ and $(201, 201)$, respectively. We assumed that the emitter velocity was in the range of $[-20, 20] \times [-20, 20] \text{ m/s}$. We discretized the velocity plane into 401×401 pixels where $[-20, -20, 0] \text{ m}$ and $[20, 20, 0] \text{ m}$ correspond to the pixels $(1, 1)$ and $(401, 401)$, respectively.

Both the receivers and emitters were assumed to be on the same plane, $z_3 = 0$. A moving emitters with unit reflectivity was assumed to be located at $[2.5\text{e}3 \ 2\text{e}3 \ 0]^T \text{ m}$ moving with velocity $[5 \ 5 \ 0] \text{ m/s}$. We assumed that there were ten receivers distributed around the scene. We considered a specular reflecting wall located at $z_1 = 0$. Thus, the multiple-scattering environment was modeled with the shoot-and-bounce model with one extra path.

In all the experiments, the noise was simulated as the additive white Gaussian process.

We perform the image reconstruction using a waveform that has high Doppler resolution. Practical examples for such waveforms include Frequency Modulated (FM) radio and television signals, waveforms used by Global System for Mobile Communication (GSM), wireless network (or WiFi) signals, etc. We simulated a high-Doppler resolution waveform as a continuous-wave (CW) with 4GHz carrier frequency, 0.1s duration. Such a waveform provides about 0.375 m/s radial velocity resolution in monostatic operations.

We reconstructed the four-dimensional test-statistic image in $(\mathbf{z}, \mathbf{v})$ (or (z_1, z_2, v_1, v_2)) coordinates. However, in order to facilitate visualization and performance evaluation, we

generated three two-dimensional images from the original four dimensional image: The first image, which we refer to as the peak-value image, was generated by choosing the maximum value of the four-dimensional image for each velocity (v_1, v_2) . We then, choose the maximum value, $\tilde{v}$, of the peak-value image as the estimate of the velocity. The second image, which we refer to as the position-image, is the cross-section of the four-dimensional image, for $\mathbf{v} = \tilde{\mathbf{v}}$. We take the maximum value, $\tilde{z}$, of the position-image as the estimate of the emitter's position. The third image, which we refer to as the velocity-image, is the cross-section of the four-dimensional image, for $\mathbf{z} = \tilde{\mathbf{z}}$.

B. Results

Fig. 1 presents the peak-value image. We see that there is a peak located at $[5, 5, 0] \text{ m/s}$ in the image, which corresponds to the moving emitter.

Fig. 1. The peak-value image with the largest value marked by the circle, corresponding to the velocity estimate: $[5, 5, 0] \text{ m/s}$.

Fig. 2(a) shows the position image reconstructed using the estimated velocity, $[5, 5, 0] \text{ m/s}$. We see that the location where the maximum value of the position image is located, i.e., $\tilde{\mathbf{z}} = [2490, 1995] \text{ m}$, provides a good estimation of the emitter location, as marked by the black circle in Fig. 2(a). Note that the estimation accuracy is limited by the discretization step of the reconstructed scene. Fig. 2(b) shows the reconstructed velocity image for $\tilde{\mathbf{z}} = [2490, 1995] \text{ m}$. The results show that our method is capable of reconstructing moving emitters in multiple-scattering environments.

Fig. 3(a) and Fig. 3(b) show the reconstructed position and velocity images of the RF emitter located at $\mathbf{z} = [2500, 2000, 0] \text{ m}$ moving with velocity $[5, 5, 0] \text{ m/s}$ in free-space. Comparing Fig. 2(a) and Fig. 2(b) with Fig. 3(a) and Fig. 3(b), we find that the strength of the reconstructed emitter increases when we exploited the multipath effect. However, there are also additional artifacts present in the background of the reconstructed images, which degrades the contrast of the images.

V. CONCLUSION

We presented a method for imaging of moving RF emitters in multiple-scattering environments using sparsely distributed

Fig. 2. The reconstructed position and velocity images. (a) The position image when $\mathbf{v} = [5, 5]$ m/s. The estimated position $\hat{\mathbf{z}} = (2490, 1995)$ m. (b) The velocity image when $\hat{\mathbf{z}} = [2490, 1995]$ m/s. The circles indicate the estimated position and velocity.

Fig. 3. The reconstructed position image and velocity image of the emitter located at $\mathbf{z} = (2500, 2000)$ m moving with velocity $\mathbf{v} = [5, 5]$ m/s in free-space. (a) The position image when $\mathbf{v} = [5, 5]$ m/s. (b) The velocity image when $\mathbf{z} = [2500, 2000]$ m/s. The circles mark the estimated position and velocity.

stationary receivers. Our method is capable of exploiting the multiple-scattering. We developed a measurement model that relates the measurements at a given receiver to measurements at other receivers. We formed the image in position and velocity spaces using the GLRT framework. We demonstrated the performance of our method in numerical simulations.

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