Solutions to Homework Set 5

Homework Problems

1. For this question, depending on how they have interpreted it, there can be two answers.

(a)

$$(1-p)^2 p \qquad\qquad (1-p)^{i-1} p$$

OR

$$(1-p)^3 p \qquad\qquad (1-p)^{i} p$$

(10 pts)

(b)

$$(1-p)^2 p \qquad\qquad \textbf{OR} \qquad\qquad (1-p)^3 p$$

(10 pts)

(c)

Yes the process is memoryless. Let

$$A = \text{prob. } i \text{ more tosses till first head}$$

$$B = \text{prob. } j \text{ tosses without a head}$$

Then

$$\begin{aligned} P[A \mid B] &= \frac{P[A,B]}{P[B]} \\ &= \frac{(1-p)^{i+j-1}p}{(1-p)^j} \\ &= (1-p)^{i-1}p \\ &= P[i \text{ tosses till first head}] \end{aligned}$$

(10 pts)

2.

$$\lambda = 25 pkts/sec$$

$$\mu = \frac{C}{L} = 28.8 pkts/sec$$

$$\rho = \frac{\lambda}{\mu} = .868$$

$$P_N = \frac{(1-\rho)\rho^N}{1-\rho^{N+1}}$$

Solving for N by trial and error so that we get $P_N < 10^{-5}$, we get $N = 68$. ($P_{67} = 1.007 \times 10^{-5}$ and $P_{68} = 0.874 \times 10^{-5}$

(20 pts)

3.

We use the P-K formula to obtain $E[n]$ and then use Little's formula to get $E[T]$. From the P-K formula

$$E[n] = \left(\frac{\rho}{1-\rho}\right)\left[1 - \frac{\rho}{2}(1-\mu^2\sigma^2)\right]$$

From Little's formula

$$E[T] = \frac{E[n]}{\lambda}$$

For our system

$$\lambda = 750 pkts/sec$$

$$\mu = \frac{C}{L} = 1500 pkts/sec$$

$$\rho = \frac{\lambda}{\mu} = .5$$

(a) ($\sigma^2 = 0$)

$$E[T] = 1 msec$$

(7.5 pts)

(b) $\sigma^2 = \frac{1}{\mu^2} = 4.44 \times 10^{-7}$

$$E[T] = 1.33 msec$$

(10 pts)

(c) $\sigma^2 = 16 \times 10^{-6}$.

$$E[T] = 13msec$$

(7.5 pts)

4.

(a)

11001001011

Shift the original message 11001001 to the left to get 11001001000. Divide it by 1001 and remember that we are only doing XOR operations rather than pure division. The remainder of 011 is added to 11001001000 to get the final message of 11001001011.

(15pts)

(b) The receiver gets 01001001011 since the leftmost bit is inverted. The receiver divides (does XOR) it by 1001 and we get a remainder of 10. Since we get a non zero remainder at the receiver, an error is detected.

(10 pts)

Additional questions for 600 level students

1.

(a) Using the notation $\rho = \frac{\lambda}{\mu}$

$$p_0 = \frac{(1-\rho)(1-2\rho)}{1-2\rho+\rho^{K+2}}$$

and

$$p_k = \begin{cases} \rho^k p_0 & 0 \le k \le K+1 \\ (2\rho)^k \frac{1}{2^{K+1}} p_0 & k > K+1 \end{cases}$$

(25 pts)

(b) $2\lambda < \mu$

(5 pts)