Probability Related Problems

1. A fair die is tossed twice (a die is said to be fair if all outcomes $1, \cdots, 6$ are equally likely). Given that a 3 appears on the first toss, what is the probability of obtaining the sum 7 in two tosses?

2. A class of CCN students is taking a multiple choice test. For any question on the test, the fraction of examinees who know the answer is p; $1 - p$ is the fraction that guess. The probability of answering a question correctly is 1 for an examinee who knows the answer and $1/m$ for a guessee; m is the number of multiple choice alternatives. Compute the probability that an examinee knew the answer to a question given that he or she has answered it correctly.

3. Assume there are three machines, A, B and C in a semiconductor manufacturing facility that makes chips. They manufacture, respectively, 25, 35 and 40 percent of the total semiconductor chips there. Of their outputs, respectively, 5, 4 and 2 percent of the chips are defective. A chip is drawn randomly from the combined output of the three machines and is found to be defective. What is the probability that this defective chip was manufactured my Machine A? by machine B? by machine C?

4. A computer chip manufacturer finds that, historically, for every 100 chips produced, 85 meet specifications, 10 need working and 5 need to be discarded. Ten chips are chosen for inspection.

(a) What is the probability that all 10 meets specifications?

(b) What is the probability that 2 or more need to be discarded?

(c) What is the probability that 8 meet specifications, 1 needs reworking and 1 will be discarded?

5. A frequently held lottery sells 100 tickets at \$1 per ticket every time it is held. One of the tickets must be a winner. A player has \$50 to spend. To maximize the probability of winning at least one lottery, should he buy 50 tickets in one lottery or one ticket in 50 lotteries?

6. In the following pdf's, compute the constant B required for proper normalization:

(a) Cauchy ($\alpha < \infty, \beta > 0$):

$$f(x) = \frac{B}{1 + [(x-\alpha)/\beta]^2} \qquad -\infty < x < \infty$$

(b) Maxwell ($\alpha > 0$):

$$f(x) = \begin{cases} Bx^2 e^{-x^2/\alpha^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) Beta ($b > -1, c > -1$):

$$f(x) = \begin{cases} Bx^b(1-x)^c & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

7. Find the expected number of throws of a fair die till a 6 is obtained.

8. Let X be a random variable with the Cauchy distribution

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2} \qquad -infty < x < \infty$$

Show that the mean $E[X]$ is not finite.

9. The arrival time of Prof. Sikdar to his office is a continuous r.v. uniformly distributed over the hour between 12:00 p.m. and 1:00 p.m. Define the events:

A = {The prof. has not arrived by 12:30 p.m. }
B = {The prof. will arrive by 12:31 p.m. }

Find:

(a) $P[B \mid A]$
(b) $P[A \mid B]$

10. A particular TV model is manufactured in three different plants, A, B and C of the same company. Because the workers at the three plants are not equally experienced, the quality of the sets differs from plant to plant. The pdf's of the time-to-failure (i.e. the time before a TV stops working), X, in years are

$$f(x) = \frac{1}{5}e^{-x/5} \qquad x > 0, \text{ for A}$$

$$f(x) = \frac{1}{6.5} e^{-x/6.5} \qquad x > 0, \text{ for B}$$

$$f(x) = \frac{1}{10} e^{-x/10} \qquad x > 0, \text{ for C}$$

Plant A produces 3 times as many sets as B, which produces twice as many as C. The sets are all sent to a central warehouse, intermingled, and shipped to retail stores all around the country. What is the expected lifetime of a set purchased at random?