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We are required under the Health and Safety at Work Act 1974, to make available to users of this equipment certain information regarding its safe use.

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If, in specific cases, circumstances exist in which a potential hazard may be brought about by careless or improper use, these will be pointed out and the necessary precautions emphasised.

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This equipment should not be used by inexperienced users unless they are under supervision.

We are required by European Directives to indicate on our equipment panels certain areas and warnings that require attention by the user. These have been indicated in the specified way by yellow labels with black printing, the meaning of any labels that may be fixed to the instrument are shown below:

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- CAUTION - RISK OF ELECTRIC SHOCK
- CAUTION - ELECTROSTATIC SENSITIVE DEVICE

Refer to accompanying documents

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Where components are of a ‘Safety Critical’ nature, i.e. all components involved with the supply or carrying of voltages at supply potential or higher, these must be replaced with components of equal international safety approval in order to maintain full equipment safety.

In order to maintain compliance with international directives, all replacement components should be identical to those originally supplied.

Any component may be ordered direct from Feedback or its agents by quoting the following information:

1. Equipment type
2. Component value
3. Component reference
4. Equipment serial number

Components can often be replaced by alternatives available locally, however we cannot therefore guarantee continued performance either to published specification or compliance with international standards.
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WARNING:
This equipment must not be used in conditions of condensing humidity.

This equipment is designed to operate under the following conditions:

- Operating Temperature: 10°C to 40°C (50°F to 104°F)
- Humidity: 10% to 90% (non-condensing)

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This equipment, when operated in accordance with the supplied documentation, does not cause electromagnetic disturbance outside its immediate electromagnetic environment.

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Notes
1 Familiarisation

1.1 Objectives

- To learn how to navigate the software.
- To become familiar with the circuit blocks available on the workboard.
- To become familiar with the interconnection of the workboard and PC.
- To determine that the set-up is functioning as required.

1.2 The Workboard – an Introduction

The Servo Fundamentals 33-125 workboard contains a number of circuit blocks that may be interconnected in many ways to demonstrate the principles of analogue servomechanisms, and by extension to those of closed-loop systems more generally.

![Figure 1-1: The Servo Fundamentals 33-125 workboard](image)

The workboard is designed to operate with the Mechanical Unit, 33-100, and to be used in conjunction with a personal computer.
The workboard requires USB connection to the PC.

Interconnection between the various circuit blocks on the workboard is by 2mm stackable patch leads. It is recommended that no more than two leads be stacked, as more than this is mechanically vulnerable and can lead to damage of the lead or the workboard.

1.3 Control Systems

What is an automatic control system?

It is a system in which we are controlling the state of a process, say the width and thickness of strip being rolled in a steel mill. In setting up the system we need to know what the required width and thickness are, and to set up reference or input signals to represent these values.

We are able, by means of transducers, to generate similar signals to represent the actual values at the output of the process. We can then compare the actual width and thickness of the strip produced with those required.

If there is a difference or error, the system must be able to send modifying signals to an actuator, in this case the motor and gearing controlling the roller setting.

1.4 Closed-Loop Control System

The difference or error signal may be thought of as producing effects which move forward, from the point of comparison to the resulting action. The comparison itself depends on a signal which is fed back from the output of the process to be compared with the reference or input signal. The forward flow and feedback of signals form a loop around which information flows as shown in Figure 1-2.

Such a system is therefore called a closed-loop system.

![Figure 1-2: The Closed Control Loop](image)

Various names are given to the signals in different industrial or other contexts. The following terms are commonly used to describe the input, the output and the difference between them:
Where the system is electrical, the state will normally be represented by signals expressed in volts; in our example it might be, for the width, a signal representing ten inches per volt. In this manual, the difference in the comparison will be called the error signal and the part of the system that carries out the comparison is the error channel.

There is usually a power amplifying device to drive the Actuator (which is shown below as the geared motor).

It is usual for control engineers to describe their systems in a block diagram form. Figure 1-3 describes the type of system we shall be using in the assignments. Here there is a comparison by the error channel of the input and output, the error is then amplified to drive a motor and gearing in the forward path so that the speed or position of the output shaft can be modified.

Figure 1-3: Block Diagram of an Analogue Closed-Loop System

1.5 Analogue and Digital Systems

In the system above, it is assumed that the input and output are measured as voltages and lead to an error voltage which is amplified to operate the motor. This system has an analogue error channel since input and output are measured as continuous voltages.
However it is common practice to use digital techniques to generate the error signal in digital form, either by digitising the input and output by an analogue-to-digital (A/D) converter or by direct digital measurement techniques. The error signal is then processed in a computer to generate a digital signal to drive the motor. The motor may then be driven from a digital-to-analogue (D/A) converter or digitally by switching techniques. Thus the system may take the general form of that shown in Figure 1-4.

![Figure 1-4: Block Diagram of a Digital Closed-Loop System](image)

The digitising of inputs may be within the system or in an internal computer interface. The computer-generated motor command will be digital and may be converted to analogue form in the computer interface or within the system. Alternatively the command may be used to drive the motor by a switching technique.

### 1.6 Practical 1: Navigating the Discovery Software

#### 1.6.1 Objectives and Background

Although the Discovery Laboratory environment is very easy to operate, these notes will help you use all its facilities more quickly.

If there is a demonstration assignment, slider controls in the software perform functions that would normally be performed on the hardware. In normal assignments, if any of the hardware systems fail to initialise the system reverts to demonstration mode. This means that none of the test equipment will be available.

#### 1.6.2 Perform Practical

This Practical requires no workboard patching connections and there are no measurements to be taken.
The assignment window opens when an assignment is launched. The assignment window consists of a title bar across the top, an assignment side bar at the right-hand edge, and the main working area. By default, the overall assignment objectives are initially shown in the main working area whenever an assignment is opened. The assignment window occupies the entire screen space and it cannot be resized (but it can be moved by ‘dragging’ the title bar, and it can be minimised to the task bar). The title bar includes the name of the selected assignment. The side bar contains the Practicals and any additional resources that are relevant for the selected assignment. The side bar cannot be repositioned from the right-hand edge of the assignment window. An example of an assignment window is shown in Figure 1-5.

![Figure 1-5: The Assignment Window](image)

The precise appearance of the assignment window will depend on the ‘skin’ that has been selected by your tutor. However, the behaviour of each of the buttons and icons will remain the same, irrespective of this.

The clock (if you have one active) at the top of the side bar retrieves its time from the computer system clock.

There are a number of resource buttons available in the assignment side bar. These are relevant to the selected assignment. In general, the resources available will vary with the assignment. For example, some assignments have video clips and some do not. However, the Technical Terms, Help and Auto Position buttons have identical functionality in every assignment. You can click on any resource in any order, close them again, or minimise them to suit the way you work.

Practicals are listed in numerical order in the side bar. When you hover the mouse over a Practical button, its proper title will briefly be shown in a pop-up tool-tip. There can be up to four Practicals in any assignment. You can have only one Practical window open at any time. See Figure 1-6.
To perform a Practical, left-click on its button in the assignment side bar. The assignment objectives, if shown in the main working area, will close, and the selected Practical will appear in its own window initially on the right-hand side of the main working area, as shown below. You can move and resize the Practical window as desired (even beyond the assignment window) but its default size and position allows the test equipment to be displayed down the left-hand side of the main working area without overlapping the instructions for the Practical.

![Figure 1-6: The Practical Window](image)

Again, the precise appearance of the Practical window can be determined by your tutor but the behaviour of each of the buttons and icons will remain the same, irrespective of this. Whatever it looks like, the Practical window should have icons for the test equipment, together with buttons for Objectives & Background, Make Connections, Circuit Simulator and Test Equipment Manuals. These resources are found in side bar, located on the right-hand edge of the Practical window. The resources will depend on which Practical you have selected. Therefore not all the resources are available in every Practical. If a resource is unavailable, it will be shown greyed out. To open any resource, left-click on its icon or button. Note that when you close a Practical window, any resources that you have opened will close. You may open any resource at any time, provided it available during the Practical. The Circuit Simulator will only be available if you have one loaded.

Note that if the hardware is switched off, unavailable, or its software driver is not installed, all the test equipment is disabled. However, you can open any other window. If you switch on the hardware it will be necessary to close the assignment window and open it again to enable the test equipment.

**Resource Windows.** These are standard, browser-like windows that may be moved, resized and scrolled. You may minimise or maximise them. The system defaults to ‘Auto Position’, which means that as you open each resource window it places it in a convenient
position. Most resource windows place themselves where the main lab window opens out. Each one lays over the previous one. You can select which one is on top by clicking the tab at the top of each window (to the right of the blue and white ‘On Top’ arrow). You can see how many windows you have open from the number of tabs. If you want to see several at once then drag them to where you wish on the screen. If you close a window it disappears from the resources tab bar.

If you want to return all the windows to their default position simply click the Auto Position button in the assignment side bar.

**Make Connections Window.** This movable and resizable window shows the wire connections (2mm patch leads) you need to make on the hardware to make a practical work. Note that some of the wires connect the monitoring points into the data acquisition switch matrix. If this is not done correctly the monitoring points on the practical diagram will not correspond with those on the hardware. The window opens with no connections shown. You can show the connections one by one by clicking the Show Next button or simply pressing the space bar on the keyboard. If you want to remove the connections and start again click the Start Again button. The Show Function button toggles the appearance of the block circuit diagram associated with the Practical.

**Test Equipment.** The test equipment will auto-place itself on the left of the screen at a default size. You may move it or resize it at any time. Note that below a useable size only the screen of the instrument will be shown, without the adjustment buttons. Each piece of test equipment will launch with default settings. You may change these settings at any time.

You may return to the default settings by pressing the Default button on each piece of test equipment. If you wish to return all the equipment to their original positions on the left of the screen click Auto Position on the side bar of the assignment window.

Note that if you close a piece of test equipment and open it again it returns to its default position and settings.

If you want more information on how a piece of test equipment works and how to interpret the displays, see the Test Equipment Manuals resource in the Practical side bar.

On slower computers it may be noticeable that the refresh rate of each instrument is reduced if all the instruments are open at once. If this is an issue then only have open the instrument(s) you actually need to use.

**Test Equipment Cursors.** If you left click on the display of a piece of test equipment that has a screen, a green cursor marker will appear where you have clicked. Click to move the cursor to the part of the trace that you wish to measure. If you then move the mouse into the cursor a tool-tip will appear displaying the values representing that position. Note if you resize or change settings any current cursor will be removed.

**Perform Practical Window.** This window contains the instructions for performing the practical, as well as a block, or circuit, diagram showing the circuit parts of the hardware board involved in the Practical. On the diagram are the monitoring points that you use to explore how the system works and to make measurements. The horizontal divider bar between the instructions and the diagram can be moved up and down if you want the
relative size of the practical instruction window to diagram to be different. Note that the aspect ratio of the diagram is fixed.

**Information Buttons on Practical Diagrams.** On many of the symbols on the diagram you will find a button that gives access to new windows that provide more information on the circuit that the symbol represents. Note that these windows are *modal*, which means that you can have only one open at a time and you must close it before continuing with anything else.

A Further Information point looks like this 😲

**Probes.** The practical diagram has probes on it, which start in default positions. These determine where on the hardware the signals are being monitored.

**Selecting and Moving the Probes.** Probes are indicated by the coloured icons like this 📈.

If this probe is the *selected probe* it then looks like this 📈 (notice the black top to the probe). You select a probe by left clicking on it.

Monitor points look like this 😵.

If you place the mouse over a monitor point a tool-tip will show a description of what signal it is.

You can move the selected probe by simply clicking on the required monitor point. If you want to move the probe again you do not have to re-select it. To change which probe is selected click on the probe you want to select.

You can also move a probe by the normal ‘drag-and-drop’ method, common to ‘Windows’ programs.

**Probes and Test Equipment Traces.** The association between probes and traces displayed on the test equipment is by colour. Data from the blue probe is displayed as a blue trace. Yellow, orange and green probes and traces operate in a similar way. Which piece of test equipment is allocated to which probe is defined by the practical.

Note that the phasescope shows the relative phase and magnitude of the signal on its input probe using another probe as the reference. The reference probe colour is indicated by the coloured square to the top left corner of the phasescope display.

**Practical Buttons.** On some Practicals there are buttons at the bottom of the diagram that select some parameter in the practical. These can be single buttons or in groups. Only one of each button in a group may be selected at one time.

**Slider Controls.** Where slider controls are used you may find you can get finer control by clicking on it and then using the up and down arrow keys on your keyboard.
1.7 Practical 2: Identification of Component Parts

1.7.1 Objectives and Background

This Practical is an exercise to get you conversant with the Mechanical Unit components and circuit blocks that are available on the Servo Fundamentals workboard. There is no patching or measurement associated with this Practical.

At this stage, do not worry if you don’t understand the description or function of the circuit blocks. As you progress through the assignments, their functions and operation should become clearer.

1.7.2 Perform Practical

This Practical requires no workboard patching connections and there are no measurements to be taken.

Read through the descriptions below and identify each of the components and circuit blocks described.

33-100 Mechanical Unit Description

External power supplies of +15V and -15V at 1.5A and of +5V at 0.5A are required. The input sockets (4mm) are protected against accidental misconnection of supplies, though misconnection may blow a fuse.

Motor Shaft. This carries the brake disk, together with a 2-phase speed track and tacho-generator.
**Brake Disk and Magnet.** The brake is applied by the lever projecting at the left. The lever scale is provided to enable settings to be repeated. **Note:** Ensure that the plastic tie-wraps, used to protect the brake assembly in transit, have been removed before the Mechanical Unit is operated.

**Speed Tracks and Readers.** These provide two-phase, 0–5V square waves at 8 cycles per revolution. These signals are available on the 34-way socket but are not used in the Analogue system.

**Motor Check Switch.** This enables the motor to be rotated as an initial check. See initial check Procedure in Chapter 2.

**Armature Current Signal.** This is a voltage waveform indicating the armature current with scale of 1V/A.

**Input Shaft.** This carries the input potentiometer and scale and gives a signal $\theta_i$ in the range ±10V.

**Test Signal Frequency and Range Switch.** These control the internal oscillator to provide ±10V square, triangular and sine waveforms with nominal frequency 0.1 to 10Hz in two ranges. The square and triangular waveforms are connected to the 34-way socket.

**Output Shaft.** This carries the output potentiometer and digital angular measurement tracks. The potentiometer provides $\theta_o$ in the range ±10V.

**Digital Measurement and Readers.** The digital tracks give 6 bit Gray code (64 locations) information and are read by infra-red readers. The 6-bit information is supplied as 0 or 5V to six pins on the 34-way socket.

**Index Pulse.** At one pulse per revolution this provides an output shaft reference point for incremental control connected to a pin on the 34-way socket.

**Output Speed Display.** This provides a direct reading of output shaft speed in r/min in the range 00.0 to 99.9, derived from the tachogenerator. Since the reduction ratio is 32:1, a motor speed of 1000 r/min gives 31.1 r/min at the output shaft.

**33-125 Workboard Description**

Connects to the Mechanical Unit through a 34-way ribbon cable which carries all power supplies and signals enabling the normal circuit interconnections to be made on the Servo Fundamentals using the 2mm patching leads provided.

The unit enables a basic system as in Figure 1-8 to be configured and contains facilities to introduce compensation to investigate improvement in overall system performance.

![Figure 1-8: Servo Fundamentals 33-125 Workboard Block Diagram](image-url)
Figure 1-9 shows the Control & Instrumentation Principles 33-125 workboard. Interconnections are made by 2mm plug leads.

**Power amplifier.** This drives the motor. The two inputs drive the motor in opposite directions for a given input. The zero offset adjustment enables the motor to be rotated with no amplifier input.

**DC motor.** This is in the Mechanical Unit and drives the brake disk and tacho-generator directly, and the output shaft through a 32:1 belt reduction.

**Tacho generator.** This is mounted on the motor shaft and provides a voltage proportional to motor speed; the voltage is available with reversed polarity.

**Incremental encoder.** The incremental encoder produces digital pulses as the motor shaft rotates, allowing measurement of relative position of shaft.

**Mechanical Reduction.** This is a gearbox in the Mechanical Unit and provides a 32:1 reduction on the output shaft.

**Gray encoder.** 6 bit Gray code (64 locations) information received from Mechanical Unit via 34-way socket displayed on led indicators.

**360 degree potentiometer.** This is an output potentiometer in the Mechanical Unit and gives a signal $\theta_o$ in the range ±10V approximately.

**Input potentiometer.** This is an input potentiometer in the Mechanical Unit and gives a signal $\theta_i$ in the range ±1V approximately.

**Sweep function generator.** Frequency sweeps up and down between set points.
**Summing & error amplifier.** The Summing amplifier algebraically adds two (or more) signals or voltages to form the sum of those signals. The error amplifier is most commonly encountered in feedback unidirectional voltage control circuits where the sampled output voltage of the circuit under control is fed back and compared to a stable reference voltage.

**Analogue controller.** This contains operational amplifiers with associated networks to enable various compensating and control circuits to be introduced to improve the performance of a basic system.

**Digital controller.** This consists of an A/D converter (converts analogue inputs to machine readable digital format) and D/A converter (converts digital outputs to a form that can be used in closed loop control).

**Instrumentation inputs.** Signals present at any of the sockets available on the workboard may be measured and displayed on a PC using the Discovery software that accompanies the product.

The points to be monitored must be patched to the Instrumentation Input sockets that are to be found at the top centre of the workboard. The figures associated with these sockets correspond to the numbers on the monitoring points as seen on the diagrams associated with each Practical activity.

1.8 Practical 3: Functional Check

1.8.1 Objectives and Background

In this Practical you will perform a very simple operational check to confirm that the PC, Mechanical Unit and the workboard are communicating with each other and that the set-up is ready to perform further Practicals.

1.8.2 Perform Practical

This Practical requires no workboard patching connections and there are no measurements to be taken.

With the power supply switched off, connect the power supply outputs to the rear of the 33-100 Mechanical Unit.

On the Mechanical Unit, set the brake fully upwards and set the RPM/DVM switch to the RPM position.

**Note:** in all the other practicals, it will be assumed that the brake is fully upwards and the RPM/DVM switch is set to the RPM position initially.

Do not connect the Control & Instrumentation Principles workboard.

Switch the power supply on.

The motor should remain stationary – there may be a slight movement when the supply is actually switched.
The output shaft speed display should show: 0.00. This indicates that the 5V supply is operating.

Next hold the motor check switch to the V+ position and the motor should run clockwise and the output speed display should indicate approximately 15 rpm.

Hold the switch to the V- position and the motor should run anti-clockwise with approximately the same speed. This test indicates that the ±15V supplies are operating.

Hold the motor check switch in either position and gradually lower the brake to maximum. The motor should slow down. This test indicates that Power Supply and Mechanical Unit are operating correctly.

Switch the power off.

Connect the Control & Instrumentation Principles workboard to the Mechanical Unit by the 34-way ribbon cable.

Raise the brake fully.

Switch the power on.

Rotating the power amplifier zero offset adjustment on the Control & Instrumentation Principles workboard should enable the motor to be driven in both directions up to about the same speed as before.

Zero the amplifier to stop the motor.

Overall the tests indicate that the system is working correctly.

The workboard requires USB connection to the PC. If you do not have an available USB socket on your PC, an external hub will have to be used. It may be either powered or un-powered.

For correct operation the PC must have the relevant Discovery software and product drivers installed. If it does not, you will need to consult your tutor.
If the Discovery software has been installed the workboard should automatically be recognised on switch-on and the system will be ready for use.

Ensure that you have connected the equipment as described above.

Launch the Discovery software associated with the product.

If the hardware has not been connected properly, a Discovery warning message is immediately displayed on the screen as shown in Figure 1-11.

![Discovery Warning Message](image)

If this warning message is shown, you must acknowledge it by clicking the OK button before you can continue. In this event, it is recommended that you resolve the problem before attempting to perform the assignment. You will need to close the assignment, correct the hardware problem and then restart the assignment.

On the screen shot of the assignment window, notice the three red indicators within the sidebar. These are marked ‘F’, ‘H’ and ‘A’. These are warning indicators. If any one of them is visible on your screen then you have a fault condition, as follows:

- **F** indicates that there is a firmware communications error;
- **H** indicates that the hardware is incorrectly connected, probably your workboard is incorrectly connected to your PC, or that the workboard driver is not installed correctly;
- **A** indicates that there is a data acquisition error.

If you do not see any of these warning indicators on your screen then your set-up is correct and you may perform any of the Practicals in the assignment. You can still open a Practical when a fault condition exists, but you will not be able to use any test equipment that may be required to perform that Practical. The hardware must be correctly connected before starting an assignment in order to use the test equipment in any of the Practicals within that assignment.

### 1.9 Practical 4: Displaying Signals

#### 1.9.1 Objectives and Background

In this Practical you will become familiar with using the test equipment available in the Discovery 3 software, and the signals that can be displayed.
1.9.2 Block Diagram

Figure 1-12: Block Diagram for Practical 4

1.9.3 Perform Practical

Figure 1-13 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale.

Set the sweep function generator min freq control to approximately 20% scale.

This arrangement enables the square wave test signal to be applied to the power amplifier when the sweep function generator output amplitude control is adjusted.

Set the sweep function generator output amplitude control to half scale.

The motor should rotate in both directions.

Open the Data Logger.

The Data Logger should now display the speed as in Figure 1-14 (b), the yellow trace being the input and the blue the output.
Drag the yellow pointer from monitor point 2 to monitor point 3. This provides the Data Logger with a triangle waveform on the X input.

Select the X-Y mode on the Data Logger. Now a diagram similar to (b) Figure 1-14 (a) should be displayed.

Figure 1-14 (a) should be displayed.

Increase the sweep function generator min freq control and note the effect in X-Y mode and with X-Y deselected. Also adjust the sweep function generator output amplitude control and note the effects in both modes.

Open the Bar Display. Observe the same signals displayed on this piece of test equipment.

Open the Voltmeter.

Observe the display change by varying the position of the variable dc control on both the Voltmeter and Bar Display (orange trace).

This signal can also be monitored on the Data Logger (set to 4 channel).
Notes
2 Operational Amplifier Characteristics

2.1 Objectives

- To learn that an operational amplifier is a dc amplifier providing a very high negative gain.
- To learn that an operational amplifier is invariably used with feedback, the nature of which almost completely determines the amplifier's behaviour.
- To learn that operational amplifiers may be used to provide scaling of analogue signals and/or summation of several such signals.

2.2 Operational Amplifier

Signals—commonly voltages—are scaled or summed to produce another signal. This process can be carried out very conveniently by an amplifier circuit termed an operational amplifier since the circuit can carry out precise mathematical operations on voltages. The general arrangement of an operational amplifier is given in Figure 2-1.

The amplifier has negative gain. That is, (if the resistors are ignored) a positive $V_e$ gives a negative $V_o$ and vice versa.

In symbols:

$$V_o = -AV_e$$
If $V_1$ is considered to move positive, $V_e$ tends to move positive and hence $V_o$ to move.

The essential principle of the circuit is that the same current $i$ must flow in both $R_1$ and $R_0$ since the amplifier has almost infinitely high input impedance and thus draws virtually no current. This means that if $V_1$ is applied, $V_o$ will automatically take such a value that the current $i$ is drawn off through $R_o$ and into the amplifier.

Thus $V_e$ will have a small positive value

$$V_e = \frac{V_o}{A}$$

where $V_o$ is the magnitude of the output. The sloping line represents the voltage to ground moving along $R_1$ and $R_0$ from $V_1$ to $V_o$.

An operational amplifier is a dc amplifier having a gain which is very large and negative.

An operational amplifier is always used with external components which apply feedback around the amplifier. These almost entirely determine the amplifier's behaviour.

If the gain is sufficiently high (which it usually is) the amplifier's input terminal will always be kept at nearly zero potential, which is called a virtual earth.

It is possible to apply several inputs, each via a separate resistor. The amplifier with these resistors and the feedback resistor connected to the virtual earth forms a summing amplifier.

Because of this the input terminal of the amplifier is also referred to as a summing junction.

### 2.3 Practical Aspects

It has been assumed that the gain of the amplifier is so high that its input voltage $V_e$ is always reduced to zero by the feedback process.

In practice, unbalance between components in the amplifier may cause its output to go to zero for some non-zero value of $V_e$.

This will have the same effect as adding a spurious input to a perfect amplifier, so that the output will be offset. A zero adjustment is often provided where this effect is likely to be serious.

### 2.4 Scaling

In an operational amplifier, if the open loop gain ($A$) is very large, in the order of $10^4$ to $10^6$, the voltage $V_e$ is quite negligible compared with $V_1$ and $V_o$, and can be considered as zero.

Hence:

$$I = \frac{V_1}{R_1} \quad \text{Also } i = \frac{-V_o}{R_0}$$

$$V_o = -V_1 \frac{R_0}{R_1}$$
where $V_o$ is reversed in sign with respect to $V_1$, but multiplied by a ratio determined only by $R_o$ and $R_1$. This process of multiplication by a constant is termed scaling and is a very important concept.

In operation, as $V_1$ varies then $V_o$ varies correspondingly, depending on the ratio $R_o/R_1$. If $V_1$ is reversed then $V_o$ changes sign but the voltage distribution along $R_1$ and $R_o$ remains a sloping straight line pivoting about the amplifier input point with $V_e$ effectively zero. Thus the overall behaviour is similar to a 'see-saw' and the diagram is sometimes termed the see-saw diagram.

### 2.5 Inverting

If $R_o = R_1$, then the scaling factor becomes -1 and we have a 'sign reverser' or inverter used to change the sign of a voltage.

Note two levels of simplification of the circuit diagram shown in Figure 2-2 which are commonly applied:

- The earth line is not shown, since no, or very few, connections are made to it from the feedback network.
- An inverter may be simply shown as an amplifier symbol with '-1' written inside it.

### 2.6 Virtual Earth

In an operational amplifier circuit working as an inverter or summing amplifier as shown in Figure 2-3, since the signal $V_e$ at the resistor junction and the amplifier input is substantially zero, this point is called a virtual earth point, and enables the overall circuit to give an output which is the sum of several inputs.

Of course there are small voltage changes at this point caused by the fact that the amplifier does not have infinite gain. It is a point through which currents flow so it is not the same as the "earth" or "ground" in the rest of any associated circuits.

It is a very important and useful property of operational amplifier circuits.
2.7 Summing

Consider the circuit in Figure 2-3:

If two separate inputs are applied the output $V_o$ will take such a value that the current drawn through $R_o$ exactly equals the sum of the input currents $i_1$ and $i_2$, that is

$$i_o = i_1 + i_2$$

$$V_o = -V_1 \frac{R_0}{R_1} - V_2 \frac{R_0}{R_2}$$

$$= - \left( V_1 \frac{R_0}{R_1} + V_2 \frac{R_0}{R_2} \right)$$

showing that the output is the sum of $V_1$ and $V_2$ (with reversed sign) each with a scaling factor. If all resistors are equal then:

$$V_o = - (V_1 + V_2)$$

giving direct addition.

Since the virtual earth point or summing junction is substantially at zero potential more resistors can be added, such as $R_3$ shown dotted.

Each such resistor $R_n$ will make a further contribution to the current in $R_o$, so that

$$V_o = -V_1 \frac{R_0}{R_1} -V_2 \frac{R_0}{R_2} - V_3 \frac{R_0}{R_3} - ...$$

$$= - \left( V_1 \frac{R_0}{R_1} + V_2 \frac{R_0}{R_2} + V_3 \frac{R_0}{R_3} + ... \right)$$

This can be expressed alternatively as

$$V_o = -R_o \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + ... \right)$$
showing that various voltages can be adjusted in relative proportion by the input resistances $R_1...R_n$ before being added, while $R_o$ acts as a common gain control to alter the scale of the result of the summation.

2.8 Practical 1: Op-amp Scaling & Virtual Earths

2.8.1 Objectives and Background

In this practical an operational amplifier is used to invert and scale a voltage signal using the input potentiometer as a source. The amplifier is connected as a unity gain inverter and then as an inverting amplifier with a gain equal to the ratio of two resistors.

2.8.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 2-4: Block Diagram for Practical 1

2.8.3 Perform Practical

Figure 2-5 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the input potentiometer on the Mechanical Unit to zero (arrow pointing upwards).

Open the Data Logger, Bar Display and Voltmeter.

Turn the input potentiometer to apply a voltage of 0.3 volts (blue trace). Note that the output voltage (orange trace) is inverted when compared to the input voltage while the virtual earth (yellow trace) remains very close to zero.

The unity gain is the result of both the input resistor and the output resistor being equal in value.

Remove connection 2 and add connection 6. This changes the feedback resistor from 100k to 330k. The gain of the circuit is now the ratio of the two resistors.

Calculate the ratio and then confirm that the output voltage is the opposite polarity to the input voltage multiplied by the ratio of the resistors. The input and output voltages can be read off the Data Logger and Bar Display, or the input can be read off the Voltmeter and then the output by dragging the blue pointer to monitor point 3.
2.9 Practical 2: Summing dc Signals

2.9.1 Objectives and Background

In this practical the behaviour of an operational amplifier used as a summing amplifier of two dc signals will be investigated. The two input signals are provided by the input potentiometer and variable dc control. The output voltage will be the sum of each input voltage multiplied by the ratio of their individual input resistors and the feedback resistors.

2.9.2 Block Diagram

![Diagram showing the connections for Practical 2]

Figure 2-6: Block Diagram for Practical 2

2.9.3 Perform Practical

Figure 2-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Open the Bar Display.

Set the input potentiometer (blue bar) and variable dc control (yellow bar) outputs to zero using the Bar Display.

Observe the output of the op-amp (green bar).

Set the input potentiometer to 0.5v
Observe the output of the op-amp (green bar).
Set the variable dc control to 0.3v
Observe the output of the op-amp (green bar).

Remove connection 3 and add connection 4 which has the effect of increasing the input resistor (R2) for the variable dc control signal by a multiple of 3 approximately (from 100k to 330k), the effect of this is it reduces the op-amp gain for the variable dc control signal which was originally 1. So the output voltage = -(V1 + (V2/3)) approximately.

So the input potentiometer signal has a gain of one thus for the 0.5v input there is -0.5v output add to this -1/3 of the 0.3v input from the variable dc control which should give approximately -0.6v output.
Note: The virtual earth (orange) remains very close to zero.

2.10 Practical 3: Summing ac and dc Signals

2.10.1 Objectives and Background

In this practical the behaviour of an operational amplifier used as a summing amplifier of an ac and dc signal will be investigated. The two input signals are provided by the input potentiometer and variable dc control (dc) or the sweep function generator (ac). The output voltage will be the sum of each input voltage multiplied by the ratio of their individual input resistors and the feedback resistors.

2.10.2 Block Diagram

![Block Diagram for Practical 3](image)

Figure 2-8: Block Diagram for Practical 3

2.10.3 Perform Practical

Figure 2-9 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical the input potentiometer is used to apply a dc signal to the op-amp, and the sweep function generator provides either sine-wave or square-wave depending on setup.

Open the Data Logger and Bar Display.

Set the input potentiometer on the Mechanical Unit to zero, arrow pointing upwards (blue).

Set the sweep function generator output amplitude control to half scale.

Set the sweep function generator min freq control to minimum.

Select 4 channel mode on the Data Logger.

With the input potentiometer at zero, note that the output signal (green) of the op-amp is simply the sweep function generator (Square wave) input signal (yellow) inverted. Adjust the input potentiometer making this input voltage either positive or negative, confirm the output signal equals the sum of the two input signals inverted.

Add connection 5 which adds a second 100k resistor in parallel with the input resistor associated with the input potentiometer signal. This has the effect of halving the input resistor (R1in) and so increasing the gain for the input potentiometer signal by a factor of two. Note the output voltage for various settings of the input potentiometer.
Note: If the input potentiometer is adjusted over a certain point, the output from the op-amp will go over the maximum (±1.25v) value that can be displayed. Also the virtual earth (orange) remains very close to zero.

Remove connection 1 and add connection 6. This applies a sine-wave from the sweep function generator to the input of the op-amp instead of the square-wave.

Observe the output of the op-amp on the Bar Display and Data Logger.
Chapter 2

Control & Instrumentation Principles

Notes
3 Analogue Transducers

3.1 Objectives

- To learn that a dc tacho-generator provides a signal representing motor speed.
- To learn about delay in the motor response to an applied input signal.
- To learn that a 360° potentiometer provides a signal representing motor position.

3.2 Potentiometer

This term is slightly confusing due to its origin. It derives from the name of an instrument used to measure the “potential” voltage of a source. This had to be achieved by measuring the voltage when no current was flowing in the source. In modern times this is easy due to the availability of electronic voltmeters with very high input resistances. In the early days of electrical research voltmeters had quite low resistances and hence the voltage they measured depended on the resistance of the voltmeter and the internal resistance of the source.

To measure the potential of the source a device was arranged to supply a variable second voltage which was connected to the source and the voltage adjusted so now current flowed. By calculating the fraction of the maximum second supply used to match the source the potential could be measured. Such a device was called a potentiometer, i.e. a device for measuring potentials.

This device was essentially a resistor that had a third connection which could be moved up and down the total resistance.

![Figure 3-1: The Potentiometer](image-url)
Figure 3-1 shows the general idea. In the context of modern electronics the term potentiometer is used to describe a device like this but they are very rarely used to measure potentials. In many cases they are simply used as variable resistors to make some circuit adjustment or to take a proportion of a signal such as the audio level on a television or radio.

In this equipment one is used to measure the rotational position of a shaft by connecting a fixed voltage across the ends and measuring the voltage at the variable connection the position of which is moved by the shaft. The voltage then represents the angle of the shaft.

Many potentiometers such as those used to adjust audio level for example do not rotate 360 degrees. The one used here to measure motor position is special and has a full 360 degrees of rotation. Of course there is a problem at the ends where the voltage suddenly changes from maximum to minimum. Additionally as it changes from maximum to minimum there may be a few degrees where the output is zero.

3.3 Tacho-generator

A tacho-generator is a device that produces a dc voltage the amplitude and polarity of which is proportional to the speed of its input shaft. Hence it can be used to measure the rotation speed. It is in fact simple a dc permanent magnet motor which means it also has this property. However devices designed for this purpose are specified to have low friction and to have the output voltage as linear as possible with respect to speed. Unlike a dc generator their power output is not important as they usually feed a high input resistance amplifier.

3.4 Power Amplifiers

A power amplifier is an amplifier the output of which is generally specified as a power rather than simply a voltage or current. It is used when a signal is needed to do work on a device that requires energy to operate it. An obvious example is an electric motor however it might be a heater, a light or even the output stage of a radio transmitter.

The design and specification of such amplifiers can be quite complex particularly if a large amount of power is required.

3.5 Motor

A motor is a rotary actuator which turns an energy supply into rotary motion. The energy supply can be many forms such as hydraulic, pneumatic or electrical. Electric motors exist that can operate on ac and dc. The choice of motor types is a complex compromise of performance and cost.

The motor in this equipment is a dc electric motor and is a permanent magnet type. It can be represented in idealised form as in Figure 3-2 (a), where $R_a$ is the armature resistance and $T_1, T_2$ are the actual motor terminals.
If the motor is stationary and a voltage $V_a$ is applied, a current $I_a$ flows which causes the motor to rotate. As the motor rotates a back emf $V_b$ is generated. As the motor speeds up—remember the general characteristics obtained in the Familiarisation assignment—the back emf increases and $I_a$ falls.

In an ideal (loss free) motor, the armature current falls to substantially zero and $V_b$ approximately equals $V_a$. Thus if $V_a$ is varied slowly in either polarity, the motor speed is proportional to $V_a$, and a plot of motor speed against $V_a$ would have the form of Figure 3-2 (b).

In the 33-100 the armature voltage $V_a$ is provided by a power amplifier. A power amplifier is necessary, because although the voltages in the error channel may be of the same order as $V_a$, the motor current may be up to 1A, while the error channel operates with currents of less than 1mA and could not drive the motor directly. The amplifier has two input sockets, enabling the motor rotation direction to be reversed for a given input.

### 3.6 Practical 1: Speed Measurement using a Tacho-generator

#### 3.6.1 Objectives and Background

To observe the output of the tacho-generator and understand how it relates to motor speed, and to learn about delay in motor response to an applied input signal, by comparing the input signal to the power amplifier that drives the motor, to the output signal of a tacho-generator.
3.6.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 3-3: Block Diagram for Practical 1

3.6.3 Perform Practical

Figure 3-4 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

Set the output amplitude control on the sweep function generator to minimum.

Set the min freq control on the sweep function generator to minimum.

Open the Data Logger and the Bar Display.

The input signal can be seen on the blue trace, with the yellow trace representing the output from the tacho-generator.

Adjust the variable dc control and observe the resulting output from the tacho-generator.

Remove connection 1, and add connections 2 and 3.

This arrangement enables the square wave test signal to be applied to the power amplifier when the sweep function generator controls are adjusted as follows.

Set the output amplitude to half scale.

Set the min freq control to give a frequency of about 0.2Hz using the Data Logger display.

The motor should rotate in both directions, giving speed displays as shown in Figure 3-5.
Examine the effect of increasing or decreasing the test frequency using the sweep function generator min freq control.

This practical shows that there is a delay in the motor response to an input, which is due to the mechanical inertia of the armature. All motors exhibit this general characteristic, which has very important consequences for control system design. Special armature design can greatly reduce the inertia for small motors.

3.7 Practical 2: Position Measurement using a Potentiometer

3.7.1 Objectives and Background

To observe the output of the 360° potentiometer and understand how its output relates to motor position.
3.7.2 Block Diagram

![Block Diagram for Practical 2](image)

3.7.3 Perform Practical

Figure 3-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

Open the Data Logger and Bar Display.

Turn the variable dc control clockwise until the motor is running at approximately 5 RPM (Mechanical Unit display).

Observe the 360° potentiometer output (yellow trace).

The potentiometer used here to measure motor position is special and has a full 360 degrees of rotation. At the end of a full rotation the voltage suddenly changes from maximum to minimum. As it changes from maximum to minimum there may be a few degrees where the output is zero. Because of this, the 360° potentiometer produces an output in the form of a ramp signal.

Slowly turn the variable dc control clockwise to increase motor speed. Note the effect on the 360° potentiometer output signal.
4 Motor and Brake Characteristics

4.1 Objectives

- To learn that the steady speed of the motor is ideally proportional to the applied voltage, less an amount proportional to load torque.
- To learn the effect of loading on a motor.
- To learn that the response of the motor to a change of input is not immediate, but may be expressed as a time constant.

4.2 Prerequisites

Before commencing this assignment, you should understand the concept discussed in the earlier section:

- 3.5 Motor

4.3 Brake Characteristics

The magnetic brake consists of a permanent magnet which can be swung over an aluminium disk. When the disk is rotated, eddy currents circulate in the area of the disk within the magnet gap. These currents react with the magnetic field to produce a torque which opposes rotation. This gives an adjustable torque speed relation of the form of Figure 4-1, and provides a very convenient load for the motor.

![Figure 4-1: Characteristic of Magnetic Brake](image)

The overall characteristics of a motor may be considered from two aspects. These aspects are:
• **Steady-state**, which are concerned with constant or very slowly changing operating conditions, and

• **Transient**, corresponding with sudden changes.

Both are important in control system applications.

### 4.4 Practical 1: Voltage to Motor Speed Characteristics

#### 4.4.1 Objectives and Background

In this practical the motor is operated in a range of steady-state conditions. By doing this you will observe the motor, with no load, runs at a speed almost proportional to the applied voltage.

#### 4.4.2 Block Diagram

![Block Diagram for Practical 1](image)

**Figure 4-2: Block Diagram for Practical 1**

#### 4.4.3 Perform Practical

Figure 4-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

Open the Voltmeter.

Rotate the variable dc control clockwise to increase motor speed.

**Note:** Rotating the variable dc control anticlockwise will make the motor turn in the opposite direction.
Make a plot of motor speed (read off the Mechanical Unit) against amplifier input, in the range ±1.25V, scaling the vertical axis in units of 1000 r/min. The plot should have the general shape of Figure 4-4.

Initially the motor speed increases substantially linearly with the voltage to the amplifier because the motor back emf \( V_b \), approximately equals the amplifier output, but finally the amplifier limits before the full ±1.25V input is reached.

**Note:** Since the reduction to the output shaft is 32:1, the motor speed is calculated by multiplying the r/min reading by 32, e.g. a reading of 31.25 = a motor speed of 1000 r/min.

![Figure 4-5: Characteristics of Tacho-generator](image)

The tacho-generator provides a voltage proportional to speed, which is required for various aspects of control system operation.

Plot the tacho-generator characteristics by setting the motor speed to various values by altering the variable dc control position and measuring the tacho-generator output voltage. The plot should be a straight line with the general form shown in Figure 4-5.

Open the Bar Display.

For each measurement first place the blue pointer on monitor point 1 to read the input voltage, then move the pointer to monitor point 2 and take the tacho-meter output reading, or use the Bar Display to obtain the readings.

An important parameter in the use of tacho-generators is the tacho-generator factor in volts per 1000 r/min.

Determine the tacho-generator factor by measuring the change in generated volts for a speed change of 1000 r/min.

The factor should be approximately 0.35V per 1000 r/min.
4.5 Practical 2: Effect of the Brake

4.5.1 Objectives and Background

Considering the idealised motor shown in Figure 4-6 (a), when the motor is unloaded the back emf $V_b$ substantially equals the applied voltage $V_a$, the armature current being very small.

When the motor is loaded the speed falls, the back emf falls, and the armature current increases and the voltage drop in the armature resistance $V_r (= I_aR_a)$ added to $V_b$ matches $V_a$, that is:

$$V_a = V_r + V_b$$

$$= I_aR_a + V_b$$

Hence, if the motor is loaded so that the speed falls, the armature current increases, the general characteristic being as the solid lines in Figure 4-6 (b). If the armature resistance is low, which is the situation for a normal motor, the current increases greatly, as shown dotted, for a small change in speed.

The proper operating range of the motor would be up to a load corresponding with a few percent drop in speed, perhaps to the point when the dotted current line crosses the speed line.
4.5.2 Block Diagram

![Block Diagram](image)

Figure 4-7: Block Diagram for Practical 2

4.5.3 Perform Practical

Figure 4-8 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Connection 2 on the Make Connections diagram should be connected between Instrument input 1 and the Armature current output (1V/A) socket on the Mechanical Unit.

Set the variable dc control to half scale (motor static).

Open the Voltmeter.

**Note:** Since the reduction to the output shaft is 32:1, the motor speed is calculated by multiplying the r/min reading by 32. e.g. a reading of 31.25 = a motor speed of 1000 r/min.

Adjust the variable dc control anti-clockwise to set the motor speed to 2000 r/min (62.5 r/min at output), with the brake fully upwards.

Set the brake to each of its six positions in turn and for each setting record and plot the speed and armature current. The plot should have the general form of Figure 4-9 below.
4.6 Practical 3: Motor Transient Response

4.6.1 Objectives and Background

A motor cannot change speed instantly due to the *inertia* of the armature and any additional rotating load (the brake). This effect has very important consequences for control system design.

If $V_a$ for an ideal motor has a step form as in Figure 4-10 (a), initially a large current will flow, limited only by the armature resistance. As the motor rotates and speeds up the back emf increases and the current is reduced to nearly zero in an ideal motor.

This is shown in the left-hand portion of Figure 4-10 (b). If $V_a$ is suddenly reduced to zero the back emf still exists, since the motor continues to rotate, and drives a current in the reverse direction dissipating energy and slowing the motor. This is illustrated in the right-hand portion of Figure 4-10 (b).
The motor shows a speed characteristic approximating to Figure 4-10 (b), but the power amplifier is arranged to limit the maximum armature current which does not show the ideal pulse characteristic.

4.6.2 Block Diagram

Figure 4-11: Block Diagram for Practical 3

4.6.3 Perform Practical

Figure 4-12 shows the required connections on the hardware.
Set the sweep function generator output amplitude control to minimum.
Set the power amplifier zero offset fully clockwise to run the motor in one direction.
Set the sweep function generator min freq control to minimum.
Adjust the sweep function generator output amplitude control until the motor is stationary for one half cycle.
Open the Data Logger.
Set the test signal frequency to 0.2Hz by adjusting the sweep function generator min freq control.
This corresponds with $V_a$ in Figure 4-10 (b).
The Data Logger will now display the speed corresponding with $V_a$ in Figure 4-10 (b).
Drag the blue pointer from monitor point 1 and place on monitor point 3.
Select X-Y on the Data Logger. Note the displayed signal on the Data Logger.
4.7 Practical 4: Motor Time Constant

4.7.1 Objectives and Background

The delay in response of a motor is of great importance in control system design and is expressed as the time-constant.

This is the time that would be required for the motor speed to change between any steady values if the initial rate of speed change was maintained. This is the dotted line in Figure 4-13 (a), while the actual speed response is shown as a solid line.

![Figure 4-13: Speed Response](image)

It can be shown that the speed changes by 0.63 of the final change during the time constant. The time constant can be measured from a display of the speed against time.
4.7.2 Block Diagram

![Block Diagram for Practical 4](image)

Figure 4-14: Block Diagram for Practical 4

4.7.3 Perform Practical

Figure 4-15 shows the required connections on the hardware.
Set the sweep function generator output amplitude control to minimum.
Set the power amplifier zero offset fully clockwise to run the motor in one direction.
Set the sweep function generator min freq control to minimum.
Adjust the sweep function generator output amplitude control until the motor is stationary for one half cycle.
Open the Data Logger.
Set the test signal frequency to 0.2Hz by adjusting the sweep function generator min freq control.
This should correspond with $V_a$ in Figure 4-10 (b).
The Data Logger will now display the speed corresponding with $V_a$ in Figure 4-10 (b).
With the square wave frequency of 0.2Hz, the time across the trace is 2.5s.
Use the capture mode on the Data Logger to estimate the time constant by considering the initial slope and maximum speed. The value should be in the region of 0.5s.
Notes
5 Closed Loop Control and Feedback Polarity

5.1 Objectives

- To learn the basic configuration of a Closed Loop System and how it functions.
- To learn that the polarity of the feedback in a closed-loop system must be negative for the system to work correctly.
- To learn that care must be taken over polarities in order to ensure that when the setting (or demanded value) is changed, then the output (actual or measured value) changes in the appropriate direction.
- To learn how input and output shaft rotation relate to each other depending on system setup.

5.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 1.3 Control Systems
- 1.4 Closed-Loop Control System
- 1.5 Analogue and Digital Systems

5.3 Error Accuracy

An ideal potentiometer would provide a voltage from the slider which varies linearly with rotation giving the heavy line in the diagram below, and if set at mid-track would give exactly 0V.
Practical potentiometers have a volts/rotation characteristic as shown dotted departing from the ideal.

The extent of the departure non-linearity depends on the individual potentiometer but is reduced by more expensive construction methods.

The effect of the non-linearity is that if two potentiometers are set to the same angle the voltages will be slightly different, the difference varying with the angle. The control system will always rotate the output shaft and potentiometer to give zero error.

The effect is that the output shaft does not exactly follow the input shaft but has a misalignment error depending on the shaft angle. The error may vary from less than a degree to several degrees depending on the potentiometers used. The misalignment can always be set to zero at some point (commonly 0°).

For rotations exceeding one revolution it is usual to employ synchros, which are energised by ac. These enable an ac error signal to be obtained, with the facility for continuous rotation. Then either the system must operate with an ac motor or the error signal must be converted to dc for a dc motor system.

Synchros are more expensive than potentiometers, but are very accurate.
5.4 Practical 1: A Simple Closed Loop System

5.4.1 Objectives and Background

A control system with input and output shafts requires some method of measuring the input and output shaft angles and determining the difference or error between them. The error must then produce a voltage, or may be measured directly as a voltage, suitable to drive the power amplifier.

A very convenient method to measure the shaft angles electrically is to attach a potentiometer to each shaft, as in Figure 5-2. The signals $V_i$ and $V_o$ can then be combined in an operational amplifier to produce an error signal to operate the power amplifier.

![Figure 5-2: Closed-Loop Control System](image)
5.4.2 Block Diagram

![Block Diagram](image)

Figure 5-3: Block Diagram for Practical 1

5.4.3 Perform Practical

Figure 5-4 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the input potentiometer on the Mechanical Unit to 0°.

Open the Data Logger.

In this practical the power amplifier drive signal comes from the summing & error amplifier section. The signal supplied to the power amplifier is called the error signal, which is the difference between the desired position of the motor set by the input potentiometer and the actual position of the motor given by the 360° potentiometer output.

Rotate the input potentiometer clockwise by 90° on the Mechanical Unit. The output shaft should rotate anti-clockwise and finally align 90° from its original starting position.

Observe the input potentiometer signal (blue), the 360° potentiometer output signal (yellow) and power amplifier input signal (orange) on the Data Logger while rotating the input potentiometer.

Note that the output 360° potentiometer, error amplifier, power amplifier, motor and drive to the output shaft form a loop, and the system is arranged to have negative feedback round the loop, which reduces the error.
When the input potentiometer signal is positive, the 360° potentiometer signal is negative or vice versa, these two signals added together using the summing & error amplifier give the difference between them, which should reduce to zero after a short period of time. This is the usual operating condition for a simple negative feedback closed loop system.

5.5 Practical 2: Feedback Polarity

5.5.1 Objectives and Background

Practical 1 demonstrated the effect of negative closed loop feedback on a system; this practical will show the effect of positive feedback on the same system. This will show that for correct system operation it is essential that the error signal rotates the motor in the appropriate direction to reduce the error – this being negative feedback.

If the error signal rotates the motor to increase the error, this is positive feedback and the system is useless.

5.5.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 5-5: Block Diagram for Practical 2

5.5.3 Perform Practical

Figure 5-6 shows the required connections on the hardware.
Figure 5-6: Make Connections Diagram for Practical 2

Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the attenuator control to minimum.

Open the Bar Display and Data Logger.

Set the input potentiometer to 0°.

Use the power amplifier zero offset control to set the 0° on the output scale pointing to the right.

Slowly turn the attenuator control until the motor just rotates and the output shaft will rotate clockwise and stop vertically with the arrow pointing downwards.

The system now has positive feedback round the loop and increases the error.

The motor stops because the slider arm of the potentiometer has moved into the gap between the ends of the track and $V_o$ becomes zero.

Turn the attenuator control to full scale and use the power amplifier zero offset control to move the output shaft and the system will probably go into a sustained oscillation.

The oscillation occurs because if the slider moves slightly to the left the signal $V_o$ becomes -1v, which drives the slider to the right, and then $V_o$ changes to +1v, driving the slider to the left, and the whole process repeats.
This can be seen on the Bar Display and Data Logger, with the yellow bar showing the 360° potentiometer output signal $V_o$, the blue being the input potentiometer and the orange showing the error signal input to the power amplifier.

5.6 Practical 3: Input and Output Rotation

5.6.1 Objectives and Background

The previous practical has shown that it is essential to have negative feedback round the loop for the system to operate properly. In addition, there are other considerations concerning relative rotation directions of shafts.

The results show that the system was following, but with reversed shaft rotation. The reasons can be seen in Figure 5-7 (a), representing the potentiometers and error amplifier.

If $\theta_i$ is rotated anti-clockwise $V_i$ is negative, hence the error signal $V_e$ will increase positively, eventually causing $V_o$ to increase and $V_e$ to fall to zero.

To make the shafts rotate in the same direction, various changes could be made, such as:

i. reverse the supply polarity to the input potentiometer, or

ii. introduce a gain of -1 in the $V_i$ line

However only changing one of these will reverse the feedback polarity to give positive feedback, corresponding with the oscillatory situation, and the system will not operate properly.

However, introducing two changes will cause the feedback to remain negative.

This can be arranged as in Figure 5-7 (b), where a gain of -1 is introduced in the $V_o$ line and the input socket on the power amplifier is reversed.

In this configuration the output shaft does follow the input shaft rotation.
5.6.2 Block Diagram

![Block Diagram for Practical 3](image)

Figure 5-8: Block Diagram for Practical 3

5.6.3 Perform Practical

Figure 5-9 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the input potentiometer to 0°.

This should move the output shaft to 0°.

Open the Bar Display and Data Logger.

Whilst turning the input potentiometer to 90°, make a note of the direction the output shaft rotates.

Repeat this while observing the system signals on the Bar Display and Data Logger. Input potentiometer (blue), inverted 360° potentiometer (yellow) and the error signal (orange).

Note how these signals and the system operation differ to those in the first practical.
6 Stability and the Effect of Loop Gain

6.1 Objectives

- To learn the measurement of transient response within a closed loop control system for step inputs.
- To learn the effects of loop gain on a system's stability.

6.2 Closed-Loop Control

A control system with input and output shafts requires some method of measuring the input and output shaft angles and determining the difference or error between them. The error must then produce a voltage, or may be measured directly as a voltage, suitable to drive the power amplifier.

A very convenient method to measure the shaft angles electrically is to attach a potentiometer to each shaft, as shown in Figure 6-1. The signals $V_i$ and $V_o$ can then be combined in an operational amplifier to produce an error signal to operate the power amplifier.

Figure 6-1: Closed-Loop Control System
A general system has the form shown in Figure 6-2, where an input \( x \) and an output \( y \) are compared to give an error \( e \) with the relation

\[ e = x - y \]

The process of comparison is represented by a conventional symbol as in the diagram, where the input and output may not necessarily be voltage signals. For a purely electrical system the comparator may be an operational amplifier.

The error operates on the forward path, which includes everything between the error and the final output \( y \). Thus the forward path may contain a facility to convert the error to a voltage, followed by a gain and a power amplifier driving a motor, and then some reduction gear to operate the output shaft.

The design and performance characteristics of systems are often considered in terms of the response to a step or ramp input as shown in Figure 6-2 (a), with possible responses shown in Figure 6-2 (b) where the input is shown in shadow. The response to a sinusoidal input, the frequency response (amplitude and phase), also has a very important application in control system design, but requires significant mathematical background theory and appropriate test equipment.

Figure 6-2: A general system with (a) Test Signals and (b) Typical Responses
In application the system will probably operate from a generalised input as at Figure 6-2 (a) bottom, which is not practical as a design input, so that the simple inputs above are used for design purposes.

6.4 Step Response

The step response of a system gives useful information about the general system characteristics and is often required to meet some requirements in the system performance specification. To generate the step response a very simple input is required, and for these reasons much consideration is given to the step response.

For a given system the form of the step response is greatly affected by the system gain. The gain essentially determines how much power is applied to move the output for a given error.
A purely electrical position control system may be represented as in Figure 6-3 (a), where the input \( V_i \), output \( V_o \) and error \( V_e \) are all voltages and the forward path gain \( G \) is shown separately, the voltage applied to the power amplifier being \( GV_e \).

If a step \( V_i \) is applied, the initial value of the error is equal to \( V_i \) as in (b), since \( V_o \) is zero. If the gain is 1 the power amplifier input is initially \( V_i \) and as the motor rotates the output gradually aligns with the input, with the motor slowing up as the error decreases.

If the gain is 2, the initial input to the power amplifier is \( 2V_i \), causing the motor to move faster and although the error decreases, the motor may overshoot the required final position due to the delay in the motor.

When the motor finally stops the error is reversed in sense, so that the motor reverses and the system aligns or may undershoot, but will finally settle. This is shown in Figure 6-3 (c).

The gain values refer to relative gains associated with the comparator. The overall effective gain depends on many factors, including the reduction ratio in the output system.

If the gain is increased further, the system may take several oscillations to settle, as at Figure 6-3 (d). If there are two delays in the system the result may be a steady oscillation at the output, or even an increasing oscillation. Systems with the characteristics of Figure 6-3 (d) are useless for control purposes.

An additional effect that must be considered is the magnitude of the input signal. In the hypothetical example described above, doubling the gain speeds up the response. However if \( V_i \) increases, the power amplifier drive \( 2V_i \) also increases and the amplifier may limit and the motor may not be able to move fast enough to give the response at Figure 6-3 (c) for an increased \( V_i \).

### 6.5 Practical 1: Measuring Transient Response

#### 6.5.1 Objectives and Background

To measure the transient response of a control system, simple test signals need be applied. The most common is a step signal, as is used in this practical.
6.5.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 6-4: Block Diagram for Practical 1

6.5.3 Perform Practical

Figure 6-5 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the sweep function generator output amplitude control to minimum.

Set the input potentiometer on the Mechanical Unit to 0°. The output shaft should be aligned with the input potentiometer.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the attenuator to approximately 20% of scale.

Open the Data Logger.

Note the signals displayed on the Data Logger. Step input signal = blue, 360° potentiometer signal = yellow and power amplifier input (error) = orange.

Select Capture to observe the transient response more clearly.
6.6 Practical 2: The Effect of Gain on Stability

6.6.1 Objectives and Background

A closed loop control system response to a step input is sluggish if the gain is low. An increase in the gain causes a faster response, but if too much gain is applied the response may overshoot and take several oscillations before settling. Further increase in gain may cause sustained oscillations to build up.

6.6.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 6-6: Block Diagram for Practical 2

6.6.3 Perform Practical

Figure 6-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the sweep function generator output amplitude control to minimum.

Set the input potentiometer on the Mechanical Unit to 0°. The output shaft should be aligned with the input potentiometer.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the attenuator to approximately 20% of scale.

Open the Data Logger.

Step input from function generator = Blue, Inverted 360° potentiometer signal = yellow, error signal = orange.

Whilst observing the signal on the Data Logger, rotate the attenuator control so that the system just overshoots (yellow signal) and estimate the time to alignment $T_a$. It is useful to examine the output of the error amplifier which passes through zero at $T_a$.

Next set the attenuator to full scale and note the reduced $T_a$ since the motor moves faster but gives increased overshoot.
When the gain is increased further, it is useful to examine the error amplifier output, which is \( GV_e \), to check that the forward path is not being overloaded. The initial value of the error depends on the total change of input, so that a \( \pm 0.625V \) input gives an initial error \( (V_e) \) of 1.25V.

Remove connection 4 and add connection 11.

This sets the amplifier feedback resistor to 330k\( \Omega \) giving Gain = 3.3.

The time to alignment is much reduced but the response is too oscillatory for a practical system.

The feedback resistor can also be set to 1M\( \Omega \) giving \( G = 10 \).

The response is even more oscillatory, but the time to initial alignment will be reduced further from the value obtained with \( G = 3.3 \).

These general results illustrate a fundamental problem for simple control systems, in that increasing the gain to give a faster response leads to more oscillation, and no advantage may be gained.
7 Velocity Feedback

7.1 Objectives

- To learn that introducing velocity feedback into a closed-loop position control can make it more stable.
- To learn that with improved stability more gain may be used.
- To learn that excessive velocity feedback makes the system slow acting.

7.2 Prerequisites

Before commencing this assignment, you should understand the concept discussed in the earlier section:

- 6.2 Closed-Loop Control

7.3 Velocity Feedback

The result of increasing gain on a control system is that it increases the oscillatory tendency in the response, which is undesirable. There are good practical reasons for using a high gain. An important one is that, due to the brushes and other factors, all practical motors have a constant friction (called coulomb friction).

Also usually an increased amount of friction force (called stiction) has to be overcome to start the motor from rest. Therefore a minimum voltage has to be applied to the motor before rotation starts.

This means that there is a minimum input below which the system will not respond; this is termed dead-band. If the gain is high the dead-band is reduced, which is advantageous, but the system may display unwanted oscillation in the response.

\[ V_c = V_s - K_t V_o \]

Figure 7-1: Velocity Feedback System
The form of the system response with high gain can be much improved by applying a feedback signal to the input proportional to the output shaft velocity. This arrangement is termed *velocity feedback*, and is illustrated in Figure 7-1 and Figure 7-2.

![Diagram showing velocity feedback](image)

In the first diagram it is assumed that the input and output signals are available as voltages and a voltage $V_s$ proportional to output shaft speed is available from a tacho-generator. A fraction of this voltage $K_t V_s$ is subtracted from $V_e$ to give $V_c$, which is the control voltage applied to the power amplifier giving:

$$V_c = V_e - K_t V_s$$

If a step input is applied to the system the error will initially equal the input step and decrease as the motor speeds up. As the motor speed increases the velocity signal $V_s$ increases and subtracts from $V_e$ to give a drive voltage $V_c$, which is less than the error. This is illustrated in the second diagram, and the motor drive goes to zero and reverses before the error goes to zero. This means that the motor begins to slow up before the initial alignment, greatly reducing or even preventing any overshoot.
7.4 Tacho-generator

A tacho-generator is a device that produces a dc voltage the amplitude and polarity of which is proportional to the speed of its input shaft. It is in fact simple a dc permanent magnet motor which means it also has this property. However devices designed for this purpose are specified to have low friction and to have the output voltage as linear as possible with respect speed. Unlike a dc generator their power output is not important as they usually feed a high input resistance amplifier.

7.5 Practical 1: Using the Tacho-generator to add Velocity Feedback

7.5.1 Objectives and Background

Velocity feedback is feedback proportional to the velocity of the output shaft. A tacho-generator is often provided for the purpose.

In a position servo it serves the purpose of modifying the (position) error signal so that the modified signal goes to zero before the desired position is reached.

This prevents the motor from accelerating right up to the desired position, thus reducing the inertia that would normally cause it to overshoot.
7.5.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 7-3: Block Diagram for Practical 1

7.5.3 Perform Practical

Figure 7-4 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the attenuator control for the tacho-generator feedback signal to minimum.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the attenuator for the summing & error amplifier signal to full scale.

Open the Data Logger.

The error amplifier feedback resistor to should be set to 100k, giving $G = 1$. A slightly oscillatory response should be obtained, and can be observed on the Data Logger.

The square-wave input signal = blue, Tacho-generator = yellow, 360° potentiometer = orange and the power amplifier input (error) = green.

Slowly turn the attenuator control for the tacho-generator feedback signal clockwise. If the overshoot decreases the velocity feedback polarity is correct. If the overshoot increases connect to the other (non-inverted) socket.

When adjusting the tacho-generator feedback signal a dead beat response will be obtained when the system aligns in the least possible time, but with no overshoot as shown in Figure 7-5.
Additional velocity feedback will cause an over-damped response, in which the system slowly moves into alignment.

![Figure 7-5: Dead-beat Response](image)

Increase the tacho-generator feedback signal until a deadbeat response is obtained, this occurs when the motor just does not reverse.

To apply additional feedback disconnect the input potentiometer signal (connection 5) and connect the tacho-generator signal to that input for as well (connection 14).

Increase the feedback resistors to 330k, $G = 3.3$; 1M$\Omega$, $G = 10$, remove connection 4 and add connections 15 and 16. In both cases deadbeat response should be obtainable.

These results show that velocity feedback is a very powerful technique to improve transient response when gain is increased.

Note that reversing the polarity of velocity feedback can make a system unstable.
8 Following Error

8.1 Objectives

- To learn that a simple system follows a ramp with an error.
- To learn that increasing the gain reduces the error but leads to a deterioration of the transient response.
- To learn that velocity feedback can improve the transient response but increases the following error.

8.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 6.2 Closed-Loop Control
- 7.3 Velocity Feedback

8.3 System Following Error

The step response of a is an important general indication of system performance. Another important characteristic is the system response to a steadily changing input requiring the output to move at a constant speed.

This is sometimes termed the response and is represented in Figure 8-1, where all signals are assumed available as voltages.
This general situation could correspond with the requirement for the cutting tool of a lathe to move at a constant speed along a workpiece, or a radar dish to sweep at a constant velocity.

If the output is to move at a steady speed then, when the system has settled, there must be an appropriate constant voltage applied to the motor. This voltage can only be obtained from the error.

Since the error $V_e$ depends on the difference between input and output $V_e = V_i - V_o$ the system will show a constant following error, that is the output always lags the input. In terms of the two examples mentioned above, this means that the cutting tool is never exactly where it is commanded to be, or the dish does not point exactly where it is intended to point.

If the forward path gain $G$ is increased, then the error for a given speed falls inversely, that is doubling the gain halves the error, however, the system response becomes more oscillatory and the settling time may increase.
8.4 Velocity Feedback and Following Error

Since velocity feedback improves the step response transient, it could be supposed that the following error transient response would also be improved. This is correct, but there is the disadvantage that the use of velocity feedback increases the steady following error.

![Figure 8-2: Increased Following Error with Velocity Feedback](image)

The application of velocity feedback is represented in Figure 8-2, where $V_c$ is the control voltage applied to the power amplifier.

If the system is following at the same steady speed the motor drive $V_c$ must remain constant and hence the signal immediately before the amplifier $G$ must have the same value as the original error ($V_{eo}$) without velocity feedback. However the actual error between $V_i$ and $V_o$ ($V_{ev}$) must increase so that when $K_t V_s$ is subtracted, the original error is available as input to $G$, thus:

$$\text{error with velocity feedback (}V_{ev}\text{)} - K_t V_s = \text{original error (}V_{eo}\text{)} \text{ or } V_{ev} = V_{ev} + K_t V_s$$

8.5 Practical 1: The Effect of Following Error

8.5.1 Objectives and Background

This assignment will investigate following error, which can be an important performance requirement for a control system.

In order to be able to examine the following error and see how it changes with gain, it is necessary to arrange the system so that the error $V_e$ is directly available as $V_e = V_i - V_o$ from the error operational amplifier as in diagram (a) below.
Any additional gain G must be introduced as a separate amplifier, see Figure 8-3 (a), so that for any gain value the direct error is always available. If the system gain G is incorporated in the error operational amplifier as in Figure 8-3 (b) by using an output resistance GR, the amplifier output is GV_e.

For the above reason it is convenient to use the general system of Figure 8-3 (c). It is important to note that if an additional operational amplifier is introduced to provide gain G, this also introduces an additional sign reversal in the loop, giving positive feedback unless some additional sign reversal is introduced.

In the Analogue controller section of the Control & Instrumentation Principles workboard there is an operational amplifier which enables convenient values of gain to be introduced, and changing the power amplifier input socket provides an additional sign reversal.

The additional amplifier has input resistors of 100kΩ and feedback resistors of 100kΩ hence with various different arrangements gains of 1, 2, 5, 20 and 25 can be obtained.
8.5.2 Block Diagram

![Block Diagram](image)

**Figure 8-4: Block Diagram for Practical 1**

8.5.3 Perform Practical

Figure 8-5 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Open the Data Logger and select X-Y mode.

Drag the Data Logger window to increase the display window.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the attenuator control to half scale.

Note the steady following error with controller operational amplifier gain of 1, i.e. input and output resistance of 100kΩ and error amplifier attenuator control at half scale.

Increase the gain to 2 by adding connection 9 (2 x 100kΩ input in parallel and 100kΩ output).

Now increase the gain to 5 by removing connections 7 and 9, and adding connection 10 (100kΩ input and 500kΩ output).

Note in each case that the error decreases inversely with increasing gain, but that the transient response deteriorates as expected.
8.6 Practical 2: The Effect of Velocity Feedback on Following Error

8.6.1 Objectives and Background

The effect of velocity feedback on following error is as the velocity feedback is increased the transient error will improve, but steady following error $V_{ev}$ will increase.

8.6.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 8-6: Block Diagram for Practical 2

8.6.3 Perform Practical

Figure 8-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Open the Data Logger and select X-Y mode.

Drag the Data Logger window to increase the display window.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the left-hand attenuator (summing & error amplifier) control to half scale.

Set the right-hand attenuator (tacho-generator) control to minimum.

Note that the controller amplifier gain is now set to 5.

Gradually turn the right-hand attenuator control up to full scale, which introduces velocity feedback. The error transient will improve, but the steady following error $V_{ef}$ will increase.
9 System Time Constant and Instability

9.1 Objectives

- To learn that an additional time constant causes a system transient response to deteriorate.
- To learn that with high gain the system may become unstable.

9.2 Additional Delay (Longer time constant)

Most additional delays have the general characteristics of a time-constant, represented by the RC circuit of diagram (a) below, showing the delay in the step response as the capacitor charges through the resistor.

This circuit characteristic may be obtained by the operational amplifier circuit of Figure 9-1 (b), where there is a capacitor in parallel with the output resistor. If a step is applied, there is a constant input current since the amplifier input is a virtual earth point.

When the step is applied:
the capacitor current \( (i_c) \) = input current \( (i) \)
the resistor current \( (i_r) \) = 0

Since the input current \( (i) \) is constant, thus as the capacitor charges:
- \( i_c \) reduces
- \( i_r \) increases

and finally:
- \( i_c = 0 \)
- \( i_r = i \)

This gives a voltage output which has a time constant form, with time constant RC, identical with that of the RC circuit of Figure 9-1 (a), except that the output has reversed polarity.

### 9.3 Practical 1: Adding a Longer Time Constant

#### 9.3.1 Objectives and Background

Unstable System Motor characteristics are investigated in previous practicals, and it is shown that there is a delay in the speed response of a motor to a sudden change of supply voltage. If a step voltage is applied the speed response would be generally as in Figure 9-2 (a).

![System Characteristics due to Delay](image)

It is also shown in the closed-loop step response investigation, the effect of Gain on stability assignment that the delay in the motor response can cause the system to overshoot and then settle with reducing oscillation as in Figure 9-2 (b) above.
An additional separate delay in the system can cause more marked overshoot because the motor has been able to move further before the drive is reversed. The additional delay may lead to sustained or increasing amplitude oscillation, as in Figure 9-2 (c) above.

It is very important to avoid significant additional delay in a system even though various procedures used in a system, such as filtering to eliminate noise on signals or signal processing may introduce delays. If such delays become comparable with those inherent in the system then, at least, the transient response will deteriorate.

So any additional delay in a system will cause the transient pressure to deteriorate because the motor can overshoot more before the drive signal is reversed.

An additional delay frequently has the characteristics of a time constant and can be represented by an operational amplifier circuit.

The combination of additional delay and high gain can cause a system to become unstable.

### 9.3.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 9-3: Block Diagram for Practical 1

### 9.3.3 Perform Practical

Figure 9-4 shows the required connections on the hardware.
In order to investigate the effect of an additional time constant, the error amplifier has a capacitor available to enable the circuit in Figure 9-1 (b) to be made.

Open the Data Logger.

Set the sweep function generator min freq control to minimum.

Adjust the sweep function generator output amplitude control to full scale then decrease until the yellow trace on the Data Logger shows no signs of clipping.

Since:
- \[ C = 0.5\mu F \] (nominal)
- \[ R = 1M\Omega \]

The time constant should be about 0.5 sec. Check this against the Data Logger display by using the capture facility.

Select the X-Y mode on the Data Logger and note the display.

If the 1M\Omega resistor is replaced with a smaller resistor the time constant decreases. Check the effect of using the \( R = 330k\Omega \) and \( R = 100k\Omega \) both in normal mode and X-Y mode on the Data Logger. Replace connection 5 with connections 6 and 7 in turn to obtain 330k\Omega and 100k\Omega respectively.
9.4 Practical 2: Instability at High Gains

9.4.1 Objectives and Background

It is has been shown in a previous practical that additional gain to a system can cause the system to overshoot and then settle with reducing oscillation.

An additional delay frequently has the characteristics of a time constant and can be represented by an operational amplifier circuit.

In this practical the combination of additional delay and high gain is demonstrated to cause a system to become unstable.

9.4.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 9-5: Block Diagram for Practical 2

9.4.3 Perform Practical

Figure 9-6 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the sweep function generator min freq control to minimum.

Set the summing & error amplifier output signal attenuator control to half scale.

Set the sweep function generator output amplitude control to 20% of scale.

Open the Data Logger and select the X-Y mode.

Note the output on the Data Logger.

Next add connection 9. This adds the capacitor into the circuit.

Note that introducing the capacitor has little effect on the transient, since the time constant (0.05 secs) is small compared with the approximate motor time constant (0.4–0.5 sec).

Remove connection 9.

Remove connection 3 and add connection 10 this sets the feedback resistor to 330kΩ. Observe the output. Then add connection 9. The capacitor will now have a marked effect on the transient. Note this effect in normal and X-Y mode on the Data Logger.

With the capacitor still in circuit set the sweep function generator output amplitude control and the summing & error amplifier output signal attenuator control to minimum.
Slowly turn up the summing & error amplifier output signal attenuator control up to full scale, the motor should be static. Then move the input potentiometer about ±10°. The error amplifier time constant, 0.165 sec, is now appreciable compared to the motor time constant. The system should eventually maintain self-oscillation.

Again, set the summing & error amplifier output signal attenuator control to minimum.

Remove connection 10 and add connection 11, this sets the feedback resistor to 1MΩ.

Slowly turn up the summing & error amplifier output signal attenuator control. The error amplifier time constant, 0.5 sec, equal to the motor time constant.

The system will go into self-oscillation without an input. This is due to the input potentiometer position and the 360° potentiometer signals differing only slightly enough to produce a small error, which is amplified by a high gain factor by the op-amp and the effect of the additional time constant.

Check that the system can be stabilised by using velocity (tacho-generator) feedback.
10 Closed Loop Speed Control

10.1 Objectives

- To learn that velocity feedback can be used (without position feedback) to enable a speed to be closely regulated. The polarity of the feedback is important (as for position feedback).
- To learn that effectiveness of the control depends mainly on the gain employed.

10.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 1.3 Control Systems
- 1.4 Closed-Loop Control System
- 1.5 Analogue and Digital Systems
- 6.2 Closed-Loop Control
- 7.3 Velocity Feedback

10.3 Speed Control

The previous assignments have been concerned with position control, but an important aspect of closed-loop control is speed control, which has many industrial applications, varying from heavy industrial, such as paper mills or steel rolling mills, to tape or video transport mechanisms.

![Diagram of Closed Loop Speed Control](image)

Figure 10-1: Essential Features of a Closed Loop Speed Control

The essential principle of closed-loop speed control is similar to position control, as illustrated in the diagram above, except that the feedback signal is now an output velocity signal $V_s$, normally from a tacho-generator, which is compared with a reference voltage $V_r$ to give an error:

$$V_e = V_r - V_s$$
In operation the reference is set to a required value, which drives the motor to generate $V_s$, which reduces the error until the system reaches a steady speed.

If the motor is loaded, e.g. with the magnetic brake on the 33-100 Mechanical Unit, the speed falls; this tends to increase the error, increasing the motor drive and thus reducing the speed fall for a given load. Note that this implies negative feedback around the loop.

The speed fall with load, sometimes termed *droop*, is a very important characteristic in speed control systems.

The rotation direction can be reversed by reversing the reference voltage, though many industrial speed control systems are required to operate in one direction only.

**Figure 10-2: Speed Control System**

10.4 Practical 1: Closed Loop Speed Control with Brake Loading

10.4.1 Objectives and Background

This assignment shows the general principle of speed control and that increasing the velocity feedback and the system gain, can give the system less speed fall at full load.
10.4.2 Block Diagram

Figure 10-3: Block Diagram for Practical 1

10.4.3 Perform Practical

Figure 10-4 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

Open the Voltmeter.

Set the summing & error amplifier attenuator control to full scale.

Set the attenuator control for the tacho-generator signal to minimum.

The amplifier feedback resistor is set to 100kΩ, this gives $G = 1$.

Adjust the variable dc control (rotate clockwise) to run the motor at 1000 r/min (31.25 r/min at output).

Turn up the attenuator control for the tacho-generator signal slightly. If the speed decreases the loop feedback is negative (as required). If the speed increases, however, use the other tacho-generator polarity.

Note that if the system has negative feedback and both the tacho-generator polarity and the power amplifier input are reversed, the system still has negative feedback, but the motor runs in the opposite direction.

Set the attenuator control for the tacho-generator to minimum and plot the speed against the 6 brake settings to full brake load. The general characteristic should be as in Figure 10-5.
Note that the armature current is displayed on the Voltmeter. The signal for this comes from connector 8, as shown in Figure 10-4, which connects from the Control & Instrumentation Principles workboard to the Mechanical Unit armature current output. The signal presents 1v/Amp. It can be seen that the armature current increases with loading, as shown in a previous practical.

Drag the blue pointer on connection point 1 and place it on point 2. The Voltmeter will now display the error signal input into the power amplifier.

Set the attenuator control for the tacho-generator to full scale and readjust the variable dc control to give 1000 r/min with the brake off. Replot the speed characteristic and error (power amplifier input) up to full brake load. Change the feedback resistor to 330kΩ so G = 3.3 (remove connection 2 and add connection 3), adjust the variable dc control to give 1000 r/min with no load, and replot the load characteristic. The droop should be reduced.

Repeat with G = 10 (remove connection 3 and add connection 4, adjusting the variable dc control as required, and the droop should be less.

In some cases a tacho-generator output can contain a ripple component which will be amplified in the forward path, and with high gain could saturate the power amplifier.

Note the ripple can be reduced by connecting a capacitor across the error amplifier output resistor, but the will introduce a time-constant and reduces the response to fast signals as seen in a previous assignment.
Notes
11 Deriving Velocity Feedback from the Error Signal

11.1 Objectives

- To learn that the derivative of error, is a measurement of the rate of change of the error.
- To learn that differentiation of a signal can be achieved by using operational amplifier.
- To learn that the effect of error derivative on a system is similar to the effect of velocity feedback.

11.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 6.2 Closed-Loop Control
- 7.3 Velocity Feedback

11.3 Derivative of Error

Figure 11-1: Derivative of Error
When a step input signal is applied, the error signal will typically respond as shown in the upper diagram of Figure 11-1(c). The derivative or rate of change of error corresponds graphically with the slope of the error graph.

If the derivative is measured, the general form will be as shown in the lower diagram of Figure 11-1 (c). Initially the slope is zero, reaches a maximum negative value (corresponding with rapidly decreasing error) shortly before the system initially reaches alignment and then falls to zero when the system reaches maximum overshoot.

If the derivative signal is added to the error signal, the combination of error and derivative goes to zero before the system aligns (dotted Ve graph). Thus if the power amplifier drive signal Vc is formed from Ve by:

\[ V_c = \text{error} + \text{derivative of error} \]

\[ = V_e + \frac{dV_e}{dt} \]

then Vc will reverse the motor drive before alignment of the output shaft is reached, much improving the transient response. The arrangement is illustrated in Figure 11-1 (b), where the amount of derivative signal is adjustable by Kd to give:

\[ = V_e + K_d \frac{dV_e}{dt} \]

The general effect of error derivative is similar to velocity feedback, but is obtained by operating on the error only.
11.4 Differentiation by Operational Amplifier

Suppose that a ramp voltage is applied to a capacitor as in diagram (a) above. Since the voltage across the capacitor rises steadily a constant current must flow into the capacitor. The current is proportional to the capacitance $C$, being given by the relation:

$$i = C \frac{dv}{dt}$$

If the capacitor is used as the input element for an operational amplifier as in (b), the amplifier input will be a virtual earth point and the amplifier output will be given by

$$V_o = i R_o$$

and since

$$i = C \frac{dv}{dt}$$

finally

$$V_o = -CR_o \frac{dv}{dt}$$

giving the constant output voltage of (b). This indicates the derivative of the input with a scaling factor $-CR_o$. 

Figure 11-2: Differentiation by Operational Amplifier
Although this circuit in principle measures the derivative, there is a limitation in practical application. If the input signal contains noise or disturbance components which are small but rapidly changing, these may cause currents in the capacitor comparable to those of slower changing signals for which the derivative is required.

These unwanted components in the input are emphasised and may even saturate the amplifier or some later stage in the amplifying system.

The effect of unwanted rapidly changing high frequency components can be limited by a resistor in series with the capacitor as in diagram (c) above. If the input is changing slowly the input current is largely determined by the capacitor, but if the input is changing fast the current is limited by the resistor giving an overall gain of $R_o/R$.

In frequency response terms the gain of the ideal differentiator in (c) rises continuously with increasing frequency, noise corresponding with high frequency components.

The introduction of an input resistor, called a limited derivative, gives a gain initially rising with frequency, representing correct derivative action, but finally becoming constant preventing emphasis of high frequency components.

### 11.5 Practical 1: Deriving Velocity Feedback from an Error Signal

#### 11.5.1 Objectives and Background

The purpose of this practical is to observe how an operational amplifier can be used to differentiate an input signal, which can be used to derive velocity feedback from an error signal.
11.5.2 Block Diagram

Figure 11-3: Block Diagram for Practical 1

11.5.3 Perform Practical

Figure 11-4 shows the required connections on the hardware.
Open the Data Logger.

Set the sweep function generator output amplitude control to maximum.

Turn up the analogue controller derivative control until the input triangle waveform (blue) is ±1V.

Set the triangle test frequency to about 1Hz using the sweep function generator min freq control.

Select the X-Y mode.

A display should be obtained as in Figure 11-5, the steady value being CR(dV/dt). The time-constant start of the waveform is due to the effect of the 10kΩ resistor.
For a frequency of 1Hz, the test waveform changes through 2V in 0.5 second, hence:

\[ \frac{\Delta V}{\Delta t} = 4 \text{ V/s.} \]

Since the capacitor is 1µF, a voltage rate of change of 1V/s would give \( i = 1\mu\text{A} \), so 4V/s gives \( i = 4\mu\text{A} \), where \( i \) is the current.

If \( R_o = 200\text{k}\Omega \), the steady output voltage would be approximately:

\[ \pm 4\mu\text{A} \times 200\text{k}\Omega \text{ (i.e CR} \frac{\Delta V}{\Delta t} \text{ )} = \pm 0.8\text{V} \]

Adjust the sweep function generator min freq control. Note that reducing the frequency will produce a proportionate fall in the steady value.

Remove connection 3 and add connection 4.

This reduces the capacitance to 0.5µF (both capacitors in series), the amplifier output will fall to 50%, but the time to establish the steady value will also fall to 25%.

11.6 Practical 2: Using Derivative of Error

11.6.1 Objectives and Background

Previous assignments have considered system performance and how this is affected by gain.

In principle a higher gain leads to improved performance in respect of reduction of dead-band and following error and also to a reduction of droop with increasing load for a speed control system.

The disadvantage of high gain is that the transient response deteriorates, giving overshoots or oscillations. This can be corrected by the use of velocity (tachogenerator) feedback, but that increases the steady following error.

A more general method to improve system performance is to arrange that the drive signal to the motor or other output element is a combination of the direct error, with components of the derivative (rate of change), and integral of the error.

This practical deals with the use of the derivative of error component that is similar to velocity feedback.
11.6.2 Block Diagram

Figure 11-6: Block Diagram for Practical 2

11.6.3 Perform Practical

Figure 11-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the derivative gain control to minimum.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to 20% of scale.

Open the Data Logger.

Observe the display on the Data Logger. Step input = blue signal, error signal = yellow signal, derivative of error = orange signal and the green signal = error + derivative of error signal.

Set the derivative gain to half scale and note the change in the displayed signals as well as the effect on the motor behaviour.

Set the derivative gain to full scale, this should have the similar effect as introducing velocity feedback into a closed loop control system.
Notes
12 Integral Control

12.1 Objectives

- To learn that a very versatile control signal can be obtained by combining the error, the derivative of the error, and on the integral of the error signals.
- To learn that with a capacitor in the feedback path, an operational amplifier can act as an integrator.

12.2 Integral of Error

Suppose that a simple position control system is following a ramp input, the steady error ($V_{es}$) will be exactly that required to drive the motor to make the output speed match the input speed. This is investigated in the System Following Error assignment, and is illustrated in Figure 12-1.

![Figure 12-1: Error and Integral Control of Error](image)

Suppose that the error is integrated by some method. At any time the value of the integral is the area under the error. This is indicated for a particular time $t_1$ in Figure 12-1. If the error has reached the steady value $V_{es}$, the integrator output would rise steadily.

If the integrator output is added to the error to form $V_c$, as in Figure 12-2, the motor control voltage is given by:

$$V_c = V_e + K_i \int V_e \, dt$$

Initially when the ramp is applied the error will occur as in the upper diagram, but the integrator output will gradually build up. This will cause the motor to speed up slightly, reducing the error, but as long as there is any error the integrator output will continue to increase. The only situation that will give a constant integrator output is that the error has fallen to zero, and this is what happens.
The integrator final output becomes $V_{es}$, as in Figure 12-2, being the signal required to drive the motor at the speed to match the input.

### 12.3 Integration by Operational Amplifier

The voltage across a capacitor is given by the integral of the current through the capacitor:

$$v = \frac{1}{C} \int i \, dt$$

This is illustrated in Figure 12-3 (a). A constant current gives a steady increase of voltage, the voltage representing the area under the current /time plot. The illustrated current pulse waveform, comprising a positive pulse, a negative pulse and then zero current, gives a final voltage value, which is the overall time integral of the current.
Integration can be obtained by an operational amplifier if the feedback resistor is replaced by a capacitor as in Figure 12-3 (b) above. If a voltage V is applied an input current i will flow through R and the amplifier output will change to hold the amplifier input point (virtual earth point) substantially at zero. This means that the amplifier output will steadily increase to maintain the current i through the capacitor. The input current is:

\[ i = \frac{1}{C} \]

The voltage across the capacitor will be:

\[ V_c = \frac{V}{R} \int V \, dt \]

giving the amplifier output as:

\[ V_o = \frac{1}{CR} \int V \, dt \]

which is the negative scaled integral of the input.
If the applied voltage \( V \) becomes zero, the current \( i \) is also zero and the integrator holds indefinitely whatever output has been obtained, since the amplifier does not ideally draw any current at the virtual earth point. If the input is a general waveform the output is correspondingly scaled integral.

It is often required to set the output of an integrator to zero before the start of integration, and this can be arranged by a switch, mechanical or electronic, connected across the capacitor, as shown dotted, which discharges the capacitor. Integration does not start until the switch is open, irrespective of a possible input signal.

An ideal integrator will hold an accumulated signal indefinitely; however an operational amplifier may draw a very small current at the virtual earth input, which will cause the output to drift very slowly. The drift may or may not be important and depends on the amplifier used.

### 12.4 Integral Action

An important application of integral control is to eliminate steady following error. Integral control is not necessary for the steady alignment (i.e. step response) of a system with a motor output, but can eliminate the effect of some disturbances.

An ideal motor can be represented, as above, by a time constant giving the relation between armature voltage and speed, and an integration giving the relation between speed and output shaft angle, since the output angle will increase however small the velocity.

Thus a motor contains an inherent integration which for an ideal motor will ensure alignment. However, as mentioned above, the inclusion of a separate integrator may improve the steady alignment against external disturbances.

A simple position control system is illustrated below with an integrator that can be introduced in parallel with the gain to give \( P + I \) control. Suppose that the input is at a constant value and the system is aligned (output vertical).
If an external torque is applied to the output, which might be wind loading on a rotatable antenna system, the output will be deflected from correct alignment.

When this deflection occurs an error signal is produced and the motor will produce a counteracting torque. However, the counteracting torque can only be produced if the system is misaligned. Thus the system will be misaligned to an angle depending on the torque and the system characteristics.

If the integrator is switched on, then when the misalignment error occurs the integrator output will increase and the motor torque will increase, pulling the system back towards alignment. As long as there is any error the motor torque will increase until there is zero error. This effect is shown dotted in Figure 12-5 and finally the integrator provides the input to the motor.

In addition to a direct output disturbance, an integrator can counter any other disturbance that may enter the system after the integrator and cause system misalignment. Such a disturbance might be a power amplifier supply variation, which might cause the motor to rotate slightly.
12.5 Integral Control – Summary

It has been shown that an important application of integral control is to eliminate steady following error. In this application integral control alone can give a poor transient response, which can be much improved by derivative control.

The integral and derivative control adjustments which give a satisfactory following error transient may not be those that give the best step response, so that some compromise in adjustments may be necessary.

Integral control is not essential for system static alignment, because a system with a motor output will in principle always align to give zero error due to the inherent integration characteristic of a motor.

However, a useful feature of integral control is that it can counter the effect of output member disturbances that may cause misalignment and can also counter some internal system disturbances.

This assignment has shown the effect of integration in causing a system to align, i.e. for a speed control system to make the output \( V_s \) equal to the input \( V_{ref} \).

In the case of position control system, there is an inherent integration in the motor which causes alignment. A speed control does not have an inherent integration and hence only aligns when a separate integrator is introduced.

12.6 Practical 1: Deriving an Integral Error Signal

12.6.1 Objectives and Background

The purpose of this practical is to observe how an operational amplifier can be used to integrate a signal.
12.6.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 12-7: Block Diagram for Practical 1

12.6.3 Perform Practical

Figure 12-8 shows the required connections on the hardware.
Set the variable dc control to minimum.

On the Mechanical Unit, set the RPM/DVM switch to the DVM position.

Open the Voltmeter.

Short circuit the integrator capacitor by the switch (slide it to the right-hand position) and adjust the integrator gain control so the voltage reads approximately -1V on the Voltmeter.

Whilst observing the DVM display on the Mechanical Unit, open circuit the integrator capacitor (slide switch to its left-hand position). The output should move positively and can be reset to zero by closing the switch.

If the input voltage is 1V and the input resistor is 200kΩ, then the input current i is:

\[
\frac{1V}{200k\Omega} = 5\mu A
\]

If the capacitor is 1µF, 5µA gives 5 volts per second

2µF, 5µA gives 2.5 volts per second

Hence the output should take about four seconds to move from 0V to +10V.
The integrator will operate in both directions depending on the polarity of the input. Discharge the integrator capacitor by sliding the switch to its right-hand position and then back to its left-hand position. Then, as the output reaches 5V, turn the integrator gain control to minimum and note the output stops increasing.

Remove connection 2.

Now any current drawn at the virtual earth point will pass through the capacitor and be integrated.

Discharge the capacitor by the switch and then open the switch and estimate the time required for the output to reach 10V – it may be some minutes.

The input current can then be determined, since 1µA gives 0.5 volt/second with 2µF. The value should be much less than 1µA.

12.7 Practical 2: The Effect of the Integral Signal on Following Error

12.7.1 Objectives and Background

In this practical the use of an integrator in the forward path, which is called integral control, gives the characteristic that any steady error is reduced to zero with the integrator output established at exactly the value required to provide the motor drive to maintain the error at zero.

Integral control is commonly applied to speed control systems and if the measured speed does not exactly match the required speed due to loading, the integrator will develop an output to reduce the error to zero.

A problem that may arise with integral control is that the integrator takes some time to develop the required output and if the system operating condition suddenly changes, the integrator requires time to readjust. This may lead to a slow and undesirable oscillatory transient response.
12.7.2 Block Diagram

Figure 12-9: Block Diagram for Practical 2

12.7.3 Perform Practical

Figure 12-10 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the sweep function generator min freq control to minimum.

Set the sweep function generator output amplitude control to half scale.

Set the analogue controller integral gain to half scale.

Set the capacitor discharge switch to its right-hand position – leave it in this position.

Open the Data Logger.

Drag the Data Logger window to make the display larger.

Select Overlay.

Note the error waveform (yellow) compared to the input signal (blue), in particular the following error.

Now set the integrator capacitor circuit to its left-hand position.

Observe the change in the error signal by introducing the integrator signal to the control loop.
What can be seen is that although the following error is reduced, the problem with integral control is that the integrator takes some time to develop the required output and if the system operating condition suddenly changes, the integrator requires time to readjust. As can be seen this leads to a slow and undesirable oscillatory transient response.
13 Application of Three Term Control

13.1 Objectives

- To learn that Proportional with Derivative control improves transient response.
- To learn that Proportional with Integral control eliminates steady errors but may give a slow response.
- To learn that the combination of proportional, derivative and integral control gives the best overall response.

13.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 12.4 Integral Action
- 12.5 Integral Control – Summary

13.3 Derivative Control Application

The general characteristics of derivative control and the operational amplifier realisation of derivative action are given in the initial portion of the Introduction to 3-Term Control assignment.

The general effect of derivative control is similar to velocity (tacho-generator) feedback in providing improved transient response. The effect arises because derivative control can reverse the drive to the power amplifier before the error is zero. Derivative control has the advantage over velocity feedback that the steady following error is not increased.

A useful method to investigate transients is to make an X-Y display between various components. Such displays are termed phase-plane displays, since they show the continuous phase or state of the system. The display or path of the system state is usually termed a trajectory.
13.4 Following Error with Derivative Control

Although derivative control has an effect similar to velocity feedback in improving transient response, derivative control does not increase the steady following error. This arises because velocity feedback introduces a constant component proportional to output velocity while derivative control does not introduce any constant component.

![Figure 13-2: Increased Following Error with Velocity Feedback](image)

13.5 Speed Control System with Integral Control

A simple speed control system with the general form above is investigated in the Speed Control System assignment. The tests demonstrate that increasing the forward path gain reduces the speed fall with load (droop).

A possible problem associated with increased gain is that the ripple component on the tacho-generator output which could be amplified to cause power amplifier saturation, though this ripple can be filtered out.

The forward path gain of a system also affects the relation between the reference voltage $V_{\text{ref}}$ and the speed voltage $V_s$. This relation

$$V_e = V_{\text{ref}} - V_s$$

can be rearranged as

$$V_{\text{ref}} = V_e + V_s$$

and represented by the voltage diagram in Figure 13-4.
If the forward path gain is low a large $V_e$ may be required so that $V_e$ is much smaller than $V_{ref}$. As the gain increases $V_e$ approaches $V_{ref}$, and if the forward path gain is infinite then $V_e$ becomes zero and:

$$V_e = V_{ref}$$

which may be desirable.

If the comparison of $V_{ref}$ and $V_e$ is carried out in an operational amplifier in the circuit of diagram (a), then the corresponding equation is strictly

$$V_e = -(V_{ref} + V_s)$$

and the condition corresponding with infinite gain, i.e. $V_e$ is zero, becomes

$$V_s = -V_{ref}$$

so that the magnitudes are equal but with opposite signs.

### 13.6 X-Y Display of Error Components

It is possible to make an oscilloscope display that gives a good demonstration of derivative control operation. The general principle of derivative control is the combination of error and derivative results in the power amplifier drive $V_c$ reversing before the system has aligned.
To illustrate this, consider an X-Y display with the error horizontal X and the power amplifier drive vertical Y as above.

If the derivative is set to zero, the power amplifier drive is the proportional component (5 × error). If the X and Y channel gains are in the ratio of 5:1, the step response display will be an oscillatory signal along a straight line, since both signals have the same form.

If derivative control is introduced the power amplifier drive is not the same form as the error and the display ceases to be a straight line. If the error gain is adjusted to give deadbeat response, a display of the form of (b) will be obtained. This display shows that the power amplifier drive (vertical) has reversed before the error has reached zero, so that the motor is slowing up as alignment is approached.

### 13.7 Practical 1: Three Term Controller Test

#### 13.7.1 Objectives and Background

We have considered system performance and how this is affected by gain.

In principle a higher gain leads to improved performance in respect of reduction of deadband and following error and also to a reduction of droop with increasing load for a speed control system.

The disadvantage of high gain is that the transient response deteriorates, giving overshoots or oscillations. This can be corrected by the use of velocity (tacho-generator) feedback, but that increases the steady following error.

A more general method to improve system performance is to arrange that the drive signal to the motor or other output element is a combination of the direct error, with components of the derivative (rate of change), and integral of the error.

Since the final drive signal contains three components or terms, the process is called *Three Term Control*.

In this practical a circuit is used that generates from an input signal a combination of that input signal summed with its derivative and its integral. A signal is applied to the input and the output examined with various proportions of the three terms.
13.7.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 13-5: Block Diagram for Practical 1

13.7.3 Perform Practical

Figure 13-6 shows the required connections on the hardware.
Set the sweep function generator min freq control to about 20% and the output amplitude to half scale.

Set the derivative and integral gain controls to minimum.

Set the capacitor discharge switch to its right-hand position.

Open the Data Logger and the Bar Display. Note the output (yellow) simply follows the input signal (blue) as the proportional path has an effective gain of +1.

Increase the derivative gain and note that the derivative output (green) is a small pulse corresponding to the edges of the square wave. Note that the output is a combination of the proportional signal plus the derivative on the output signal.

Set the capacitor switch to its left-hand position so the capacitors are not shorted out.

Increase the integral gain. Note the integral output (orange) is a triangle wave which is the integral of a square-wave. The output now contains the proportional signal, the derivative signal and the integral signal in ratios set by the derivate and integral gain controls.

Note that the integrator will slowly drift up or down due to small zero offsets in the sweep function generator output. This offset is small but is integrated over time and will cause the integrator output to limit. Momentarily setting the capacitor discharge switch to its right-hand position zeros the integrator.
Figure 13-6 shows the typical output signals from derivative, integral, proportional circuits and the summed output signal.

13.8 Practical 2: Three Term Position Control

13.8.1 Objectives and Background

In this practical the behaviour of an analogue position control system is investigated. The controller is a PID block such that the relative amounts of derivative and integral control may be varied.

Of particular interest is how the derivative control affects the stability of the system and how the integral control affects following error. In this context, note that there is no perfect solution for a controller and it turns out to be a compromise depending on which of the performance parameters are of most importance.
13.8.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 13-8: Block Diagram for Practical 2

13.8.3 Perform Practical

Figure 13-9 shows the required connections on the hardware.
Figure 13-9: Make Connections Diagram for Practical 2

Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

The input is initially a triangle wave.

Set the sweep function generator output amplitude control to minimum and the min freq control to about 5%.

Set the derivative and integral gain controls to minimum.

Set the capacitor discharge switch to its right-hand position.

Set the attenuator control for the main proportional gain to half scale.

Open the Data Logger and the Bar Display. Drag the Data Logger window to enlarge the display.

Increase the sweep function generator output amplitude control to about half scale. The position control loop should start to generally follow the triangle input signal. Check that the period of the input signal (blue) is approximately 6 seconds and adjust the sweep function generator min freq control if necessary.

Change the proportional gain (attenuator control) and note that decreasing the gain increases the following error (orange) and increasing the gain decreases the following error but at the same time the system becomes more oscillatory.
Remove connection 1 and add connection 23.
This changes the input waveform to square-wave and the instability at high gain becomes very clear. At low gain the system response time to the square-wave becomes slow.

Using the square-wave, input set the proportional gain (attenuator control) to full scale and note that there is considerable overshoot (yellow). Increase the derivative gain and the overshoot becomes significantly less.

Adjust the derivative gain set such that the overshoot is almost negligible and note that the system response time has not been significantly affected.

Remove connection 23 and replace connection 1.
Do not adjust the derivative gain. Note the following error.

Set the capacitor discharge switch to its left-hand position.

Increase the integral gain and notice how the following error is integrated out. Set the integral gain to the minimum value that integrates out the following error at each end of the waveform amplitude.

Remove connection 1 and add connection 23.
This changes the input back to a square-wave. Note that the system response has become slower and more oscillatory than previously.

13.9 Practical 3: Three Term Speed Control

13.9.1 Objectives and Background

In this practical the behaviour of an analogue speed control system is investigated. The controller is a PID block such that the relative amounts of derivative and integral control may be varied.

Of particular interest is how the derivative control affects the stability of the system and how the integral control affects following error. In this context, note that there is no perfect solution for a controller and it turns out to be a compromise depending on which of the performance parameters is of most importance.
13.9.2 Block Diagram

Figure 13-10: Block Diagram for Practical 3

13.9.3 Perform Practical

Figure 13-11 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

The input is initially a square wave.

Set the sweep function generator output amplitude control to minimum and the min freq control to minimum.

Set the derivative and integral gain controls to minimum.

Set the capacitor discharge switch to its right-hand position.

Set the attenuator control for the main proportional gain to half scale.

Open the Data Logger and the Bar Display.

Increase the sweep function generator output amplitude control to about half scale. The speed control loop should start to generally follow the input signal (blue). Check that the period of the input signal is approximately 6 seconds and adjust the sweep function generator min freq control if necessary.

Note that the measured speed (yellow) never reaches the set speed because the motor speed is purely proportional to error. This means that if the error were zero there would be no drive to the motor.

Change the proportional gain and note that the final error decreases with increasing gain (orange).
Set the attenuator control for the main proportional gain back to half scale.

Set the capacitor discharge switch to its left-hand position.

Increase the integral gain. Note how the error is reduced to zero by the integrator supplying the drive to the motor even when the error is zero but that the response is quite slow.

Remove connection 2 and add connection 22. This changes the input signal to be a fixed voltage supplied by the input potentiometer.

Set the speed to be about positive one third of the input potentiometer range and set the integral control to be about half scale.

Set the capacitor discharge switch to its right-hand then left-hand position to discharge it. The error should fall to zero as the required speed matches the measured speed.

Apply the brake and note how the speed falls but then recovers as the integrator supplies more drive to the motor. Remove the brake and note that the motor drive signal falls to maintain zero error.

Figure 13-12: Motor Loading Characteristics
Notes
14 Single Amplifier Control Circuits

14.1 Objectives

- To learn that a single amplifier can provide P + D or P + I or P + D + I control.
- To learn that single amplifier circuits are not as versatile as a full 3-term controller.

14.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 1.3 Control Systems
- 1.4 Closed-Loop Control System
- 1.5 Analogue and Digital Systems

14.3 Single Amplifier P + D Control

The circuit of a P+D controller is given in Figure 14-1. Since the input to A2 is a virtual earth point (see the Operational Amplifier Characteristics assignment), a unity gain proportional component can be added to A2 output by an input resistor as in Figure 14-2. The output from A2 could then drive the power amplifier directly.
It is important to note that the controller of Figure 14-1 has two amplifiers in each path to the PA input, while the simpler circuit in Figure 14-2 has only one and hence causes a sign reversal, which must be compensated by reversing the power amplifier drive.

14.4 Single Amplifier P + I Control

![Figure 14-3: Single Amplifier P + I Control](image)

The proportional + integral control of Figure 14-3 (a) can also be combined in a single amplifier with the general arrangement of Figure 14-3 (b).

If a step input is considered for Figure 14-3 (b) the input current will be:

\[ i = \frac{V_i}{R_1} \]

This current passes through the series capacitor and resistor to give:

\[ V_C = \frac{1}{C} \int i \, dt = \frac{1}{CR} \int V_i \, dt \]
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Single Amplifier Control Circuits

\[ VR = iR = \frac{R}{R_1} V_1 \]

The output voltage \( V_o \) will be the sum of these two voltages giving:

\[ V_0 = (-) \left( \frac{R}{R_1} V_1 + \frac{1}{CR} \int V_1 dt \right) \]

representing proportional + integral components of \( V_l \).

If \( R_1 \) is chosen as a convenient value, the relative size of the proportional and integral components can be adjusted by choice of \( R, C \), but without the convenience of continuous adjustment available with the circuit of Figure 14-3 (a). Also Figure 14-3 (b) has an inherent sign reversal.

14.5 Proportional, Integral and Derivative Control

The combination of the three terms (proportional, integral and derivative) can be thought of as separate characteristics.

Proportional, to provide the general error driven control signal. Integral, so that there does not have to be a residual error to provide the control signal.

Derivative, to give the system stability and hence reduce overshoot.

However, in some ways the derivative and integral terms act against each other and are all controlled by one overall gain, making the analysis much more involved.

The error control channel is like this:

Expressed mathematically the motor control voltage, \( V_m \) is given by:

\[ V_m = \text{error} + \text{integral of error} + \text{derivative of error} \]
\[ = V_e + \int V_e \, dt + dV_e/dt \]

where \( V_e \) is the error voltage.
When gain factors are added then:

\[ V_m = K_p (V_e + K_i \left( \int V_e \, dt \right) + K_d (dV_e/dt)) \]

Where \( K_p \) is the proportional gain, \( K_i \) the integral gain and \( K_d \) the derivative gain

### 14.6 Practical 1: Single Amplifier Controller

#### 14.6.1 Objectives and Background

This assignment investigates single amplifier circuits to give the characteristics of 3-term control. However, the circuits examined are not as versatile as a proper 3-term controller, since they do not have facilities for continuous adjustments.

The circuits are not substitutes for a full 3-term controller, but have application in particular situations where the component values are chosen to meet system requirements.

#### 14.6.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 14-5: Block Diagram for Practical 1

#### 14.6.3 Perform Practical

Figure 14-6 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Since derivative control improves the transient response of a system and can be introduced by an RC circuit across an operational amplifier input resistor, this can be applied across the input resistor of the P + I circuit. This is shown in Figure 14-7 and represents single amplifier P + I + D control.

Open the Data Logger.
Set the summing & error amplifier attenuator control to approximately half scale.
Increase the sweep function generator output amplitude control to about half scale and the position control loop should start to generally follow the triangle input signal. Check that the period of the input signal (blue) is approximately 6 seconds and adjust the sweep function generator min freq control if necessary.
Set the capacitor discharge switch to its right-hand position and then back to its left-hand position.

The motor should now give an output of ±90°.

Note the error displayed on the Data Logger display in both normal mode and X-Y mode.

The gain of 5 provided by the controller amplifier could be incorporated in the compensating circuit by reducing the integrating capacitor by 5 and increasing the output resistor by 5.

It is useful to experiment with different capacitor combinations and observe the effect on ramp and step transients.

First try changing the 0.5µf capacitor in the derivative section of the circuit to 1µF and then to 2µF (1µf + 1µF in parallel) and note the effects of these changes.

Replace the 0.5µF capacitor.

Set the capacitor discharge switch to its right-hand position, this short circuits the capacitor, note the effect.

Set the capacitor discharge switch to its left-hand position.

Change the 4µF capacitor in the integral circuit to 2µF and then to 6µF (4µF + 2µF in parallel) and note the effects seen.

Also try setting the capacitor switch to its right-hand position and note the effect.

Set the integrator capacitor to 6µF.

Now set the input to a square-wave from the sweep function generator.

Change the derivative capacitor from 0.5µF to 1µF and then to 2µF (1µF + 1µF in parallel) and note the effects of these changes.

Replace the 0.5µF derivative capacitor.

Change the integrator capacitor to 4µF. The system should now be oscillating.

Set the derivative capacitor to 1µF or 2µF (1µF + 1µF in parallel). This should stabilise the system.

This balancing act with component values shows single op-amp PID control is not a substitute for a full 3-term controller, but they have application in particular situations where the component values are chosen to meet system requirements and do not require the flexibility of adjustment after values have been initially set.
15 Transient Velocity Feedback and Feed-forward Systems

15.1 Objectives

- To learn that Transient Velocity Feedback and Derivative Feed-forward are techniques that can be used to reduce following error.

15.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 7.3 Velocity Feedback
- 8.4 Velocity Feedback and Following Error

15.3 Practical 1: Transient Velocity Feedback

15.3.1 Objectives and Background

Velocity feedback (tacho-generator feedback) has a powerful effect in improving transient response, but also has the effect of increasing following error. The error increase is due to the steady component of the feedback, while it is the changing component during transients which really gives the transient response improvement.

For systems with a high gain in order to give a small error the increase due to velocity feedback may be undesirable.
Figure 15-1: Transient Velocity Feedback

If a capacitor is placed in the velocity feedback line as in Figure 15-1 (a), which may in practice be an operational amplifier, as in Figure 15-1 (b), the steady component is blocked, preventing increased error, but the varying component is transmitted. Hence the description transient velocity feedback.

The improvement in response may not be as marked as direct velocity feedback, which can give dead-beat response from a system with an initially very poor response. It can however give a significant improvement without increasing the following error in systems with a high forward gain to obtain a small error.

A system with transient velocity feedback is given in the practical section, where the tacho-generator signal is supplied to the proportional amplifier through an integration capacitor, which can be short-circuited by SW2. There is a gain of about 20 in A_v to give small error, but the direct error is available at the error amplifier output.
15.3.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 15-2: Block Diagram for Practical 1

15.3.3 Perform Practical

Figure 15-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the summing & error amplifier attenuator and the tacho-generator attenuator controls to minimum.
Set the capacitor switch to its right-hand position (Capacitors short circuit).
Set the sweep function generator output amplitude control to half scale.
Set the summing & error amplifier attenuator control to full scale and the system may be unstable or if perturbed by the motor check switch on the Mechanical Assembly may go unstable.

Adjust the tacho-generator attenuator so that a small amount of tacho-generator feedback stabilises the system.

Open the Data Logger. Enlarge the Data Logger display.

Adjust the sweep function generator output amplitude control to give about ±90° at the output of the motor.

Check that the period of the input signal (blue) is approximately 6 seconds and adjust the sweep function generator min freq control if necessary.

Examine the error in normal mode and X-Y mode.

Set the capacitor switch to its left-hand position (open circuit).

This will block the steady tacho-generator signal and after the initial transient the error will be reduced.

It is useful to try different values of capacitor in the tacho-generator line. Values try the capacitor with values of 4µF, 0.5µF and 1µF. The 0.5µF (1µF + 1µF series) and 1µF can be obtained using the derivative section capacitors.

These tests demonstrate the operation of transient velocity feedback.

15.4 Practical 2: Feed-forward Control

15.4.1 Objectives and Background

If a system is following a ramp input, a constant drive signal must be supplied to the motor, which is normally provided by the following error, as in Figure 15-5, where the shaded portion is ignored. The following error can be zero if integral control is used.
If the input is differentiated directly as represented by the dotted portion of the diagram above, this would provide a constant signal $V_d$ as shown, which could be added into the forward path through an adjustable gain $G_d$. If it is arranged that

$$\frac{d(npl_t)}{dt} \cdot G_d = V_e$$

then the differential signal will provide the necessary drive and:

following error = $(V_i - V_o) = 0$

If the input slope varies, requiring a different drive value to the motor, this will automatically occur, since the differentiator output will change.

### 15.4.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 15-6: Block Diagram for Practical 2

### 15.4.3 Perform Practical

Figure 15-7 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

A circuit to investigate feed-forward control is given in Figure 15-8.

The differentiation can be carried out in the derivative circuit, and the normal error \( (V_e) \) and differentiated signal \( (V_d) \) added in the Controller output amplifier and applied to the power amplifier.
Note that since there is one additional amplifier in the forward path, the power amplifier input polarity must be changed to give overall negative feedback.

Open the Data Logger. Select the X-Y mode.

Set the derivative gain to minimum and the summing & error amplifier attenuator control to full scale.

Adjust the sweep function generator output amplitude control to give about ±90° and set the output to about 0.1Hz with the max freq control.

Increase the derivative gain control and it should be possible to eliminate the following error after an initial transient.

If a larger derivative signal is required, the 100k input to the Controller output amplifier can be reduced to 50k by using another 100k input in parallel.

Note that it is possible to overcompensate by excessive derivative signal, the system then leading the input in steady state.

Note that adjusting the amplitude or increasing the triangle frequency does not affect the steady state error elimination.

Although derivative feed-forward provides a technique to eliminate following error, it requires an exact adjustment of derivative gain, while integral control, though introducing a significant transient, does not require an exact adjustment.

In addition, any noise on the input is emphasised by feed-forward, whilst integral control does not emphasise noise.
16 Analogue to Digital Conversion

16.1 Objectives

- To learn that an analogue signal can be represented in digital form (binary).
- To learn how sampling rate and resolution influence the accuracy of the equivalent digital signal.
- To learn that Pulse Width Modulation (PWM) is another way of representing an analogue signal digitally, which can be used to drive a motor.
- To learn that PWM motor control has benefits when compared with analogue signal control.

16.2 A/D Conversion

Continuous electrical signals are converted to the digital language of computers using analogue-to-digital (A/D) converters.

In addition to the converter itself, sample-and-hold circuits, an amplifier, a multiplexer, timing and synchronization circuits, and signal conditioning elements also may be on board (Figure 16-1). The logic circuits necessary to control the transfer of data to computer memory or to an internal register also are needed.

![Figure 16-1: Analogue Input Flow Diagram](image_url)

When determining what type of A/D converter should be used in a given application, performance should be closely matched to the requirements of the analogue input transducer(s) in question. Accuracy, signal frequency content, maximum signal level, and dynamic range all should be considered.

Central to the performance of an A/D converter is its resolution, often expressed in bits. An A/D converter essentially divides the analogue input range into $2^N$ bins, where $N$ is the number of bits. In other words, resolution is a measure of the number of levels used to represent the analogue input range and determines the converter's sensitivity to a change in analogue input.
This is not to be confused with its absolute accuracy! Amplification of the signal, or input gain, can be used to increase the apparent sensitivity if the signal's expected maximum range is less than the input range of the A/D converter. Because higher resolution A/D converters cost more, it is especially important to not buy more resolution than you need—if you have 1% accurate (1 in 100) temperature transducers, a 16-bit (1 in 65,536) A/D converter is probably more resolution than you need.

Absolute accuracy of the A/D conversion is a function of the reference voltage stability (the known voltage to which the unknown voltage is compared) as well as the comparator performance. Overall, it is of limited use to know the accuracy of the A/D converter itself. Accuracy of the system, together with associated multiplexer, amplifier, and other circuitry is typically more meaningful.

The other primary A/D converter performance parameter that must be considered is speed—throughput for a multi-channel device. Overall, system speed depends on the conversion time, acquisition time, transfer time, and the number of channels being served by the system:

- Acquisition is the time needed by the front-end analogue circuitry to acquire a signal. Also called aperture time, it is the time for which the converter must see the analogue voltage in order to complete a conversion.
- Conversion is the time needed to produce a digital value corresponding to the analogue value.
- Transfer is the time needed to send the digital value to the host computer’s memory. Throughput, then, equals the number of channels being served divided by the time required to do all three functions.

### 16.3 A/D Converter Options

While all analogue-to-digital converters are classified by their resolution or number of bits, how the A/D circuitry achieves this resolution varies from device to device.

There are four primary types of A/D converters used for industrial and laboratory applications:

- successive approximation,
- flash/parallel,
- integrating, and
- ramp/counting.

Some are optimized for speed, others for economy, and others for a compromise among competing priorities (Figure 16-2). Industrial and lab data acquisition tasks typically require 12 to 16 bits; 12 is the most common. As a rule, increasing resolution results in higher costs and slower conversion speed.
16.4 Successive approximation

The most common A/D converter design used for general industrial and laboratory applications is successive approximation (Figure 16-3). This design offers an effective compromise among resolution, speed, and cost. In this type of design, an internal digital-to-analogue (D/A) converter and a single comparator—essentially a circuit that determines which of two voltages is higher—are used to narrow in on the unknown voltage by turning bits in the D/A converter on until the voltages match to within the least significant bit. Raw sampling speed for successive approximation converters is in the 50kHz to 1MHz range.

To achieve higher sampling speeds, a redundancy technique allows a fast initial approximate conversion, followed by a correction step that adjust the least significant bit after allowing sufficient settling time. The conversion is therefore completed faster at the expense of additional hardware. Redundancy is useful when both high speed and high resolution are desirable.

16.5 Flash/parallel

When higher speed operation is required, parallel, or flash-type A/D conversion is called for. This design uses multiple comparators in parallel to process samples at more than 100MHz with 8 to 12-bit resolution. Conversion is accomplished by a string of comparators with appropriate references operating in parallel (Figure 16-4).

The downside of this design is the large number of relatively expensive comparators that are required. For example, a 12-bit converter requires 4,095 comparators.
16.6 Integrating

This type of A/D converter integrates an unknown input voltage for a specific period of time, then integrates it back down to zero. This time is compared to the amount of time taken to perform a similar integration on a known reference voltage. The relative times required, and the known reference voltage, then yield the unknown input voltage. Integrating converters with 12 to 18-bit resolution are available, at raw sampling rates of 10–500kHz.

Because this type of design effectively averages the input voltage over time, it also smoothes out signal noise. And, if an integration period is chosen that is a multiple of the ac line frequency, excellent common mode noise rejection is achieved. More accurate and more linear than successive approximation converters, integrating converters are a good choice for low-level voltage signals.

16.7 Ramp/counter

Similar to successive approximation designs, counting or ramp-type A/D converters use one comparator circuit and a D/A converter (Figure 16-5). This design progressively increments a digital counter and with each new count generates the corresponding analogue voltage and compares it to the unknown input voltage. When agreement is indicated, the counter contains the digital equivalent of the unknown signal.

A variation on the counter method is the ramp method, which substitutes an operational amplifier or other analogue ramping circuit for the D/A converter. This technique is somewhat faster.
16.8 Multiplexing & Signal Conditioning

As shown in Figure 16-1, A/D converters seldom function on their own but must be considered in a systems context with associated circuitry for signal conditioning, multiplexing, amplification, and other functions. Every application will dictate a unique mix of add-ons that may be implemented in a variety of physical configurations-on a PC I/O board, inside a remote transmitter, or at a local termination panel.

Multiplexing: In many industrial and laboratory applications, multiple analogue signals must be converted to digital form. And if speed is not the limiting factor, a single A/D converter often is shared among multiple input channels via a switching mechanism called a multiplexer. This is commonly done because of the relatively high cost of converters. Multiplexers also allow amplification and other signal conditioning circuitry to be time-shared among multiple channels. Software or auxiliary hardware controls the switch selection.

Sample-and-hold: It is important to acknowledge that a multiplexer does reduce the frequency with which data points are acquired, and that the Nyquist sample-rate criterion still must be observed. During a typical data acquisition process, individual channels are read in turn sequentially. This is called standard, or distributed, sampling. A reading of all channels is called a scan. Because each channel is acquired and converted at a slightly different time, however, a skew in sample time is created between data points (Figure 16-6).
If time synchronization among inputs is important, some data acquisition cards offer “burst” mode operation or “simultaneous sample-and-hold” circuitry. Burst mode, or pseudo-simultaneous sampling, acquires each channel at the maximum rate of the board, then waits a user-specified amount of time before sampling again.

True simultaneous sample-and-hold systems can sample all channels within a few nanoseconds of each other, eliminating phase and time discontinuities for all but the fastest processes. Essentially, a switched capacitor on each channel tracks the corresponding input signal. Before starting the A/D conversion process, all switches are opened simultaneously, leaving the last instantaneous values on the capacitors.

**Signal scaling:** Because A/D converters work best on signals in the 1–10 V range, low voltage signals may need to be amplified before conversion—either individually or after multiplexing on a shared circuit. Conversely, high voltage signals may need to be attenuated.

Amplifiers also can boost an A/D converter’s resolution of low-level signals. For example, a 12-bit A/D converter with a gain of 4 can digitize a signal with the same resolution as a 14-bit converter with a gain of 1. It's important to note, however, that fixed-gain amplifiers, which essentially multiply all signals proportionately, increase sensitivity to low voltage signals but do not extend the converter's dynamic range.

Programmable gain amplifiers (PGAs), on the other hand, can be configured to automatically increase the gain as the signal level drops, effectively increasing the system's dynamic range. A PGA with three gain levels set three orders of magnitude apart can make a 12-bit converter behave more like an 18-bit converter. This function does, however, slow down the sample rate.

From a systems perspective, amplifier performance should be on par with that of the A/D converter itself—gain accuracy should be specified as a low percentage of the total gain. Amplifier noise and offset error also should be low.

Other conditioning functions: Other A/D signal conditioning functions required will vary widely from application to application. Among the options:

**Current-to-voltage conversion:** A 4–20mA current signal can be readily converted to a voltage signal using a simple resistor (Figure 16-7). A resistor value of 250ohms will yield a 1–5 V output.

Figure 16-7: Conversion of 4–20 mA to 1-5 V
Filtering: A variety of physical devices and circuits are available to help separate desired signals from specific frequencies of undesirable electrical noise such as ac line pick-up and other electromagnetic/radio frequency interference (EMI/RFI). If the signal of interest is lower in frequency than the noise, a low-pass filter can be used. High-pass and notch-band filters are designed to target low frequency interference and specific frequency bands, respectively.

Excitation: Voltage supplied by the data acquisition card or discrete signal conditioner to certain types of transducers such as strain gages.

Isolation: Used to protect personnel and equipment from high voltages. Isolators block circuit overloads while simultaneously passing the signal of interest.

16.9 Single-Ended & Differential Inputs

Another important consideration when specifying analogue data acquisition hardware is whether to use single-ended or differential inputs (Figure 16-8). In short, single-ended inputs are less expensive but can be problematic if differences in ground potential exist.

In a single-ended configuration, the signal sources and the input to the amplifier are referenced to ground. This is adequate for high level signals when the difference in ground potential is relatively small. A difference in ground potentials, however, will create an error-causing current flow through the ground conductor otherwise known as a ground loop.

Differential inputs, in contrast, connect both the positive and negative inputs of the amplifier to both ends of the actual signal source. Any ground-loop induced voltage appears in both ends and is rejected as a common-mode noise. The downside of differential connections is that they are essentially twice as expensive as single-ended inputs; an eight-channel analogue input board can handle only four differential inputs.

Figure 16-8: Single-ended & Differential Analogue Input configurations
16.10 Sampling Rate and Resolution

Signals in the real world are analogue. In a digital communications system the first process is to turn these analogue signals into digital format. The signals could be anything: speech, television or representing the pH of a liquid, for example. However, the common factor linking analogue signals is that they are “time continuous”. This means that they are varying in time in a smooth manner. The graph in Figure 16-9 shows a typical time continuous varying signal.

A digital signal is a series of discrete numbers that describes the signal, where each number represents the signal at a particular point in time. This means that analogue signal has to be “sampled” at various points in time and each value converted to a digital number. This concept of sampling is very important to understand.

In order for the digital signal to be useful, three further factors have to be considered:

- the sampling has to be regular;
- the time interval between samples has to be short enough to follow the fastest changes in the analogue signal;
- in a digital signal not only is the time domain in discrete steps but so is the signal itself.

For example a signal may be represented by zero to fifteen amplitude states, which might mean that some of the finer detail may be lost. The number of steps to which the signal is digitised is an important consideration.

The terms used to describe these digitising parameters are:

- the rate at which the signal is sampled regularly is called the sampling rate;
- the number of levels in the digital signal is called the resolution;
- the resolution is often a power of two as this represents steps in the number of bits in a binary system.

For example 16 levels requires 4 bits and 256 levels requires 8 bits.

The graph in Figure 16-10 shows the same signal but sampled and digitised to 8 levels.
Note that the output steps between the available levels and is timed at the sampling points. Note also that some of the detail of the signal has been lost due to both the lack of resolution and the low sampling rate. In a digital system the choice of resolution and sampling rate must be made very carefully.

If the sampling rate is far too low, then the wrong waveshape can be produced from time repetitive signals. This effect is called aliasing and is described in a concept section.

There are several methods of implementing both the analogue to digital process.

### 16.11 Practical 1: Sampling Rate Resolution

#### 16.11.1 Objectives and Background

In this practical you will take an analogue signal, convert it to a digital one and pass it to the computer. It can then be displayed by the computer and the result observed on the Data Logger instrument.

The effects of different sampling rates, and bit resolution, on a continuous analogue signal when using an analogue to digital converter will then be demonstrated.
16.11.2 Block Diagram

Figure 16-11: Block Diagram for Practical 1

16.11.3 Perform Practical

Figure 16-12 shows the required connections on the hardware.
Set the sweep function generator output amplitude and min freq control to half scale.

Open the Data Logger.

Enlarge the Data Logger window.

The signal displayed is an analogue test signal, having been converted to digital to enable it to be displayed on the Data Logger.

Initially the sample rate is set to 125Hz.

Now select the 62.5, 41.7 and 31.25Hz sample rates, noting the change to the displayed waveform. With each reduction in the sampling rate, the waveform will have more pronounced steps than the previously.

While on the 31.25Hz setting decrease the sweep function generator min freq control to about 15% of scale, the steps on the waveform should now be less pronounced.

This shows that sampling frequency needs to be substantially higher the frequency of the signal being converted.

Select the 125Hz setting.

Set the sweep function generator min freq control to half scale.

Now select 4 bit resolution.
Note the waveform will now have steps present, similar to those seen when the sampling frequency is too low.

Select 8 bit resolution and 62.5Hz sampling rate.

Select 4 bit resolution.

The selecting 4 bit resolution will have less effect now on the sampled signal.

So this practical has shown that sampling rate and resolution are key parameters which are influenced by the characteristics of the signal to be converted.

16.12 Practical 2: Pulse Width Modulation

16.12.1 Objectives and Background

In control systems, dc motors are often used because the speed and direction are controlled by the magnitude and direction of the applied voltage. Since the signal intended to operate the motor is a low power signal, a power amplifier is required to drive the motor. This amplifier needs to be linear, i.e. the output proportional to the input, and this leads to some design problems together with cooling problems.

An alternative method uses a switching amplifier which rapidly switches full power to the motor between forward and reverse directions. The ratio of forward to reverse power can be varied, and provided the switching is sufficiently rapid, the motor only responds to the average power.

The signal applied to the motor is a square wave, the mark/space ratio of which is varied while the frequency is kept constant. This drive method is called Pulse Width Modulation (PWM).

The main advantage is that in general, and particularly at higher powers, switching circuits are easier to design than linear ones and have lower heat dissipation.
16.12.2 Block Diagram

Figure 16-13: Block Diagram for Practical 2

16.12.3 Perform Practical

Figure 16-14 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Ideally, for this practical, an external oscilloscope is required. If an oscilloscope is not available then refer to the provided movie.

In this practical you will see a pulse width modulator used to drive a motor.

A triangle signal is applied to the A/D converter and passed through the digital controller, and is output as PWM.

On the Control & Instrumentation Principles workboard, the PWM frequency is approximately 330Hz and you can hear the motor vibrating at this frequency.

Set the sweep function generator min freq and max freq control to minimum, and set the output amplitude to full scale.

Adjust the minimum frequency and amplitude control and note the effect on the motor.

Open the Data Logger.

The triangle input signal is shown in blue and the tacho-generator output in orange.
Connect the external oscilloscope (if available) to the PWM signal from the digital controller, and observe the waveform in relation to the input triangle waveform on the Data Logger. If no oscilloscope is available, view the movie ‘Pulse Width Modulation’ for this practical to see the typical PWM waveform generated. It can be seen how the mark space ratio of the square wave is proportional to the input signal.

The analogue tacho-generator signal should track the input signal although at reduced amplitude.
Notes
17 Digital to Analogue Conversion

17.1 Objectives

- To learn that an analogue signal that has been converted into a digital signal can be converted back to an analogue signal by using a D/A converter.

17.2 Digital to Analogue Converters

A digital signal is a sequence of pulses, where each pulse is represented by a digital word which has a finite number of bits (binary digits).

Digital signals derived from a computer may be required in a continuous (analogue) form, for various applications. Typical examples are motor control or audio signals in a digital communication system.

A D/A converter (or simply DAC), is a circuit that provides an analogue voltage (or current) that is the weighted sum of the bits in the digital word.

In an A/D conversion, the analogue waveform is sampled and the resulting discrete-time analogue samples are quantised by truncating or rounding, and finally these quantised values are encoded into a digital word.

Therefore, unlike the A/D conversion, the D/A conversion process is unique; that is, there is a one-to-one correspondence between a given digital word and an analogue value.

However, there are gaps between levels, arising from the existing quantised values. Thus, the converted signal has abrupt discontinuities and presents a staircase type of pattern. Hence, low pass filtering is often required to smooth the restored analogue signal by removing the high-frequency content.

An eight bit device gives 256 levels and a 10 bit one 1024 levels. The output voltage is often passed through a buffer to obtain the range required for a particular application.

17.3 D/A Conversion

Analogue outputs commonly are used to operate valves and motors in industrial environments and to generate inputs for electronic devices under test. Digital-to-analogue (D/A) conversion is in many ways the converse of A/D conversion, but tends to be generally more straightforward. Similar to analogue input configurations, a common D/A converter often is shared among multiplexed output signals. Standard analogue output ranges are essentially the same as analogue inputs: ±5V dc, ±10V dc, 0–10V dc, and 4–20mA dc.
Essentially, the logic circuitry for an analogue voltage output uses a digital word, or series of bits, to drop in (or drop out, depending on whether the bit is 1 or 0) a series of resistors from a circuit driven by a reference voltage. This ladder of resistors can be made of either weighted value resistors or an R-2R network using only two resistor values, one if placed in series (Figure 17-1). While operation of the weighted-value network is more intuitively obvious, the R-2R scheme is more practical. Because only one resistor value need be used, it is easier to match the temperature coefficients of an R-2R ladder than a weighted network, resulting in more accurate outputs. Plus, for high resolution outputs, very high resistor values are needed in the weighted-resistor approach.

![Figure 17-1: Weighted Value & Single Value Resistor Networks for D/A Conversion](image)

Key specifications of an analogue output include:

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**Settling time**: Period required for a D/A converter to respond to a full-scale setpoint change.

**Linearity**: This refers to the device's ability to accurately divide the reference voltage into evenly sized increments.

**Range**: The reference voltage sets the limit on the output voltage achievable.

Because most unconditioned analogue outputs are limited to 5mA of current, amplifiers and signal conditioners often are needed to drive a final control element. A low-pass filter may also be used to smooth out the discrete steps in output.

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17.4 Practical 1: Signals Generated by D/A Conversion

### 17.4.1 Objectives and Background

In this practical an analogue signal is converted by the digital controller into a digital signal. This is then converted by using a D/A converter to reproduce the original signal.
17.4.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 17-2: Block Diagram for Practical 1

17.4.3 Perform Practical

Figure 17-3 shows the required connections on the hardware.
Set the sweep function generator min freq control to minimum and the amplitude control to half scale.

Open the Data Logger.

The input test signal (blue) is input into the A/D on the digital controller, the digital signal produced from this sent to the D/A by the digital controller which then converts this digital signal back into an analogue signal (orange).

Compare both signals by selecting overlay. The two signals should be almost identical.

In some A/D to D/A conversions a phase shift due to conversion times may be seen, and where resolution and sampling rate are incorrectly specified in either conversion the comparison of the input and output signals may produce differences because of these factors.
18 Digital Position Measurement

18.1 Objectives

- To understand how Gray code sensors and Incremental encoders produce an output signal.
- To learn that Gray code sensors and Incremental encoders can be used to measure output shaft position.

18.2 Absolute Encoder

The absolute encoder is an encoder that gives as an output a binary value, usually Gray code, that indicates absolute position.

A pattern of tracks is printed on the absolute encoder disk, each track is represented by one bit, and a sector of tracks produces the position of the disk as a number in Gray code. This code is used in preference to pure binary, because only one bit changes between two consecutive states.

The encoder disk that is used in the Mechanical Unit is shown in Figure 18-1.

Figure 18-1: Absolute Encoder Disk

Figure 18-2 shows the codes that are represented by these patterns.
### Decimal | Pure Binary | Gray Code
--- | --- | ---
0 | 0000 | 0000
1 | 0001 | 0001
2 | 0010 | 0011
3 | 0011 | 0010
4 | 0100 | 0110
5 | 0101 | 0111
6 | 0110 | 0101
7 | 0111 | 0100
8 | 1000 | 1100
9 | 1001 | 1101
10 | 1010 | 1111
11 | 1011 | 1110
12 | 1100 | 1010
13 | 1101 | 1011
14 | 1110 | 1001
15 | 1111 | 1000

**Figure 18-2: Absolute Encoder Patterns**

The Gray code values shown on this table are constructed by beginning with the zero state and going to the next state by changing only the least significant bit, with the old and new state differing only in that bit.

Therefore, only one track changes in value between adjacent angular positions. This means that the disk can be read at any one time and not only at the centre of each position.

If pure binary coding were used, massive errors could result if all the different tracks did not change at exactly the same time.

For example, in going from binary 7 (0111) to binary 8 (1000), an output of 15 (1111) could be obtained if the most significant bit (msb) is changed first.

If the same operation were performed in Gray code, it would result in no error since 7 and 8 are represented by 0100 and 1100 respectively, which means that only the msb is required to change.

The translation from Gray code to a binary or decimal number is done by the computer using a table, as a number in Gray code has a direct equivalent in decimal.
18.3 Incremental Encoders

The Optical Incremental Encoder is less complex than the absolute encoder. It consists of a rotating disk having a track of transparent windows. On one side of the disk, there is a light source, and just opposite to it, on the other side of the disk there is a light sensor.

A photograph of the sensor that is used with the hardware is shown in Figure 18-3.

![Figure 18-3: Encoder Light Sensor](image)

The encoder outputs a voltage pulse every time a transparent window passes the light source. With this sensor, electronic circuitry must be available to count the pulses, in order to determine the angle of rotation.

This encoder is called an incremental encoder, because the generation of a pulse indicates an incremental change in position, and not the actual (absolute) position.

![Figure 18-4: Incremental Encoder Disk](image)

A reference window is required to determine absolute position. The reference window is used for initialisation, representing zero position.

In order to be able to determine the direction of rotation, a second light sensor is placed at a different window of the encoder track and is offset from the first sensor. The second sensor points at the edge of a window, when the first sensor points at the centre of a window.
Thus, for movement at a constant velocity, sensor 1 square wave output leading sensor 2 output by 90° indicates one direction of rotation; sensor 1 output lagging sensor 2 output by 90° indicates the other direction of rotation.

You can see the two encoders in the Mechanical Unit and their associated infra-red sensors. In practice they are contained in a sealed unit and are much smaller.

Most encoders are of the incremental type, because these can have greater resolution due to the simpler track arrangement and they are much cheaper.

18.4 Tacho-generator

A tacho-generator is a device that produces a dc voltage the amplitude and polarity of which is proportional to the speed of its input shaft. It is in fact simple a dc permanent magnet motor which means it also has this property. However devices designed for this purpose are specified to have low friction and to have the output voltage as linear as possible with respect speed. Unlike a dc generator their power output is not important as they usually feed a high input resistance amplifier.

18.5 Practical 1: The Gray Code Sensor

18.5.1 Objectives and Background

In other assignments, you can see how a potentiometer may be used as a shaft position transducer (sensor).

In that analogue technique, the displacement of the slider is proportional to the resistance from the beginning of the potentiometer to the position of the slider, and hence, the output voltage across this resistance represents the displacement of the shaft.

However, advanced shaft position measurements can be obtained by employing digital techniques.

In this practical a Gray code sensor (absolute encoder) operation is demonstrated for position measurement.
18.5.2 Block Diagram

Figure 18-5: Block Diagram for Practical 1

18.5.3 Perform Practical

Figure 18-6 shows the required connections on the hardware.
Set the motor speed to about 3 RPM using the power amplifier offset control.

Open the Data Logger.

Enlarge the Data Logger window.

The blue trace shows the 360° potentiometer output (analogue sensor).

The yellow trace shows the Gray encoder output (digital sensor).

The Gray encoder trace is the result of the digital signal being read in by the digital controller and which has then sent the digital value to the computer to display on the Data Logger.

Note the gray encoder output LEDs against the output signal display on the Data Logger, it should be possible to spot the 180° point by all the LEDs not being lit up.

Count the levels in the Gray’s encoder waveform for one cycle.

These should total 64, given that the Gray encoder is 6-bit therefore $2^6 = 64$ levels.
18.6 Practical 2: Incremental Sensors

18.6.1 Objectives and Background

In the previous assignments, you have seen how a potentiometer (analogue sensor) and absolute encoders (digital sensor) can be used as a shaft position transducer.

In this practical an Incremental encoder sensors operation is demonstrated for measuring position.

Incremental encoders, also known as relative encoders, are simpler in design than the absolute encoders. They can consist of up to two tracks and two sensors whose outputs are called channels A and B, although one track with the sensors offset is used in this case. As the shaft rotates an output consisting of a trail of pulses at a frequency proportional to the shaft speed is produced. Additionally the phase relationship between the two signals produced indicates the direction of rotation. The diagram below shows the typical output.

![Figure 18-7: Typical Output of Incremental Encoder](image)

The output pulses from the sensor are counted, and knowing the resolution of the encoder enables position of rotation to be calculated. If there was only one output channel then direction of rotation could not be determined.

A third output channel called Index (I), outputs one pulse per rotation, this is used to zero or synchronise the output. This is useful if the count for the other channels is lost, i.e. power loss in a system where no memory backup is provided.
18.6.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 18-8: Block Diagram for Practical 2

18.6.3 Perform Practical

Figure 18-9 shows the required connections on the hardware.
Open the Data Logger.

Using the power amplifier offset control set the motor rotating at 3 RPM in either direction.

Observe the motor shaft output and the Incremental encoder LEDs on the Servo fundamentals workboard.

As the output shaft passes the 0° point the LED labelled ‘I’ (Index) flashes.

Compare this to the output on the Data Logger. The yellow trace is from the 360° potentiometer (analogue sensor), and the blue trace is the incremental encoder (digital sensor).

The incremental encoder signal displayed on the Data Logger, this is the result of the incremental encoder output (digital) being read in by the digital controller which sends this value to the computer. This is then displayed on the Data Logger.

Now select the Index off button to inhibit the index pulse.

This means that the pulse, that indicates that a full rotation has been completed, has been disabled. Whilst there are no disturbances to cause the signals to become out of sync they will remain almost identical.

Set the hold button so the incremental count stops and then the run button. So now the incremental encoder signal is not in sync with respect to the analogue signal.
Set the Index ON button and check that the next zero crossing the index pulse resynchronises the incremental count.
19 Digital Speed Measurement

19.1 Objectives

- To learn that Incremental encoders can be used for motor speed measurement.
- To learn how the rotational direction of a motor can be obtained from incremental encoder.

19.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 10.3 Speed Control
- 17.2 Digital to Analogue Converters
- 17.3 D/A Conversion
- 18.3 Incremental Encoders

19.3 Practical 1: Incremental Encoder Speed and Direction Measurement

19.3.1 Objectives and Background

Incremental encoders, also known as relative encoders, can be used for speed and direction measurement. The sensors consist of two tracks and two sensors whose outputs are called channels A and B, although one track with the sensors offset is used in this case. As the shaft rotates an output consisting of a trail of pulses at a frequency proportional to the shaft speed is produced. Because the sensors are offset, the phase relationship between the two signals produced indicates the direction of rotation. The diagram below shows the typical output.
The output pulses can be counted, and knowing the resolution of the encoder enable speed of rotation to be calculated. Depending on which channel produces a pulse before the other, the direction of rotation can be determined. If there was only one output channel then direction of rotation could not be determined.

A third output channel called INDEX (I), outputs one pulse per rotation, this is used to zero or synchronise the output. This is useful if the count for channels A and B is lost, i.e. power loss in a system where no memory backup is provided or if a pulse has been missed throughout the course of 1 revolution of the motor.
19.3.2 Block Diagram

Figure 19-2: Block Diagram for Practical 1

19.3.3 Perform Practical

Figure 19-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

Open the Data Logger.

Turn the variable dc control slowly to full scale then to minimum, noting the effect on the Incremental encoder output signal (blue), against the tachometer signal (yellow).

When using the incremental encoder for speed and direction measurement the Index signal from the encoder is not needed, and so the digital controller has been configured not to take this signal into account.

The two signals aren’t the same amplitude but the incremental encoder signal should show similar behaviour to that of the tacho-generator.

When the variable dc control is turned below 50% scale, the motor runs in the opposite direction, and the output signal from the incremental encoder to also goes negative. The motor direction is determined on whether channel A or B of the incremental encoder produces a pulse before the other, and from this the output can be signed positive or negative accordingly.

If there was only one output channel then direction of rotation could not be determined.
20 Digital Controller with Analogue Sensors

20.1 Objectives

- To learn that analogue speed and position sensors can be used with a digital controller in a control loop.
- To learn that a digital controller can provide the same control signals as an analogue controller with analogue sensors by using A/D and D/A converters.

20.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 1.3 Control Systems
- 1.4 Closed-Loop Control System
- 1.5 Analogue and Digital Systems
- 6.2 Closed-Loop Control
- 16.2 A/D Conversion
- 16.3 A/D Converter Options
- 16.4 Successive approximation
- 16.5 Flash/parallel
- 16.6 Integrating
- 16.8 Multiplexing & Signal Conditioning
- 16.9 Single-Ended & Differential Inputs
- 17.2 Digital to Analogue Converters

20.3 Ramp/counter

Similar to successive approximation designs, counting or ramp-type A/D converters use one comparator circuit and a D/A converter (Figure 20-1). This design progressively increments a digital counter and with each new count generates the corresponding analogue voltage and compares it to the unknown input voltage. When agreement is indicated, the counter contains the digital equivalent of the unknown signal.

A variation on the counter method is the ramp method, which substitutes an operational amplifier or other analogue ramping circuit for the D/A converter. This technique is somewhat faster.
20.4 Practical 1: Position Control Loop

20.4.1 Objectives and Background

This practical demonstrates that a combination of an analogue sensor and a digital controller can be used to provide a position control loop. And that the resulting output is similar to that which would be produced from an all analogue control system.

The object of a positional control system is that the position of the output actuator should follow the set value as closely as possible.

The drive for the motor is derived from the difference between the input to the system and the actual measured position of the motor. This difference is the error signal. The error signal is amplified by a gain which means that the motor drive is simply a proportion of the error.

When the motor position value is close to the input value, the error signal will be small. With no gain the drive signal may not be large enough to keep the motor turning against friction.

The digital controller available on the Control & Instrumentation Principles workboard has the ability not only to provide the error signal (difference between set value and actual) for the position control, but also the ability to adjust the gain of this signal.

The gain of the error signal can be set by using the track bar instead of the manual gain controls on the Control & Instrumentation Principles workboard as used with an analogue controller.

Increasing the error gain will mean that a smaller error signal is required to keep the motor turning.

There is however, a finite value to the gain which can be applied. With a large gain, even a small value of error will produce a large drive to the motor. The drive value will overshoot whenever there is a large change in the input signal. This results in an unstable system.

Remember that increasing the proportional gain should give a faster response but will increase overshoot and eventually lead to instability.
20.4.2 Block Diagram

![Block Diagram for Practical 1](image)

20.4.3 Perform Practical

Figure 20-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical the digital controller is used, together with the 360° potentiometer sensor, to make a simple closed loop position control system. The triangle generator is used to provide an input to the system that the output shaft attempts to track. The input is called the set value and the A/D converter sends this to the digital controller.

The actual position of the shaft (the measured value) is sensed by the 360° potentiometer and this is also sent to the digital controller via a second A/D channel.

The difference between the two inputs (called error) is calculated and the result used to drive the motor via the D/A converter. In this example, the system is required to follow a steadily changing position demand (triangle waveform). This will cause the output shaft to move at a constant speed. The error signal between the demanded position and the actual position is called the ‘following error’.

In order for the error to be small, the error signal that drives the motor must be magnified so that small errors still correct the output; this magnification factor is called gain. The gain value has a large effect on the behaviour of the system and too much or too little both cause problems. The experiment will show both these effects.

Set the sweep function generator min freq control to minimum and the amplitude control to half scale.
Open the Data Logger.

On the Data Logger the signal are, set value (blue), digital controller error signal (orange), 360° potentiometer (yellow) and power amplifier drive signal from the D/A converter (green).

The motor should follow a steadily changing position demand (triangle waveform). The error signal displayed shows that an analogue sensor can be used with a digital controller to produce a control loop equivalent to an all analogue system as seen in previous practicals.

The digital controller also has the facility to increase the gain of the output signal. This can be increased by sliding the track bars below.

Experiment with the settings of the track bar and note the effect of the additional gain on the error signal.

**20.5 Practical 2: Speed Control Loop**

**20.5.1 Objectives and Background**

A speed control system using feedback is used when it is necessary to have a shaft rotating at a certain speed independent of load. In some applications the speed may be fixed but in others it may need to change in response to a varying set value.

As in positional control this experiment uses a simple proportional feedback system with the error signal controlling the motor. It follows that there must always be an error in the system or the motor would stop.

Multiplying the actual error by a gain and using this to drive the motor can help reduce the required minimum error. However, there is a limit to how much gain can be applied before instability occurs.

As well as instability there are other problems which can occur such as poor performance due to tachometer ripple.

More complex systems solve some of these problems by using other methods like integral control which does allow the motor to keep running with zero error.

This practical demonstrates that a combination of an analogue sensor and a digital controller can be used to provide a speed control loop. And that the resulting output is the same as that which would be produced from an all analogue control system.

The digital controller used also has the ability not only to provide the error signal (difference between set value and actual) for the speed control, but also gain of the error signal and both Derivative and Integral signals of the error signal to enable PID control.

The PID control signals are determined by the controller performing calculations on the error signal, these are then combined and the digital result of which is then sent out to the D/A converter.
The gain of the error signal and the gain of the Derivative and Integral signals are determined by the user using the track bars instead of the manual gain controls on the servo fundamentals workboard as used with an analogue controller.

Remember that increasing the proportional gain should give a faster response but will increase overshoot and eventually lead to instability.

### 20.5.2 Block Diagram

Figure 20-4 shows the required connections on the hardware.

### 20.5.3 Perform Practical

Figure 20-5 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical the digital controller is used, together with the tachometer, to make a simple closed loop speed control system.

The triangle wave generator is used to provide an input to the system that the output shaft attempts to track. The input (called the set value) is connected to the A/D converter with the result of the conversion being read by the digital controller.

The actual speed of the shaft (the measured value) is sensed by the tachometer and this is also sent to the digital controller via a second A/D channel.

The difference between the two inputs (called error) is calculated and the result used to drive the motor via the D/A converter.

In order for the actual error to be small the error signal that drives the motor must be magnified so that small errors still correct the output, this magnification factor is called gain.

Set the sweep function generator min freq control to minimum and the amplitude control to half scale.

Open the Data Logger.
On the Data Logger the blue signal represents the set value, the orange shows the digital controller error signal, the yellow is the tacho-generator output (analogue) and the green is the D/A signal driving the power amplifier.

From the error and tacho-generator signals displayed it can be seen that a tacho-generator sensor can be used with a digital controller to produce a control loop equivalent to an all analogue system as seen in previous practicals.

The digital controller also has the facility to increase the error signal gain by sliding the track bar below.

Experiment with the settings of the error signal gain by sliding the track bar.
21 Digital Controller with Digital Sensors

21.1 Objectives

- To learn that digital sensors can be used with digital controllers to implement a true digital control system.
- To learn that a digital controller can provide Proportional, Integral and Derivative (PID) speed and position control loops.
- To learn the effect of sampling rate change on Proportional, Integral and Derivative signals in a digital system.

21.2 Prerequisites

Before commencing this assignment, you should understand the concept discussed in the earlier section:

- 17.2 Digital to Analogue Converters

21.3 Proportional and Derivative Control

Derivative feedback control involves using the rate of change of the error signal to reduce overshoot.

When a step input is applied, the error initially rises to a high value and then decreases as the system nears alignment. The initial high rate of change of error results in the derivative output producing a very short positive peak.

As the motor accelerates to maximum speed, the error signal begins to decrease. Resulting in the derivative decreasing to a negative value, rising to zero when the motor has reached its maximum speed and the error becomes constant. The process then reverses producing a negative spike as the motor reverses at maximum overshoot.

In a proportional feedback system, the error signal is used to control the motor drive. The error signal does not go negative (producing reverse torque) until the step input goes negative.

The important point is that due to the overshoot of the error signal, no reverse torque can be applied via the motor to slow it down until after it has passed through alignment.

However a combination of error and its derivative becomes negative before alignment causing the motor to provide reverse torque and stop the overshoot. As the motor slows towards alignment the derivative component drops towards zero.

If too much derivative component is added the response becomes slow. The best response depends on the application but a small amount of overshoot is tolerable in exchange for a reasonable response speed.
Expressed mathematically the motor control voltage, \( V_m \) is given by:

\[
V_m = \text{error} + \text{derivative of error} = V_e + \frac{dV_e}{dt}
\]

where \( V_e \) is the error voltage.

When gain factors are added then:

\[
V_m = K_p (V_e + K_d \frac{dV_e}{dt})
\]

where \( K_p \) is the proportional gain and \( K_d \) the derivative gain.

### 21.4 Proportional and Integral Control

The reason that in a purely proportional control system there must be a residual following error is simple. As the motor is driven only by error, if there were none the motor would stop!

Hence if the system is static the error can be zero because there is no requirement to drive the motor. However, as soon as the motor is required to move, there must be following error.

Increasing the gain reduces the following error that is needed to keep the motor turning. For a fixed speed, the signal required to drive the motor is fixed. Therefore the higher the gain, the smaller the following error can be to provide that drive.

The faster the input changes, the faster the motor needs to go in order to follow the input. So, for a fixed gain, the larger the following error must become to supply the drive.

It is apparent therefore that there must always be an error present in order for a proportional control system to work. Increasing the gain reduces the error but introduces unwanted overshoot and eventually leads to an unstable system. To eliminate this error integral control action is used.

Suppose that the motor is being driven in order to follow an increasing input. The following error is a constant value. A sum of all the previous errors would be rising continuously and, if this component were added to the motor drive signal, the motor would speed up and the following error reduce.

This would in turn make the integral component level off at a value just enough to keep the motor running at the correct speed to make the error zero. The system always tries to maintain a state of zero following error.

The important point is that now even though the following error may be zero, the motor can still be driven by the integral component.

Expressed mathematically the motor control voltage, \( V_m \) is given by:

\[
V_m = \text{error} + \text{integral of error}
\]
where \( V_e \) is the error voltage.

When gain factors are added then:

\[
V_m = K_p V_e + K_i (\int V_e \, dt)
\]

where \( K_p \) is the proportional gain and \( K_i \) the integral gain.

### 21.5 Proportional, Integral and Derivative Control

The combination of the three terms (proportional, integral and derivative) can be thought of as separate characteristics.

Proportional, to provide the general error driven control signal. Integral, so that there does not have to be a residual error to provide the control signal. Derivative, to give the system stability and hence reduce overshoot.

However, in some ways the derivative and integral terms act against each other and are all controlled by one overall gain, making the analysis much more involved.

The error control channel is like this:

![Error Control Channel](image)

Expressed mathematically the motor control voltage, \( V_m \) is given by:

\[
V_m = \text{error} + \text{integral of error} + \text{derivative of error}
\]

\[
= V_e + \int V_e \, dt + \frac{dV_e}{dt}
\]

where \( V_e \) is the error voltage.

When gain factors are added then:

\[
V_m = K_p (V_e + K_i (\int V_e \, dt) + K_d (dV_e/dt))
\]

where \( K_p \) is the proportional gain, \( K_i \) the integral gain and \( K_d \) the derivative gain.
21.6 Practical 1: PID Position Control Loop

21.6.1 Objectives and Background

This experiment shows that a positional servo control system can be implemented using a digital shaft encoder instead of the potentiometer and A/D converter (analogue).

The concept is the same as in the previous practical, the only difference is that the measured value is derived from the Gray encoder or Incremental encoder.

The resolution of the measured values is limited to the number of tracks on the encoder disk. The Gray encoder has six tracks, so the resolution is $2^6 = 64$ levels. This results in the measured value having a more stepped appearance.

In this practical a PID feedback system is used for controlling the motor. It follows that there must always be an error in the system or the motor would stop.

Multiplying this signal by a gain and using this to drive the motor can help reduce the error signal. However, there is a limit to how much gain can be applied before instability occurs.

The digital controller calculates the error signal (difference between set value and actual) and from this the Derivative and Integral signals. These signals can be introduced by adjusting the gain to enable PID control as seen in previous practicals with the analogue control circuits. The signals are combined and the digital result of which is then sent out to the D/A converter which is connected to the power amplifier to drive the motor.

The gain of the error signal and the gain of the Derivative and Integral signals are determined by the user using the track bars instead of the manual gain controls on the servo fundamentals workboard as used with an analogue controller previously.

Remember that:

- Increasing the proportional gain should give a faster response but will increase overshoot and eventually lead to instability.
- Increasing the differential gain will reduce the overshoot allowing increased proportional gain.
- Increasing the integral gain will reduce following error.

When dealing with digital controllers and PID control, the sample rate of the derivative and integral is an important factor. As explained in the A/D and D/A conversion assignments, the digital controller samples input data at regular intervals using the A/D converter. Taking too few samples can inhibit the performance of the system, since the Mechanical Unit may go beyond a set point without the digital controller noticing. Taking too many samples has the potential to put a load on the digital controller, causing other processes to be potentially 'starved' of processor time.
The error signal is calculated by subtracting the set value from the measured value. If these two signals weren't sampled at a fast enough rate then data will be lost. This obviously has an affect also on the derivative and integral signals produced from the error signal, but also if the sampling rate is too low or too fast when calculating the derivative and integral signals they also will not contain the correct data even if the error signal had been originally sampled at an acceptable rate.

When designing a digital controller, a compromise between the required response and the available processing resources of a digital controller must be met.

21.6.2 Block Diagram

![Block Diagram for Practical 1](image)

**Figure 21-2: Block Diagram for Practical 1**

21.6.3 Perform Practical

Figure 21-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

Set the variable dc control to half scale (motor static).

In this practical the digital controller is used separately with the Gray encoder, and then the Incremental encoder, to make a simple closed loop position control system. The square wave generator is used to provide an input to the system that the output shaft attempts to track. The input is called the set value and the A/D converter (A/D Ch1) sends this to the digital controller.

The actual position of the shaft (the measured value) is sensed by the Gray/Incremental encoder and this is sent directly to the digital controller.

The difference between the two inputs (called error) is calculated and the result used to drive the motor via the D/A converter.

In order for the error to be small, the error signal that drives the motor must be magnified so that small errors still correct the output; this magnification factor is called gain. The gain value has a large effect on the behaviour of the system and too much or too little both cause problems. The experiment will show both these effects.

Open the Data Logger.

Set the sweep function generator min freq control to minimum and the amplitude control to half scale.
On the Data Logger the blue represents the set value, orange the digital controller error signal, yellow the Gray/Incremental encoder output (converted to analogue) and the green the power amplifier input from the D/A.

The motor should follow a fast changing position demand (square waveform).

This error signal is produced by the digital controller reading in the set value from A/D ch1 and subtracting this from the actual measured value from the Gray encoder.

The digital controller also has the facility to increase the output signal gain, and introduce derivative and integral signals to the output enabling PID control as shown in previous analogue practicals. These can be introduced by sliding the track bars below.

Experiment with the settings of the track bars. First individually, then combine elements and note the effect on the positioning performance of the control system. Some of the effects seen should replicate those seen in previous practicals involving analogue control systems and PID control.

It should be possible to reduce the overshoot and reduce settling time of the error signal.

Next set all track bars to zero except the Gain, leave this set to 1.

Note the error signal (orange). There should be slight overshoot and oscillation on the error signal. Select the different sampling rates in turn. Although the signal will be more compact the actual overshoot and slight oscillation should still be the same, showing that sampling rate doesn’t affect the proportional signal.

Select the 125Hz sampling rate. Now introduce some derivative signal so that the effect can be seen. Select the different sampling rates in turn and note the effects.

Repeat this again with integral signal applied. Note the effects of sampling rate change.

Now select the Incremental button below, this tells the digital controller to use the output from the incremental encoder.

Repeat the tests set out above. The yellow signal now represents the Incremental encode signal.

Note the output from the Incremental encoder is smoother due to its higher resolution.

### 21.7 Practical 2: PID Speed Control Loop

#### 21.7.1 Objectives and Background

This experiment shows that a servo speed control system can be implemented using a digital shaft encoder instead of the potentiometer and A/D converter (analogue).

The concept is the same as in the previous practical, the only difference is that the measured speed value is derived from the Incremental encoder.

This practical uses the PID feedback system for controlling the motor.
Multiplying the actual error by a gain and using this to drive the motor can help reduce the required minimum error. However, there is a limit to how much gain can be applied before instability occurs.

The digital controller calculates the error signal (difference between set value and actual) and from this the Derivative and Integral signals. These signals can be introduced by adjusting the gain to enable PID control as seen in previous practicals with the analogue control circuits. The signals are combined and the digital result of which is then sent out to the D/A converter which is connected to the power amplifier to drive the motor.

The gain of the error signal and the gain of the Derivative and Integral signals are set by using the track bars instead of the manual gain controls on the servo fundamentals workboard as used with an analogue controller previously.

Remember that:

- Increasing the proportional gain should give a faster response but will increase overshoot and eventually lead to instability.
- Increasing the differential gain will reduce the overshoot allowing increased proportional gain.
- Increasing the integral gain will reduce following error.

When dealing with digital controllers and PID control, the sample rate of the derivative and integral is an important factor. As explained in the A/D and D/A conversion assignments, the digital controller samples input data at regular intervals using the A/D converter. Taking too few samples can inhibit the performance of the system, since the Mechanical Unit may go beyond a set point without the digital controller noticing. Taking too many samples has the potential to put a load on the digital controller, causing other processes to be potentially 'starved' of processor time.

The error signal is calculated by subtracting the set value from the measured value. If these two signals weren't sampled at a fast enough rate then data will be lost. This obviously has an affect on the derivative and integral signals produced from the error signal, but if the sampling rate is too low or too fast when calculating the derivative and integral signals they also will not contain the correct data even if the error signal had been originally sampled at an acceptable rate.

When designing a digital controller, a compromise between the required response and the available processing resources of a digital controller must be met.
21.7.2 Block Diagram

![Block Diagram for Practical 2](image)

Figure 21-4: Block Diagram for Practical 2

21.7.3 Perform Practical

Figure 21-5 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical the digital controller is configured to use the Incremental encoder, to make a simple closed loop speed control system. The square wave generator is used to provide an input to the system that the output shaft attempts to track. The input is called the set value and the A/D converter (A/D Ch1) sends this to the digital controller.

The actual position of the shaft (the measured value) is sensed by the Incremental encoder and this is sent directly to the digital controller.

The difference between the two inputs (called error) is calculated and the result used to drive the motor via the D/A converter.

In order for the error to be small, the error signal that drives the motor must be magnified so that small errors still correct the output; this magnification factor is called gain. The gain value has a large effect on the behaviour of the system and too much or too little both cause problems. The experiment will show both these effects.

Open the Data Logger.

Set the sweep function generator min freq control to minimum and the amplitude control to 10% scale.
On the Data Logger the blue represents the set value, orange the digital controller error signal, yellow the Incremental encoder output (converted to analogue) and the green the power amplifier input from the D/A.

The motor should stay at a constant speed for each transition (square waveform).

The error signal is calculated by the digital controller reading in the set value from A/D ch1 and subtracting this from the actual measured value from the Incremental encoder.

The digital controller also has the facility to increase the output signal gain, and introduce derivative and integral signals to the output enabling PID control as shown in previous analogue practicals. These can be introduced by sliding the track bars below.

Experiment with the settings of the track bars. First individually, then combine elements and note the effect on the positioning performance of the control system. Some of the effects seen should replicate those seen in previous practicals involving analogue control systems and PID control.

Note the derivative has little or no effect on the error signal.

It should be possible to reduce the following error to zero by a combination of Gain and Integral.

Next set all track bars to zero except the Gain, leave this set to 1.

Select the different sampling rates in turn and note the effects.

It can be seen that the change in sampling rate affects the proportional signal. This is an important point with digital speed control and incremental encoders, if there is a change to the sampling rate the calculations carried out by the digital controller must be scaled accordingly. Otherwise with the sampling rate lowered the sampling period is longer thus a larger count recorded for the incremental encoder, but this is not scaled for this sampling rate, so the motor looks to be rotating faster than it really is, thus the signal error reduces which in turn lowers the D/A output to the power amplifier. Resulting in the motor rotating slower when in actual fact a change in sampling rate should have no effect on the control system if only proportional control is implemented.
22 Concepts of Transfer Function Analysis

22.1 Objectives

- To investigate the frequency response characteristics of two common control system elements, CR and Integrator circuits.

22.2 Transfer Function and Frequency Response Principles

In previous assignments an external test signal has been used of either a step or a ramp type as the input to the system on test.

These signals are very convenient since they indicate the performance of the system in the time domain in a manner which is easily appreciated and where the effects of various compensations, such as velocity feedback, three-term control, are easily demonstrated.

These test signals are not convenient when numerical design work is required, for example: what change in gain will affect the damping by a certain amount, or how much tacho-generator feedback is required to reduce excessive overshoot to an acceptable value.

In order to deal with this type of problem, more mathematical design methods are required, the most basic of which is the concept of frequency response.

This involves the determination, either practically or theoretically, of the response of a system to a steady sinusoidal input.

This method enables systems with parameters already known from theory, or from separate measurements, to be designed for a specified performance.

Alternatively for an unknown system an overall characteristic can be measured and design then to proceed based on the measured characteristic. The full background of the frequency response method is extensive and is treated in detail in all introductory books on control.

22.3 Transfer Function

In frequency response testing a sinusoidal input is applied to some object which may be a circuit or part of a control system and the relation considered between input and output signals. This is illustrated in Figure 22-1 (a) where at some frequency \((\omega_1, V_1)\), is the input and \(V_2\) is the output or response.
Chapter 22
Control & Instrumentation Principles
Concepts of Transfer Function Analysis

Figure 22-1: Transfer Function Relations

In general the output will differ in magnitude from the input and there will also be a phase difference $\phi$ in the illustration.

The ratio of the magnitudes $|V_1|$ and $|V_2|$ and phase angle can be plotted separately against frequency as in Figure 22-1 (b). The results in Figure 22-1 (a) might correspond with the points shown.

Alternatively the magnitude ratio and phase angle can be plotted as a polar plot as in Figure 22-1 (c) to give a frequency response locus. Where arrows give the direction of increasing frequency along the locus. The transfer function is the combination of magnitude and phase information:

$$|\text{Transfer}| = \left|\frac{V_2}{V_1}\right|; \quad \angle (\text{Transfer}) = \angle \phi$$

The frequency response locus plot or polar plot of a system is very important in control system design.

22.4 Integration by Operational Amplifier

The voltage across a capacitor is given by the integral of the current through the capacitor:

$$v = \frac{1}{C} \int i \, dt$$

This is illustrated in Figure 22-2 (a). A constant current gives a steady increase of voltage, the voltage representing the area under the current/time plot. The illustrated current pulse waveform, comprising a positive pulse, a negative pulse and then zero current, gives a final voltage value, which is the overall time integral of the current.
Integration can be obtained by an operational amplifier if the feedback resistor is replaced by a capacitor as in Figure 22-2 (b). If a voltage V is applied an input current i will flow through R and the amplifier output will change to hold the amplifier input point (virtual earth point) substantially at zero. This means that the amplifier output will steadily increase to maintain the current i through the capacitor. The input current is:

\[ i = \frac{1}{C} \]

The voltage across the capacitor will be:

\[ V_c = \frac{V}{R} \int V \, dt \]

giving the amplifier output as:

\[ V_o = \frac{1}{CR} \int V \, dt \]

which is the negative scaled integral of the input.
If the applied voltage $V$ becomes zero, the current $i$ is also zero and the integrator holds indefinitely whatever output has been obtained, since the amplifier does not ideally draw any current at the virtual earth point. If the input is a general waveform the output is correspondingly scaled integral.

It is often required to set the output of an integrator to zero before the start of integration, and this can be arranged by a switch, mechanical or electronic, connected across the capacitor, as shown dotted, which discharges the capacitor. Integration does not start until the switch is open, irrespective of a possible input signal.

An ideal integrator will hold an accumulated signal indefinitely; however an operational amplifier may draw a very small current at the virtual earth input, which will cause the output to drift very slowly. The drift may or may not be important and depends on the amplifier used.

**22.5 Time and Constant Integration**

Two very important system concepts are time and constant integration.

Both these have already been mentioned in the context of a motor transient response and also in connection with an operational amplifier.

The characteristics have been considered in the time domain but are very important in frequency domain analysis.
22.6 Frequency Response of a Time Constant

If the frequency response of a time constant (ideally a simple RC) circuit as in Figure 22-5 (a) is measured, the magnitude and phase characteristics will be as in Figure 22-5 (b).
The transfer magnitude is unity at dc but as the frequency increases the reactance of the capacitor falls and the output gradually decreases. It can be shown that when:

\[ \omega = \frac{1}{\tau} \]

\[ \left| \frac{V_o}{V_i} \right| = \frac{1}{\sqrt{2}} = 0.707 \]

\[ \angle \left[ \frac{V_o}{V_i} \right] = \angle \varphi = -45^\circ \]

where \( \omega \) is frequency in radians/second

(frequency in Hz = \( \frac{\omega}{2\pi} \))

This particular point is indicated on a polar plot in Figure 22-5 (c), and it can be shown that the complete frequency response is a semi-circle between +1 and the origin.

The condition where \( \omega \) is referred to as the 45°-point.

The operational amplifier representation of a time constant has been covered earlier but this representation is reversed in polarity due to the operational amplifier.

If an additional "-1" amplifier is arranged before (or after) then the overall response is identical with a time constant.
22.7 Frequency Response of Integration

The general characteristics of integration and integration by an operational amplifier are covered in the Introduction to the PID 3 Term Control assignment.

In electrical terms the value of the integral of a signal or waveform is represented by the voltage developed across a capacitor when a current corresponding with the signal or waveform is passed through a capacitor as in Figure 22-8 (a).
If a cosine current is passed through the capacitor as in Figure 22-8 (a), the first quarter cycle will give a positive voltage due to accumulation of charge, and the second quarter cycle will discharge the capacitor to give zero volts. The waveform developed will be a positive half sine wave. The process repeats for the next half cycle in the negative sense giving a negative half cycle.

If a sine wave current is applied as in Figure 22-8 (c) the voltage rises during the first half cycle and then decreases to zero during the second half cycle. Thus the output is a dc level together with a negative cosine waveform. In both cases the peak output, ignoring the dc component in Figure 22-8 (c), lags the input by 90°.

In addition in both cases, if the frequency of the input is increased, the magnitude of the maximum value of \( V_c \) falls because the current charges or discharges the capacitor for a shorter time. Conversely, if the frequency is reduced the charge or discharge lasts longer and the output increases.

The magnitude/frequency relation between current and voltage is expressed for ac signals using the reactance of the capacitor:

\[
V_2 = i \times \frac{1}{\omega C}
\]

If an operational amplifier integrator is considered as in Figure 22-8 (d), the current through the capacitor is given by:

\[
i = \frac{V_1}{R}
\]

and if this is an ac signal then:

\[
V_0 = (-) \times \frac{1}{\omega C}
\]

hence the transfer magnitude of an operational amplifier integrator, is:

\[
\left| \frac{V_2}{V_1} \right| = \frac{1}{\omega CR} = \frac{1}{2\pi f CR}
\]

where \( f \) is the frequency (in Hertz) and \( \omega \) is radians/sec.

This gives a magnitude/frequency relation as in Figure 22-9.
For $\Omega = \frac{1}{CR}$; \[
\left| \frac{V_s}{V_i} \right| = 1
\]
at very low frequencies:
\[
\lim_{\omega \to 0} \left| \frac{V_s}{V_i} \right| = \infty
\]
and at very high frequencies:
\[
\lim_{\omega \to \infty} \left| \frac{V_s}{V_i} \right| = 0
\]
The polar plot (which is a straight line) is shown in Figure 22-9 (b) illustrating the constant 90° lag. The semi-circular plot for a time constant is also indicated for comparison.

### 22.8 Frequency Response of Time Constant and Integration

An ideal motor can be represented by a time constant (relating armature voltage and speed) followed by an integration (relating speed and output shaft position).

From the frequency response point of view an ideal motor can be represented by two blocks with response loci as in Figure 22-10 (a).
For any frequency the overall magnitude and phase angle is found by multiplying the individual magnitudes and adding the phase angles, hence if an input is applied at $\omega_1$ the overall transfer is given by:

$$d = d_1 d_2 \quad \text{and} \quad \phi = \phi_1 + 90^\circ$$

The combined locus is shown in Figure 22-10 (b) and for low frequencies has a large magnitude with $90^\circ$ lag and for high frequencies has a decreasing magnitude with a lag approaching $180^\circ$.

For $\omega = \frac{1}{\tau_1}$ (where $\tau_1$ is the motor speed time constant) the overall lag is $45^\circ + 90^\circ = 135^\circ$.

22.9 Closed-Loop Frequency Response

The combined locus of Figure 22-10 (b) shows that the overall magnitude response “$d$” falls continuously with increasing frequency. However, when a feedback loop is closed round a system as in Figure 22-11 (a), which could represent a position control system, the closed-loop frequency response may be very different from the open-loop response depending on the gain.
If the input signal is sinusoidal then the signals throughout the system will be sinusoidal and the ideas developed previously can be applied. In particular the relation between error (E) and output ($\theta_o$) will vary with frequency as in Figure 22-11 (b) above which corresponds with the combined locus of Figure 22-10 (b). The error signal is given by the relation:

$$E = \theta_i - \theta_o$$

can be re-arranged as

$$\theta_i = E + \theta_o$$

which implies that if $\theta_o$ is added to E, as in Figure 22-11 (c), to give the point p the line from the origin to p represents the input $\theta_i$. This diagram indicates the relative magnitude of the signals so that the form is independent of the actual signal magnitudes. The diagram also gives the phase-angles between the signals, which again are independent of the actual magnitudes.

The operating condition at p might correspond with a frequency $\omega$, and indicate that the input and output are about equal giving:

$$|\text{System Transfer}| = \left| \frac{\theta_o}{\theta_i} \right|$$

If the frequency is reduced to $\omega_o$ with operating point q, then it can be seen that again.
and as the frequency is reduced θ₀ and θ₁ become more nearly equal so that the transfer approached unity which is correct since at dc (i.e. \( \omega \rightarrow 0 \)) the system will align perfectly.

If a higher frequency \( \omega_2 \) is considered with operating point \( r \), \( \theta_0 \) is much smaller than \( \theta_1 \) and the transfer decreases. These results show that the closed-loop transfer magnitude will have the general form of the initial gain response in Figure 22-12 (a).

If the gain K is increased, say to about 3, then all values of \( \theta_0 \) will increase by a factor of 3 in the Figure 22-11 (b), Figure 22-11 (c) and in particular for frequency \( \omega_2 \) the form of Figure 22-11 (c) will change to Figure 22-11 (d) and:

\[
|\text{Transfer}| \quad \omega = \omega_2 = \frac{\theta_0}{\theta_1} = \text{approx } 4
\]

again at very low frequencies the transfer approaches unity and at high frequencies decreases giving the general form of the "increased gain" response of Figure 22-12 (a) above.

In general terms the peak occurs because due to the phase shift in the forward path approaching 180°, together with the increased gain, the feedback signal \( (\theta_0) \) nearly provides the error signal. This implies that the system is approaching self-oscillation.

The form of the step response Figure 22-12 (b) is closely related to the corresponding frequency response. If the frequency response has no peak, as for the 'initial gain' condition, the step response has no overshoot. If the frequency response has a marked peak, as for 'increased gain', the step response has a very evident oscillatory component with frequency approximately that of the peak.

A step response overshoot of about 25%, sometimes regarded as a practical maximum, corresponds with a frequency response peak of about 1.3.
22.10 Practical 1: Transfer Function of a CR Circuit

22.10.1 Objectives and Background

In frequency response testing a sinusoidal input is applied to some object which may be a circuit or part of a control system and the relation is considered between input and output signals. Magnitude and phase between the two signals are the important factors. This relationship is mathematically represented in the form of a transfer function. A transfer function for a system can be used to calculate the output of a system for a given input.

In this practical the frequency response for a CR circuit (also known as time constant) will be recorded using the GPA instrument. This will give the magnitude ratio and phase relationship between the input and output signals over a range of frequencies. (Note that the output displayed is not in the form of a conventional Bode plot, in general a Bode plots frequency scale would be logarithmic but due to function generator limitations this is not possible).

The frequencies applied to the CR circuit are set by the max and min freq controls in the sweep function generator on the Control & Instrumentation Principles workboard. By using these settings the digital controller will apply frequencies between them to the circuit whilst recording the response of the circuit.

Note plotting the frequency response of any system will require reasonable amount of time, a progress bar on the in upper most instrument open should increment gradually showing system activity.
22.10.2 Block Diagram

Figure 22-13: Block Diagram for Practical 1

22.10.3 Perform Practical

Figure 22-14 shows the required connections on the hardware.
The CR circuit is constructed using the amplifiers in the controller and summing & error parts of the Control & Instrumentation Principles workboard (33-125) as in Figure 22-15.

Note an additional “-1” amplifier is arranged before the CR amplifier so that the overall response is identical with a passive component CR circuit.

Set function generator output amplitude control to half scale.

Open the GPA.

Set min freq control to minimum (approx 0.04Hz) will take a while for the frequency to be displayed (wait for progress bar to travel fully across in each case), select max frequency button and set the max freq control to approx 2Hz, gain the set frequency will take a while to be displayed (wait for progress bar to travel fully across in each case).
Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

Set the capacitor switch to its left-hand position.

Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again, writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end a set amount of data will be written to the screen and the sweep function generator frequency is automatically incremented by the digital controller, and then the progress bar starts over again to show that it is still active.

As the circuit under test represents a CR circuit, the frequency response (magnitude and phase characteristics) will be similar to Figure 22-16 (b).

Since:

\[ \tau = CR = 100k \times 2\mu F = 0.2 \text{ sec} \]

the 45° and -3db point will occur for:

\[ \omega = \frac{1}{\tau} \]
\[
\omega = \frac{5}{2\pi} \approx 0.8 \text{Hz}
\]

Check that the relative phase and magnitude are approximately correct with 0.8Hz applied by clicking on the GPA display which will set a pointer at that point. Then place the cursor over the bar that appeared on the display to give the phase, gain and frequency.

Select Nyquist to see how the results are represented using this type of plot, the results should be similar to Figure 22-16 (c).

Remove connection 7, and add connection 10. This will change the capacitor to 4\(\mu\)F.

Use the GPA to plot the frequency response for this CR circuit. Observe the 45° and -3db point for this circuit. Compare with the theory for these component values.

22.11 Practical 2: Transfer Function of an Integrator

22.11.1 Objectives and Background

In frequency response testing a sinusoidal input is applied to some object which may be a circuit or part of a control system and the relationship considered between input and output signals. Magnitude and phase between the two signals are the important factors. This relationship is mathematically represented in the form of a transfer function. A transfer function for a system can be used to calculate the output of a system for a given input.

In this practical the frequency response for a Integrator circuit will be recorded using the GPA instrument. This will give the magnitude ratio and phase relationship between the input and output signals over a range of frequencies. Note the output displayed is not in the form of a conventional Bode plot; in general a Bode plot's frequency scale would be logarithmic which due to function generator limitations this is not possible.

The frequencies applied to the Integrator circuit are set by the max and min freq controls on the Control & Instrumentation Principles workboard. By using these settings the digital controller will apply frequencies between these to the circuit whilst recording the response of the circuit.

Note plotting the frequency response of any system will require reasonable amount of time, a progress bar on the in upper most instrument open should increment gradually showing system activity.
22.11.2 **Block Diagram**

![Block Diagram for Practical 2](image)

Figure 22-17: Block Diagram for Practical 2

22.11.3 **Perform Practical**

Figure 22-18 shows the required connections on the hardware.
The general characteristics of integration and integration by an operational amplifier are covered in the 3-Term Control assignment.

In electrical terms the value of the integral of a signal or waveform is represented by the voltage developed across a capacitor when a current corresponding with the signal or waveform is passed through a capacitor as in Figure 22-19 (a).
If a cosine current is passed through the capacitor as in Figure 22-19 (a), the first quarter cycle will give a positive voltage due to accumulation of charge, and the second quarter cycle will discharge the capacitor to give 0 volts. The waveform developed will be a positive half sine wave. The process repeats for the next half cycle in the negative sense giving a negative half cycle.

If a sine wave current is applied as in Figure 22-19 (c) the voltage rises during the first half cycle and then decreases to zero during the second half cycle. Thus the output is a dc level together with a negative cosine waveform. In both cases the peak output, ignoring the dc component in Figure 22-19 (c), lags the input by 90°.

In addition in both cases, if the frequency of the input is increased, the magnitude of the maximum value of \( V_c \) falls because the current charges or discharges the capacitor for a shorter time. Conversely, if the frequency is reduced the charge or discharge lasts longer and the output increases.

The magnitude/frequency relation between current and voltage is expressed for ac signals using the reactance of the capacitor:

\[
V_2 = i \times \frac{1}{\omega C}
\]

If an operational amplifier integrator is considered as in Figure 22-19 (d), the current through the capacitor is given by:

\[
I = \frac{V_1}{R}
\]

and if this is an ac signal then:

\[
V_0 = (-) \frac{1}{\omega C}
\]
Hence the transfer magnitude of an operational amplifier integrator, is:

\[
\frac{|V_2|}{|V_i|} = \frac{1}{\omega CR} = \frac{1}{2\pi f CR}
\]

where \(f\) is the frequency (in Hertz) and \(\omega\) is radians/sec.

This gives a magnitude/frequency relation as in Figure 22-20.

For \(\omega = \frac{1}{CR}\); \[\frac{|V_2|}{|V_i|} = 1\]

at very low frequencies:

\[\frac{|V_2|}{|V_i|} \rightarrow \infty\]

and at very high frequencies:

\[\frac{|V_2|}{|V_i|} \rightarrow 0\]

The polar plot (which is a straight line) is shown in Figure 22-20 (b) illustrating the constant 90° lag. The semi-circular plot for a time constant is also indicated for comparison.

With \(R = 200k\) and \(C = 2\mu F\).

Since for an integrator the transfer magnitude is unity for:

\[\Omega = \frac{1}{CR} = \frac{1}{\tau}\]

and since \(\tau = 0.04\) sec, unity transfer occurs for:
Open the GPA.

Select the Set Min Freq button (should already be selected) adjust the Min frequency control on the Servo fundamentals board to 0.1Hz (wait after each adjustment for the progress bar to travel across before the new frequency is displayed). Select Set Max Freq and set this in the same way as above to 2Hz. This procedure has set the two frequencies between which the digital controller will take a set number of readings for the circuit under test.

It is important that the input amplitude is set at a low level. The best way to achieve this is to use the Data Logger.

Set the capacitor switch the short circuit position (right), then set back to open circuit. This will discharge the capacitor.

Open the Data Logger.

Wait for the progress bar to travel across the top of the Data Logger display.

Now the input and output signals of the integrator will be displayed.

Adjust the sweep function generator output amplitude control so that the blue (integrator output) signal is an unclipped sine wave. It doesn’t matter if the top half of the sine wave is off the display so long as the bottom part has a smooth curve to it. Each time you adjust you will have to wait for the progress bar to travel across before the effect of the adjustment is displayed.

Close the Data Logger.

On the GPA once the progress bar as travelled across fully the Ref dbfs value will be displayed, this is the input signal, which should be around -15db.

Before starting to plot, discharge the capacitor (switch to the right-hand position and then back to the left-hand position).

Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

Select Plot. Now the plot will take place.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end a set amount of data will be written to the screen and the sweep function generator frequency is automatically incremented by the digital controller, and then the progress bar starts over again to show that it is still active.

The plot should be similar to polar plot (b) above for an integrator.

\[ \omega = \frac{1}{0.4} = \frac{2.5}{\text{rads/sec}} \quad \text{0.4Hz} \]
The circuit incorporates a 10M ohm resistor in parallel with the feedback capacitor. If it was not there the theoretical gain at DC would be infinite. Because op-amps at low frequencies have small DC offset voltages and currents any small input offset would cause an integration that would eventually saturate the op amp.

There is a frequency where the impedance of the capacitor equals the impedance of the R(f) resistor. Over this frequency the circuit acts as an integrator.

Disconnect the feedback resistor from the circuit and run the test again to see the effect this has.

### 22.12 Practical 3: Transfer Function of a CR plus Integrator

#### 22.12.1 Objectives and Background

This practical looks at the frequency response of the CR (time constant) and Integrator circuits combined into one system.

From the frequency response point of view an ideal combination of CR (time constant) and Integration can be represented by two blocks with response loci as shown below.

![Figure 22-21: Frequency Response of Ideal Time Constant and Integrator](image)

For any frequency the overall magnitude and phase angle is found by multiplying the individual magnitudes and adding the phase angles, hence if an input is applied at \( \omega_1 \) the overall transfer is given by:

\[
d = d_1 \cdot d_2 \\
\phi = \phi_1 + 90^\circ
\]

The combined locus is shown above and for low frequencies has a large magnitude with 90° lag and for high frequencies has a decreasing magnitude with a lag approaching 180°.

For \( \omega = \frac{1}{\tau_1} \) where \( \tau_1 \) is the CR circuit (time constant) the overall lag is 45° + 90° = 135°.
### 22.12.2 Block Diagram

![Block Diagram](image)

Figure 22-22: Block Diagram for Practical 3

### 22.12.3 Perform Practical

Figure 22-23 shows the required connections on the hardware.
It is important that the Input signal amplitude is set at a low level. The best way to achieve this is open the Data Logger.

Set the capacitor switch the short circuit position (right), then set back to open circuit. This will discharge the capacitor.

Open the Data Logger.

Wait for the progress bar to travel across the top of the Data Logger display.

Now the input and output signals of the CR plus integrator circuit will be displayed.

Adjust the sweep function generator output amplitude control so that the input (blue) signal is an unclipped sine wave. It doesn’t matter if the top half of the sine wave is off the display so long as the bottom part has a smooth curve to it. Each time you adjust you will have to wait for the progress bar to travel across before the effect of the adjustment is displayed.

Close the Data Logger.

Open GPA.

Once the progress bar as travelled across once the Ref dbfs value will be displayed, this is the input signal which should be around -15db.
Select Set Min Freq button (should already be selected) adjust the Min frequency control on the Control & Instrumentation Principles workboard to 0.1Hz (wait after each adjustment for the progress bar to travel across before the new frequency is displayed). Select Set Max Freq and set this in the same way as above to 2Hz. This procedure has set the two frequencies between which the digital controller will take a set number of readings for the circuit under test.

Before starting to plot, discharge the capacitor (switch to the right-hand position and then back to the left-hand position).

Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

Select Plot. This will take some time to complete so be patient.

The Nyquist plot should be similar to Figure 22-21, which is a combination of the separate CR and Integrator practical results previously carried out in this assignment.
23 Open Loop Transfer Function

23.1 Objectives

- To investigate the open loop transfer function for a motor.
- To learn the effect on the motors frequency response by introducing derivative and integral signals in addition to proportional control signal.
- To learn system identification by observing open loop transfer function characteristics.

23.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 22.2 Transfer Function and Frequency Response Principles
- 22.3 Transfer Function
- 22.5 Time and Constant Integration
- 22.6 Frequency Response of a Time Constant
- 22.7 Frequency Response of Integration
- 22.8 Frequency Response of Time Constant and Integration
- 22.9 Closed-Loop Frequency Response

23.3 Nyquist Stability Criterion

In application the error (E) is assumed unity and the origin of the frequency response locus taken as the origin, giving Figure 23-1 (a).

Figure 23-1: Nyquist Diagrams for System Design

(a) General diagram
(b) Locus with max phase lag of 270°
(c) Oscillatory system (1) stable after compensation (2)
The closeness of the frequency response locus to the “-1” point indicates the magnitude of frequency of the frequency response peak and hence the form of the step response. If the system has an additional time constant in the forward path the maximum phase lag will be 270° with the form of Figure 23-1 (b). If the locus passes through the “-1” point, as in Figure 23-1 (c), which would occur for Figure 23-1 (b) if the gain is increased, the system oscillates.

The diagrams form the basis of the Nyquist Stability Criterion which provides a very instructive method for introductory system analysis and design.

From considerations of practical systems the frequency and step responses of closed loops previously seen indicate a major general problem. As the system gain is increased the transient response deteriorates but at the same time a high gain may be required to counter the effect of dead-band in the motor or to reduce steady following error.

The general solution is to introduce networks in the forward path giving additional transfer functions, termed compensation, which provide a high gain at low frequencies but reduce the gain at frequencies where the existing forward path introduces increasing phase shift.

These networks (a 3-term controller for example) modify or re-shape the overall frequency locus to give adequate clearance of the “-1” point, perhaps as shown dotted in Figure 23-1 (c) above. Detailed investigation of such methods is beyond the scope of an introductory manual but would be covered in any general book on control systems.

These ideas form the basis of the Nyquist Stability Criterion, originally developed in connection with the stability of feedback telephone repeater amplifiers, but applicable to any open-loop/closed-loop situation.

Other methods are available but the Nyquist method has a simple background. In addition it can be applied to situations where the detail is known (as in the system of this assignment) and the locus can be calculated, or to systems when the detail is not known but the locus can be measured experimentally.

23.4 Practical 1: Motor Transfer Function

23.4.1 Objectives and Background

In this practical the frequency response of the motor will be examined by using the GPA. First the motor is driven by a sine wave and the speed measured by the tacho-generator. The two are then plotted with respect to each other over a range of frequencies to give the frequency response of the time constant element which is part of the transfer function.

Then using the incremental encoder, with the index bit disabled, the motor is driven by a sine wave, with the two signals being plotted with respect to each other over the range of frequencies set to give the frequency response of the time constant and integration of the motor (transfer function).
23.4.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 23-2: Block Diagram for Practical 1

23.4.3 Perform Practical

Figure 23-3 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical it will be demonstrated that a dc motor’s transfer function contains both time constant and integral elements.

The first part of the practical will show the time constant of the motor transfer function. This is done by using the analogue signal from the tacho-generator and comparing it to the input signal to the power amplifier. These two signals are input into the GPA as in the previous assignment so that phase and magnitude ratio can be seen on the GPA Bode/Nyquist plot.

The complete transfer function which includes the time constant and the integration of the motor can be examined using the incremental encoder. The signal from the incremental encoder is compared to the input signal to the power amplifier. These two signals are input into the GPA as in the previous assignment so that phase and magnitude ratio can be seen on the GPA Bode/Nyquist plot.
Chapter 23
Control & Instrumentation Principles
Open Loop Transfer Function

It is possible to measure the forward path transfer by operating the system under closed-loop and measuring input, error and output as in the Figure 23-4. The forward path transfer is the same as the motor transfer in frequency characteristics except for an overall gain in the power amplifier.

But the digital controller in this case negates the need to do this as it is able to remove the offset generated when the motor rotates through many revolutions. The offset is caused by the motor creeping in one direction or the other because the number of rotations usually differs in each direction. So by using the digital controller the motor transfer function can be obtained open loop.

The digital controller sets the frequency of the sweep function generator, and sweeps between the two set min and max frequencies. Also the digital controller is set to inhibit the index count, meaning that it is possible to detect position over more than one rotation. Figure 23-5 shows the configuration used to obtain the transfer function of the motor.

Time constant measurement
The time constant can be measured by using the principles from the Concepts of Transfer Function Analysis assignment (see Chapter 22). In this practical a sinusoidal input is applied to the power amplifier and the tacho-generator signal is displayed which is proportional to speed.
The frequency is then adjusted between set points by the digital controller to determine the 45° frequency. However, for small motors, such as used in the 33-100 Mechanical Unit, the effects of brush friction and stiction which distort the display at the zero crossing, i.e. where the motor stops and reverses, may prevent a good estimate being obtained. These effects can be largely eliminated if a speed “offset” is used causing the motor always to rotate in one direction.

Set the sweep function generator output amplitude control to full of scale.

Open the GPA.

Set the min freq control to approximately 0.1Hz, the set frequency will take a while to be displayed (wait for progress bar to travel fully across after adjustment), then select max frequency button and set the max freq control to approx 1.6Hz.

Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

The motor will now be rotating in one direction and then the other.

Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end, a set amount of data will be written to the screen, and then the progress bar starts over again to show that it is still active. While this is happening the sweep function generator frequency is automatically incremented by the digital controller.
Select Nyquist to see how the results are represented using this type of plot, the results should be similar to diagram shown above and those seen for the CR circuit in the previous assignment.

The -45° shift for the motor should occur at about 0.4Hz (≈2.5 rads/sec) corresponding with a time constant of about 0.4 sec.

**Time constant and integration**

To measure the transfer function of the motor, the position of the motor needs to be read and compared with the input signal.

Select the Time constant and Integration button. This sets the incremental encoder as the position sensor but the index signal disabled to enable multiple rotations to be counted.

Remove connection 4.

Open the GPA.

Set the sweep function generator output amplitude control to half of scale.

Set min freq control to approximately 0.1Hz, the set frequency will take a while to be displayed (wait for progress bar to travel fully across after adjustment), select max frequency button and set the max freq control to approx 1.6Hz.

Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

The motor will now be rotating in one direction and then the other.

Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end, a set amount of data will be written to the screen, and then the progress bar starts over again to show that it is still active. While this is happening the function generator frequency is automatically incremented by the digital controller.

Select Nyquist to see how the results are represented using this type of plot, the results should be similar to diagram shown above and those seen for the CR circuit in the previous assignment.

With Nyquist selected the display should be similar to the locus diagram above, the -135° shift should occur at about 0.4Hz.

**Note:** Towards the end of the plot it is possible that the phase shift trace will turn red, the reason for this is that the amplitude of the signal is now so small that it is difficult to determine phase.
23.5 Practical 2: Open Loop P + D Control Transfer Function Analysis using Gain Phase Analyser

23.5.1 Objectives and Background

This practical looks at the effects of Proportional + Derivative control on the frequency response (transfer function) of a motor. By using the digital controller in conjunction with the incremental encoder it is possible to carry out this experiment in open loop mode.

23.5.2 Block Diagram

23.5.3 Perform Practical

Figure 23-8 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.
The circuit as illustrated within the patching diagram gives a system with proportional +
derivative control using the circuit equivalent of Figure 23-9 (b).

The transfer function for the motor which is made up of time constant and the integration

The circuit as illustrated within the patching diagram gives a system with proportional +
derivative control using the circuit equivalent of Figure 23-9 (b).

The transfer function for the motor which is made up of time constant and the integration
can be examined using the incremental encoder as seen in the previous practical. In order
to see the effect of adding derivative signal to the control signal the GPA is used again in
the same manner. The input to the power amplifier is compared to the signal obtained from
the incremental encoder, with phase and magnitude change plotted on the GPA. The
phase and magnitude ratio can be seen on the GPA in the form of either a Bode or Nyquist
plot.

This practical is carried out in an open loop configuration. The digital controller sets the
frequency of the sweep function generator, and is set to inhibit the index count meaning
that it is possible to detect position over more than one rotation.

Set function generator output amplitude control to full scale.

Open the GPA.

Set min freq control to approximately 0.1Hz, the set frequency will take a while to be
displayed (wait for progress bar to travel fully across after adjustment), select max
frequency button and set the max freq control to approx 1.6Hz.

Hi resolution can be selected but this will greatly increase the time to carry out the
frequency response plot, but will give a more detailed plot.

To begin with the GPA is set to provide a Bode style plot, if you require a Nyquist plot then
select this although it is possible to switch between the two whilst plotting.
Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end, a set amount of data will be written to the screen, and then the progress bar starts over again to show that it is still active. While this is happening the sweep function generator frequency is automatically incremented by the digital controller.

Select Nyquist to see how the results are represented using this type of plot, the results should be similar to Figure 23-9 (a) when comparing the motor transfer plot and this plot that has proportional and derivative control.

As the phase shift of the motor at 0.4Hz from the previous practical was shown to be -135°, it should be possible to see if the addition of the derivative signal has reduced the lag at this frequency.

For the values of Figure 23-9 (b) an estimate of the phase lead from the derivative can be made for about 0.4Hz (≈ 2.5 rad/sec) can be made as follows:

$$\text{reactance of capacitor} = \frac{1}{\omega C} = \frac{10^6}{\omega} \text{ for } 1\mu F, \quad 400\Omega$$

thus

$$i_2 = \frac{200i_1}{400*0.66 i_1}$$

hence vertical component in (c) = 0.66 x horizontal component giving $\Delta \phi$ about 30°

This change should be evident on the plot produced, where at 0.4Hz the phase shift will be approximately 30° less than when on derivative signal was introduced.

23.6 Practical 3: Open Loop P + I Control Transfer Function Analysis using Gain Phase Analyser

23.6.1 Objectives and Background

This practical looks at the effects of Proportional + Integral control on the frequency response of a motor. By using the digital controller in conjunction with the incremental encoder it is possible to carry out this experiment in open loop mode.
23.6.2 Block Diagram

Figure 23-10: Block Diagram for Practical 3

23.6.3 Perform Practical

Figure 23-11 shows the required connections on the hardware.
Figure 23-11: Make Connections Diagram for Practical 3

Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

In this practical the effect of introducing integral additionally to the input proportional signal is investigated.

Set the sweep function generator output amplitude control so the arrow points ‘←’ (approximately 15% scale).

Set the derivative and integral gain controls to zero and the capacitor discharge switch to its right-hand position.

Open the GPA.

Set min freq control to approximately 0.2Hz, the set frequency will take a while to be displayed (wait for progress bar to travel fully across after adjustment), select max frequency button and set the max freq control to approx 1.6Hz. This sets the GPA to sweep between about 0.2Hz and 1.6Hz.

Hi resolution can be selected but this will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

Currently the GPA is set to provide a Bode style plot, if you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

The motor will now be rotating in one direction and then the other.
Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end, a set amount of data will be written to the screen, and then the progress bar starts over again to show that it is still active. While this is happening the sweep function generator frequency is automatically incremented by the digital controller.

Set the integral signal attenuator to approximately 30% of scale.

Set the integral gain so the arrow points ‘←’ (approximately 15% scale) and the capacitor discharge switch to its left-hand position.

Select Plot.

Save or print the results once finished. Note the changes to the plot in both Bode and Nyquist forms compared to the plots without integral added.

**Note**: applying too much integrator gain and the integrator attenuator not adjusted correctly, can result in an unstable system. This is because the integrator has a high gain at low frequencies as seen in previous practicals.

The plot produced should have a curve that passes nearer to 180° than without the integral being introduced.

Next set the derivative control to half scale.

The plot should show a reduction in the maximum phase shift, counteracting the effect of the integral control signal as shown in Figure 23-12.

![Response Loci](image)

**Figure 23-12: Integral Control, and Integral Control with Derivative**
23.7 Practical 4: System Identification

23.7.1 Objectives and Background

The purpose of this practical is to outline the benefits of Bode and Nyquist plots when it comes to understanding and designing control systems. If an open loop transfer function plot is produced for a system either Bode or Nyquist, it is possible to determine the phase and gain margin in order to determine how much phase and gain the system in closed loop form can change before becoming unstable.

23.7.2 Perform Practical

From the results obtained in the previous practicals it is possible to identify the factors that form a systems frequency response (transfer function), be it a combination of time constant, integration and/or differentiation. From this, a designer is able to determine the type of control required to get the effect that would lead to satisfactory system performance for a particular application.

To identify whether an open loop system is likely to be stable when a closed loop is applied, a Nyquist plot is required. A unit circle needs to be applied to the plot if it is in magnitude rather than db, as shown in Figure 23-13. Otherwise, if the plot is in db, then it is the 0db point that needs to be referred to.

From the Nyquist plot it is possible to say that a closed loop system is stable if the unit circle crossing (or 0db) of the open loop frequency response is at a lower frequency than the -180° crossing of the open loop frequency response.

We can even apply this criterion to a Bode plot that if the unit circle crossing occurs when the magnitude of G(jw) is one. At that point, the gain (expressed in db) is zero db.
The -180° crossing is the frequency at which the phase becomes -180°.

A closed loop system is stable if the unit circle crossing (zero db crossing on a Bode plot) of the open loop frequency response is at a lower frequency than the -180° crossing of the open loop frequency response.

From this two important tolerances of the system can be calculated; these are the Phase and Gain margins. When the same system is closed loop, these margins have to be kept within in order for it to remain stable. These two factors can be calculated from a Bode or Nyquist plot.

The gain margin is defined as the change in open loop gain required in order for the system to become unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed loop.

The phase margin is defined as the change in open loop phase shift required in order to make a closed loop system unstable.

To calculate the phase margin using Bode/Nyquist plots, find the frequency at which the system has a gain of 1 (0db). At this frequency look at the phase; the phase margin is the difference in phase between the phase curve and -180 deg.

The gain margin is also calculated using Bode/Nyquist plots. To do this, find the frequency at which the system has a -180 degree phase shift. At this frequency look at the gain; the gain margin is the gain difference with respect to zero.

The bode plots in Figure 23-14 show how to determine the phase and gain margin.

Refer back to the Nyquist plots obtained in previous practicals. It is possible to determine whether the introduction of Derivative and Integral control will improve the stability of the system by analysing the phase and gain margins.
24 Closed Loop Transfer Function

24.1 Objectives

- To learn that closing a feedback loop round a system can have a marked effect on the overall transfer.
- To learn how gain in a closed loop system affects the transfer function.
- To have an understanding of the relationship between open and closed loop systems and the importance of being able to obtain open loop transfer functions from closed loop transfer functions.

24.2 Prerequisites

Before commencing this assignment, you should understand the concepts discussed in the earlier sections:

- 6.2 Closed-Loop Control
- 22.2 Transfer Function and Frequency Response Principles
- 22.3 Transfer Function
- 22.9 Closed-Loop Frequency Response
- 23.3 Nyquist Stability Criterion

24.3 Practical 1: Position Control Loop

24.3.1 Objectives and Background

In this practical a position control loops effect on the motor frequency response (transfer function) is observed. And additionally the effect of increased system gain. The influence of the position control and increased gain can be viewed on the Bode/Nyquist plots produced by the GPA.
24.3.2 Block Diagram

![Block Diagram for Practical 1](image)

Figure 24-1: Block Diagram for Practical 1

24.3.3 Perform Practical

Figure 24-2 shows the required connections on the hardware.
Ensure the power amplifier zero offset control is set so the motor is static (approximately half scale), when no input signal is applied to the power amplifier.

The closed loop feedback system is setup as shown in Figure 24-3.

Set the sweep function generator output amplitude control to half scale.

Open the GPA.

Set the min freq control to give a frequency of approximately 0.1Hz. The set frequency will take a while to be displayed (wait for progress bar to travel fully across after adjustment). Select max frequency button and set the max freq control to approx 1.6Hz.
Hi resolution can be selected. This will greatly increase the time to carry out the frequency response plot, but will give a more detailed plot.

To begin with the GPA is set to provide a Bode style plot. If you require a Nyquist plot then select this although it is possible to switch between the two whilst plotting.

Select Plot.

Now the plot will take place. This could take 15–20 minutes to complete due to the readings being recorded for values between the two set frequencies. When the plot is complete, it will repeat from the lowest frequency again writing over the existing plot.

While the plot is in progress, a progress bar will keep incrementing across the top of the GPA display, when the progress bar reaches the end, a set amount of data will be written to the screen, and then the progress bar starts over again to show that it is still active. While this is happening the sweep function generator frequency is automatically incremented by the digital controller.

How does this closed loop plot compare to the plot from Practical 1 of the Open Loop Transfer Function assignment? Refer to section 23.4.

With the open loop system the phase shift was over 90° at low frequencies, this is not the case with the position control closed loop. The phase shift at low frequencies is less than 45°.

Remove connection 5 and add connection 6.

This will change the gain of the system to 3.

Select Plot.

How has this gain change affected the system frequency response?

The plots produced demonstrate that an increase in gain at low frequencies has little effect on phase and gain of the system. But as frequency increases the zero db crossing point moves nearer to the 180° phase shift. Also, there is a more pronounced phase shift than previously seen.

### 24.4 Practical 2: Conversion to Open Loop Transfer Function

#### 24.4.1 Objectives and Background

Due to it being difficult to operate systems in open loop when the output is not known for a given input, it is possible to obtain the open loop transfer function from a closed loop transfer function, this can then be used to predict system responses for different inputs. This is particularly important information when it comes to control system design.

#### 24.4.2 Perform Practical

The importance of being able to extract open loop system response from a closed loop system is that it may not be possible to run the system in question open loop.
An example of this may be a power station where the output for a particular input my take days to appear. Also if the output is not known then potentially the output could be dangerous.

So there is a real need to be able to obtain the open transfer function from a closed loop system.

One method of obtaining the open loop transfer function of a system is to record the closed loop and plot this to a Nichols chart. An example is shown in Figure 24-4.

The open loop response for a system can be obtained from a Nichols Chart, where the closed loop values are plotted on the curved lines. The open loop response is obtained from the corresponding values for the points on the square axis.

Figure 24-4: A Nichols Chart
Notes
Using the Test Equipment

General Notes

Any of the instruments can be resized and moved at any time using conventional ‘drag-and-drop’ mouse techniques. If you make an instrument small enough then only the display area will be shown; you must increase its size again in order to restore the controls. If you close any of the instruments and open them again they will return to their default settings. Each instrument has a Defaults button which returns the equipment to its default settings (equivalent to closing and re-opening the instrument). If you want to return all the instruments (and any other resource windows) to their default size and position simply click the Auto Position button in the assignment side bar.

Some instruments allow you to place a cursor (by clicking the mouse) at any position on their display; the cursor reveals information regarding the point at which it is located. You will have to reactivate this cursor each time you change the settings, size or position of the instrument.

The Oscilloscope

The Discovery oscilloscope has many of the functions that you would find on a conventional or computer-driven scope. Its fundamental purpose is to show varying waveforms plotted against time. It is a dual trace scope, which means that it can display two separate waveforms at the same time.
The scaling of the Y (voltage) axis is determined by the practical, but is scaled to represent the true voltage levels on the hardware. Either only one channel can be displayed or both channels. The Y2 Show tick box determines whether the second channel is shown. In two-channel mode, if the Overlay box is ticked, the two traces are superimposed on the same scale as for one trace. If Overlay is not ticked the display area is divided into two and each trace is displayed half-size. The Y1 dc and Y2 dc tick-boxes determine if the inputs are dc coupled or not (ac coupled). If the signal has a large dc offset then ac coupling can be useful.

The X (time) axis is set to a default value by the practical but you may change it by using the ^ button for a faster timebase and the v button for a slower timebase. The <> and >< buttons provide a means of further expanding the trace if the highest, or lowest, timebase is in use. If you have the X scale expanded and select a lower timebase speed then the X scale automatically returns to its default setting.

An anti-alias feature automatically switches the time-base speed up if you select a rate that may produce a misleading display due to aliasing. If this feature has increased the timebase rate then the ^ button is coloured red.

The oscilloscope can also be operated in X-Y mode, where data from channel 1 is in the vertical axis and data from channel 2 is in the horizontal axis. Because the oscilloscope is a digital sampling scope, in X-Y mode the time base settings are still relevant and determine the sampling rate for both channels. Also in X-Y mode the traces have persistence and stay on the screen longer than one trace refresh.

Note that you can switch off the anti-aliasing feature from the main laboratory screen.

Triggering takes place when the selected trace crosses the zero volt level. If the Y2 Trig box is ticked, then the trigger source is Channel 2. Otherwise, Channel 1 is used. The Neg trig box enables only negative transitions to trigger the scope. Normally only positive ones do.

If the signal has a large dc offset, ac triggering can be useful.

You can return to the default settings by pressing the Default button. The Auto Position button on the Discovery laboratory window moves all the test instruments back to their default positions and sizes on the screen but does not affect their settings.

A cursor is available to make more accurate measurements. Left click on the display area to activate it. The green cursor can be moved to anywhere on a waveform. Move the mouse away and back into it to allow a tool-tip window to open with the measurement data displayed for that point.

You have to reactivate the cursor if you change the settings, size or position of the oscilloscope.
The Spectrum Analyser

The spectrum analyser enables you to look at signals in the frequency domain. In common with many modern test instruments, it uses DSP to transform time domain data into frequency domain data. The mathematics to do this is called a Fourier transform.

The Y (amplitude) scale is calibrated in Decibels relative to an arbitrary dotted line near to the top of the screen. The dB scale is linear and the number of dB per division is shown in the box. The minimum level that you can see is determined by the assignment, and ultimately by the noise in the system.

The analyser has the capability of showing two channels at the same time. Click Ch2 Show button to show channel 2 as well as channel 1.

The X (frequency) axis is calibrated in MHz, kHz or Hz per division, as appropriate. The default scale is set by the practical but you may change it by using the Higher Frequency and Lower Frequency buttons.

The anti-alias feature will operate if you try to set the frequency too low. The Higher Frequency button is shown red if this feature has increased the frequency. Note that if a new frequency component appears such as noise, the anti-alias feature may operate suddenly. The Alias Hi tick-box allows you to increase the threshold at which the anti-alias feature operates. This allows signals to be examined that have larger amounts of harmonic content. The default setting for this is off.

A cursor is available to make more accurate measurements. Left click on the display area to activate it. The green cursor can be moved to anywhere on a waveform. Move the mouse away and back into it to allow a tool-tip window to open with the measurement data displayed for that point.

You will have to reactivate the cursor if you change the settings, size or position of the spectrum analyser.
The Phasescope

The Phasescope is a special instrument that compares two signals in phase and amplitude (magnitude). The two signals are referred to as the reference and the input. The display is in polar format, i.e. the phase is in the form of a circle and the amplitude as the radius. The use of a circle is possible because phase is a continuous function repeating every 360 degrees. The display can be seen as Polar, as the one orthogonal axis represents the real component and other the imaginary part. The convention here is that the real axis is the X axis, which means that zero degrees is straight up or at 12 on a clock face. +90 degrees is at 3 on the clock face and –90 at 9. It is important to note that in terms of phase +180 degrees is the same as –180 degrees.

The radius scale has one circle at radius = 1 (the outermost circle) i.e. the two signals are of the same amplitude. Further inner circles are at 0.707, 0.5 and 0.25.

The circle at 0.5 has a square associated with it, the corners of which are at 0.707. This represents the case when two orthogonal vectors of amplitude = 0.5 are added.

In some cases only the phase is of interest so, if you click the Phase Only box, the radius is set to 1.

The conventional display is that of a vector i.e. a line joining the point to the centre. However, in some cases it is much easier to interpret the display if only a point is drawn. Where the amplitude and phase is varying between discrete values they are shown as a pattern of dots resembling stars, hence the term constellation display. This mode can be selected by ticking the Constellation box. In constellation mode, the persistence of the display can be varied. By selecting the Persistence tick box, traces stay on the screen for a number of trace refreshes before being removed. By selecting Hi Persist this time is extended.

If the two signals are of different frequencies the result is a continuously rotating vector, rotating at a rate equal to the difference in frequency. The direction depends on the sign of the frequency difference. If the rate is fairly fast, the instrument may only be able to show a limited number of discrete values.
In many cases the reference input will not be at exactly zero degrees with respect to the theoretical zero degrees of the input signal. This causes the display to be rotated. In some cases this may be important to know, but where it is not the Phi Offset control gives the ability to rotate the display for easier interpretation.

The coloured indicator (Ref Ch) to the top left of the display tells you which probe is being used as the reference channel.

A cursor is available to enable more accurate measurement. Click the display to use it.

**The Voltmeter**

The meter is simply an ac and dc voltmeter that displays the value in digital form. It can be used in ac mode by clicking ac p-p, in which case the value represents the peak to peak value. If the waveform has a high crest factor the results can be slightly surprising. In dc mode, if there is an ac component present, the average value is displayed.

**The Frequency Counter**

This has the facilities of a conventional frequency meter/counter. It will display in either frequency or time. If the input amplitude is too low a warning message will be displayed. Like all frequency counters, it can produce misleading results if the waveform is complex or contains many frequencies.

**The Gain Phase Analyser (GPA)**

The GPA displays graphically the gain and phase characteristics between a selectable range of frequencies, either as Bode plots (the default mode) or as a Nyquist plot.
Whenever the GPA is opened, the Set Min Freq button on the GPA is initially selected. The minimum frequency that you set in the circuit whilst this button is selected will represent the low end of the range to be plotted. After setting the minimum frequency, select the Set Max Freq button and adjust the maximum frequency in the circuit to the high end of the range. When either the Set Min Freq or Set Max Freq buttons are selected, the frequency will be shown numerically on the GPA display. Finally, select the Plot button to plot the gain and phase between the frequencies you have chosen. Note that the GPA takes time to plot at low frequencies.

You can verify (and change) the minimum or maximum frequencies at any time by selecting the Set Min Freq or Set Max Freq buttons as required (and adjusting the low or high frequency if desired). You will then need to select the Plot button again. Note that only one of these three buttons can be selected at any time.

Tick the Hi Resolution box to get a better frequency resolution of the plot, although plotting will take longer.

Bode plots are the most widely used means of displaying and communicating frequency response information. Bode diagrams are presented as two separate graphs: one showing magnitude and one showing phase, both plotted against frequency. Because the axes are logarithmic, they condense a wide range of frequencies (horizontal axis) and a wide range of gains (vertical axis) into the graphical area. In Bode plots, commonly encountered frequency responses have a shape that is simple and easy to recognise.

A Bode diagram shows two traces, representing the magnitude and phase, from which you can see variations with frequency. You are able to measure the gain and phase at different frequencies by clicking the cursor on the GPA display. It is much easier to use the GPA than the oscilloscope for this purpose.

Tick the Magnitude only box if you prefer to display the gain trace without the phase trace. This box is disabled when you select a Nyquist plot.
The Nyquist plot of a system is simply the polar representation of the Bode plots. This plot combines the magnitude and phase on a single graph, with the frequency as a parameter along the curve. Nyquist plots are particularly helpful for stability analysis in control system design.

On the GPA, tick the Nyquist box to create a Nyquist plot (you should already have set the minimum and maximum frequencies as described above). Use the vertical slider bar at the left-hand side of the GPA display to move the cursor to any point along the plotted curve. Then read off the frequency, magnitude and phase by hovering the mouse over that cursor. The convention here is that the zero degrees radial is at 3 on a clock face.

Untick the Nyquist box to return to Bode plot mode. You can switch between Bode and Nyquist modes at any time.

The Data Logger

This displays as a moving image four channels of data much as a conventional paper data logger / chart recorder does. The four full scale values are shown in the lower boxes. In the example shown the full scale is 1.25 volts which means that the display is centred on zero and the range of voltages shown is plus 1.25 volts to minus 1.25 volts. The X Cal Rate box shows how many milliseconds between each vertical timing line. In overlay mode all four channels are displayed on top of each other. If this mode is not selected then the available screen height is divided separate sections for each trace.

In four channel mode all four channels are shown. If this mode is not selected then only the first two channels are shown.
In order to measure exact values a capture facility is provided and is activated by the Capture button. This displays a separate window of stationary data. Successive clicks of the capture button overlays further traces. The Clear button clears the capture window. Cursors are available for the capture window. The main cursor simply allows the reading of both time and amplitude data. If the Delta cursor button is clicked a second cursor appears and all readings for it are relative to the normal cursor.

The record facility enables the recording of results from each of the four channels. The results obtained are saved to a .CSV type file.

To record, first select the ‘Set file name’ button. Select a path where you wish to save the recording file. Enter a new file name or select one from those available.

Note: Selecting an existing file name and clicking ‘Save’ will erase the current file and all data will be lost.

Now select ‘Start’ to begin recording. Once all the data required has been recorded select the ‘Stop’ button. The recording can be restarted by selecting record again. The additional data will be added to the existing file.

The .CSV file created will contain the name of the product you were using and practical name and number along with the time and date the recording was started.

The first of the columns contain the time from the point of the recording starting to each of the timing lines. After the first timing line value they will be spaced approximately by the X Cal Rate setting.

The other four columns contain the data from each of the channels. If no data was recorded from a particular channel then all the entries will be zero.

If a recording was stopped and then started again the data will be separated by the name of the product you were using and practical name and number along with the time and date that section of the recording was started.
The Bar Display

This simply displays in real time the amplitudes of data connected to its four channels. In bipolar mode it displays values up to its full scale value either side of zero. If this mode is not selected it only shows positive values.

If the Show Values box is ticked then the numerical values of the data is displayed underneath each bar.
Buffer Amplifiers

A buffer amplifier is simply an amplifier that has a high input impedance, a low output impedance and a voltage gain of 1.

The purpose is to isolate the previous stage in a circuit from the effect of driving another stage in the circuit. This is often simply the loading effect on the source due to input impedance but may also, for example, prevent a signal being fed back.

Many buffer amplifiers are ac coupled and actually have a dc shift between input and output. Some are dc coupled. An operational amplifier may be connected as a buffer and is then sometimes referred to as a voltage follower.

The circuit fragment below shows how an operational amplifier is connected as a voltage follower.

![Buffer Amplifier Circuit](image)

There are also buffer amplifiers that invert the signals, hence the terms **inverting** and **non-inverting** buffers. The act of inversion may be required for certain applications.
Notes
Function Generator

A function generator is a type of signal generator. The name *function* is derived from the fact that they usually generate three types of waveform, a sine wave, a square wave and a triangle wave. These wave shapes can be represented by simple mathematical functions and are used to apply test signals such that a system response to these functions can be evaluated. Hence the term *function generator*.

Although many signal generators produce sine waves, which is one of the simple function the term function generator is usually reserved for generators that produce all three.

Because of the difficulty of producing accurate wave shapes at high frequency function generators are usually fairly low frequency devices.
Notes
Gain Phase Analyser/Transfer Function Analyser

This generally refers to an automatic system that measures the relative phase and amplitude of a system’s input and output at a number of frequencies. The results are usually displayed in graphical form either as an plot of amplitude against frequency plot plus a phase against frequency plot (commonly called a Bode plot) or as a single polar plot of phase and amplitude with frequency as the locus parameter (a Nyquist plot).

These plots can be most useful in analysing a control system for various performance criteria, particularly stability.
Notes
Incremental Code Disk

This type of sensor is a digital sensor using optics to read the angular position of shaft. It may also be used to measure speed by calculating change of position in a set time.

It has three optical tracks arranged in a 1-0-1-0 pattern around the disk. By counting the pattern the fact that the disk is moving can be sensed. The two main tracks are arranged to be out of phase by half a pattern width which means that by sensing the state of one track when the other changes the direction of rotation can be sensed.

A counter is incremented and decremented to measure position.

The reason that these sensors are called *incremental encoders* is that while the fact that the position has changed and by how much can be sensed there is no information about the absolute position. A third track called the index track provides a single bit to reset the counter to a known value at a particular point in the rotation.

The number of pulses per revolution determines that resolution of the measured position and can vary from a few to several thousands depending on the encoder.
Notes
Three Term (PID) Controller

A general controller combines integral and derivative actions, with the direct error as shown above:

\[ V_c = G (\text{error} + K_d \frac{dV}{dt} + K_i \int V_e \, dt) \]

where there is an adjustable overall gain G, and the integral and derivative components are individually adjustable.

The controller is often referred to as a PID (signifying Proportional + Integral + Derivative) controller.

The processes required in a Three Term Controller, the generation of derivative and integral signals and their combination in adjustable proportions are very easily realisable in analogue form with operational amplifiers if the signals are available as voltages.

However, it is possible to realise Three Term Controllers in a non-electrical medium such as pneumatics.
Notes