Purpose: The objectives of this experiment are to gain some experience with the tools we use (i.e. the electronic test and measuring equipment and the analysis software) and to gain some fundamental understanding of voltage dividers.

Background: Before doing this experiment, students should be able to

- Apply Ohm’s Law to determine the current through a resistor
- Determine the values of series and parallel combinations of resistors
- Download and install software on a Windows machine

Learning Outcomes: Students will be able to

- Build, test and simulate a simple resistive voltage divider and demonstrate conditions under which measurement devices (e.g. DMM or oscilloscope) significantly affect the operation of the circuit. Then, use the changes in voltages caused by the measurement devices to determine the resistance of the measurement device.
- Be able to build simple resistive circuits driven by constant voltages using a small protoboard (aka breadboard).
- Use an oscilloscope to measure and display the voltages in a simple resistive circuit driven by a sinusoidal voltage from a function generator.
- Articulate a series of questions posed about simple circuits and answer the questions using fully annotated data obtained from physical experiments.
- Develop the circuit model of a physical battery using an ideal voltage source and an ideal resistor.
- Calculate the power delivered by a battery and dissipated in a resistor.

Equipment Required

- Analog Discovery With Digilent Waveforms
- Oscilloscope Analog Discovery
- Function Generator Analog Discovery
- DC Power Supply Analog Discovery & Batteries
- DVM Analog Discovery
- Two 100 Ohm resistors, two 1M Ohm resistors and two 1k Ohm resistors.
- Protoboard

Pre-Lab

Required Reading: Before beginning the lab, at least one team member must read over and be generally acquainted with this document and the other reading & video materials.

Hand-Drawn Circuit Diagrams: Before beginning the lab, hand-drawn circuit diagrams must be prepared for all circuits either to be analyzed using a SPICE program or physically built and characterized using your Analog Discovery board.
Experiment

Summary

You should now know how to set up voltage signals with the Wavegen feature of the Discovery Board and how to use the Channel 1 (and Channel 2) inputs to measure voltage.

Part A – Voltage Dividers and Measuring Equipment

In this part of the experiment you will be learn that equipment isn’t ideal and that “real” behavior must be taken into account when making measurements. You will look at batteries and measure the effective internal resistance; they aren’t ideal voltage sources. You will also look at the behavior of two voltage dividers when a DC voltage and an AC voltage are applied. You will use circuit analysis to examine the behavior of these circuits.

Background

Series and parallel circuits (repeated from Laboratory 2): Another fundamental concept we need to understand in order to analyze the circuits we will build is how to mathematically combine resistances. If any number of resistances are connected in series, you simply add them to find the total resistance. If any number of resistances are wired in parallel, the total resistance is the reciprocal of the sum of the reciprocals of all of the resistances. This is summarized in Figure A-1.

![Figure A-1](image)

Series: \[ RT = R_1 + R_2 + \ldots + R_n \]

Parallel: \[ \frac{1}{RT} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \]

\[ RT = \frac{R_1 \cdot R_2}{R_1 + R_2} \quad \text{(2 only)} \]

Impedance: Every piece of electrical equipment has an effect on the circuit you connect it to. Just as it is impossible to design a dynamic mechanical system without friction (that resists motion), it is impossible to design an electrical system without impedance (that resists the flow of electrons). Impedance has two effects on an electrical system. It changes its magnitude (the value of the voltage) and its phase (voltage behavior over time). If the impedance affects only magnitude, then we call it resistance. Each electrical measurement device has an internal impedance, and this is also true for the Analog Discovery. The impedances we will concern ourselves with in this class are listed in Table A-2 below: (These values aren’t exactly correct, but they still can be used to make the point.)

<table>
<thead>
<tr>
<th>Device</th>
<th>Impedance (magnitude only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scope Analog Discovery</td>
<td>1MegΩ</td>
</tr>
<tr>
<td>DMM (DC voltage)</td>
<td>10MegΩ (typical)</td>
</tr>
<tr>
<td>DMM (AC voltage)</td>
<td>1MegΩ (typical)</td>
</tr>
<tr>
<td>function generator Analog Discovery</td>
<td>Negligible (typical)</td>
</tr>
<tr>
<td>DC power supplies (any)</td>
<td>Negligible (typical)</td>
</tr>
<tr>
<td>Batteries</td>
<td>0.4 to 32Ω (typical)</td>
</tr>
</tbody>
</table>

Table A-1 Approximate impedances associated with common circuit’s equipment
Note that presently we are only concerned about the effect of the equipment on the magnitude (resistance component) of the impedance. Also note that the devices are designed to have minimal effect on any circuit they are connected to. In this part of the experiment, we will examine how much of an effect the equipment has.

*Voltage dividers:* In order to analyze the effect of the equipment, we need to understand a fundamental concept of circuit analysis called a voltage divider. Basically, when a voltage in a circuit is applied across two or more resistances, it divides up in a manner proportional to the resistances. That is, a larger resistance will have a larger voltage drop and that voltage drop will be proportional to the size of the resistance divided by the total resistance of a circuit.

![Voltage Divider](image)

In Figure A-2 above, $V_{in}$ is divided between $R_{1}$ and $R_{2}$. Mathematically, this can be expressed:

$$V_{in} = V_{R1} + V_{R2}$$

$$V_{R1} = \frac{R_1}{R_1 + R_2} V_{in}$$

$$V_{R2} = \frac{R_2}{R_1 + R_2} V_{in}$$

Note that $R_1 + R_2$ is the total resistance of the circuit. We can use a voltage divider to determine how much effect a device has on a circuit, or in this case, the effect that a circuit has on a device. In the simple electrical model of the battery shown in Figure A-3, the internal resistance of the battery depends on the battery size and chemistry. This is a simple model that ignores much of the internal chemistry including changes as the battery is discharged. The default assumption normally is that the voltage output of a battery doesn’t change with the load. We will investigate how this works in an actual circuit.

![Battery Model](image)

The output of the battery is measured using the Analog Discovery with and without a load resistance. Remember that $R_{bat}$ represents the internal model of the battery. **You do not add this resister to the circuit.** $R_{load}$ represents the load, or combined resistance of whatever circuit you place on the source. Using the voltage divider rule, we know that the voltage drop across the load is given by:

$$V_{measured} = \frac{R_{load}}{R_{bat} + R_{load}} V_{bat}$$

Note that the voltage divider rule applies only to series circuits. Any time we use our measuring devices to measure the voltage across a device, as illustrated in Figure A-4, we are combining that device in parallel with the resistance we are measuring. So just connecting the oscilloscope will affect the quantity to be measured. In this case the effective
load resistance on the battery is \( R_{\text{total}} \) and it is the parallel combination of the scope impedance (1 Meg\( \Omega \)) with the resistance of the load resistor (\( R_{\text{load}} \)). This results in total load resistance, \( R_{\text{total}} \).

\[
\text{Figure A-4.}
\]

Once you have the total load resistance, \( R_T \), you can use the voltage divider rule to find the internal resistance of the battery. Note that, since the voltage drop across any number of resistors in parallel is the same, \( V_{R_{\text{total}}} \) is equal to \( V_{R_{\text{load}}} \).

**Experiment**

**A.1) Some DC Measurements**

We will look at what happens when we apply a load to a battery. We will be using batteries extensively in this course, so understanding their basic electrical properties is critical. We will be making DC measurements, like we do with a typical multi-meter. For this section, shut off the scope, go to the main Waveforms window and select Voltmeter from the ‘More Measurements’ drop down menu. When this is enabled, it will use the inputs for the scope channels, but it is better to have the scope off to avoid confusion.

- Measure the voltage of a 9V “Heavy Duty” battery without any load. Simply connect the battery to the protoboard and connect the leads from the protoboard to the 1+ (Scope Channel 1 Positive (orange)) and 1- (Scope Channel 1 Negative (orange-white)). Note that, when we make most measurements in this course, they will be single-ended (referenced to ground). Then you need to touch the 1+ wire to the point of interest. To do this, the negative input 1- and GND must be connected. When we make what are called differential measurements, we use the two wires but do not connect the ground. We will return to that in a future experiment. In this case the load is an open circuit (infinite resistance) because we have added no load to the battery; the input resistance of the Analog Discovery is also so large compared to the range of battery resistance listed above, that it can also be ignored. Record the value, \( V_{\text{bat}} \), of the voltage you measure.

- Now add a load to the battery, as in **Figure A-5**. The load is two 100\( \Omega \) resistors in series. We will discuss why two resistors are used a little later. Set up the circuit so that you can add and remove the load quickly, leaving it disconnected unless you are making a measurement. This just means wire it so that it is easy to pull out and reinstall one end of one resistor. You should only connect the load to the battery for a short moment (a second or two) long enough to make the measurement. If you leave the resistors connected, your battery will drain down quickly and will definitely not last a full semester. Record the voltage displayed in the Voltmeter window of the Analog Discovery with the resistive load disconnected. Then connect the resistors, quickly record the new voltage, and quickly disconnect the load. You may want to repeat this a few times to find the typical change of voltage with and without the load. Remember to record the unloaded battery voltage as well as the change in voltage. In the figure, A1+ is 1+ and A1- is 1- on the Analog Discovery Board.

\[
\text{Figure A-5.}
\]
Use the results from this experiment to determine the value of \( R_{\text{bat}} \). We can apply the circuit in Figure A-5 to the voltage divider equations shown above. We are measuring the voltage across \( R_1 + R_2 \) which are equivalent to \( R_2 \) in the voltage divider equation. When \( R_1 + R_2 \) is an open circuit, we are effectively measuring \( V_{\text{bat}} \), which is \( V_{\text{in}} \) in the voltage divider equation. When \( R_1 + R_2 = 100\Omega + 100\Omega \), the result is \( V_{R_2} \) in the voltage divider equation. We can then find \( R_{\text{bat}} \), which is \( V_{R_1} \) in the voltage divider equation.

Repeat the experiment with a different battery from a plastic case the TAs will place on the center table. Choices are a 9V Alkaline, a pack of 2 AA alkaline batteries in series, and a watch battery. If you do a low voltage battery it may be wise to load the battery with only one 100\( \Omega \) resistor.

Share data with other teams so that you have numbers for at least 4 battery types.

As noted under Figure A-5, the information you have just obtained about each battery using a voltage divider can be used to determine the internal resistance of the battery. What is the internal resistance of the two batteries characterized by your team?

A.2) Some more Voltage Divider Measurements

The part above showed that the load can affect the equipment, in this case a battery. Now we will look at how the instrument can affect the circuit. The Analog Discovery oscilloscope can load the circuit and affect the circuit to be measured.

- Use the function generator, W1, of the Analog Discovery to output a DC signal of 1V. Build the circuit in Figure A-6. Use \( R_1 = 1k\Omega \) and \( R_2 = 1k\Omega \). Connect the Channel 1 and Channel 2 inputs as shown in the figure. Using the voltmeter, measure the voltage across \( R_1 + R_2 \) (combined) using Channel 1 and the voltage across \( R_2 \) using Channel 2. Record your measurements.

The Function Generator output is W1. Only one ground connection has to be made from the Analog Discovery because the Function Generators (W1 & W2) are connected internally to ground.

- Calculate the ratio of the voltage measured on Channel 2 to the voltage on Channel 1. Using the source voltage of 1V, verify that the voltage divider equation applies to the circuit.

- Repeat the experiment using \( R_1 = R_2 = 1\text{Meg}\Omega \) resistors. Again, record your measurements. Check the voltage divider expression. You should notice that it does not seem to be correct.

- The more exact model of this measurement is given in Figure A-7, were \( R_{\text{channel1}} \) and \( R_{\text{channel2}} \) represent the effective internal input resistances of the analog input channels of the Analog Discovery. The effective input resistance of Channel 1 can be ignored (Do you know why?), but the input resistance of Channel 2 affects the measurement. Using the measurements above, expressions for parallel circuit (\( R_2 \) and \( R_{\text{channel2}} \)) and the voltage divider expression, estimate \( R_{\text{channel2}} \).

Using the measurements above, expressions for parallel circuit (\( R_2 \) and \( R_{\text{channel2}} \)) and the voltage divider expression, estimate \( R_{\text{channel2}} \).
A.3) Potentiometer (Variable Resistor)

- Replace R1 and R2 with the one of the potentiometers from your parts kit. A 5kΩ is shown above. The value is obtained using the ‘502’ indicated on the case, where the three digits are read in the same manner as a resistor color code, 50 x 10². When connecting the potentiometer to the circuit, notice that there are three ‘legs’. The ‘middle leg’ is equivalent to the middle arrow of the schematic shown in Figure A-9. It does not which way you orientate the ‘end legs’ Measure the voltage between the ‘end legs’ and between the ‘middle leg’ and one ‘end’.
- Apply 5V using Wavegen. Make the two indicated voltage measurements. Apply the voltage divider expression to determine the resistance between the ‘middle leg’ and the ‘end leg’. Note, the resistance between both ends is the resistance value of the potentiometer, 5kΩ for Figure A-8.
- Rotate the pin at the end of the potentiometer and take new measurements, once again determining the resistance. Repeat for two or three rotations.

A.4) Power Calculations and Impedance Matching

Now we will look at the power associated with the battery circuit

- Power Ratings: In part A.1 you used two resistors in series. The effective resistance of resistors in series is simply the sum of the resistances. So why use two 100Ω resistors in series when we could use one 200Ω resistor? Power rating is the answer. \( P = IV = I^2R = V^2 / R \) where P is the power, V, I, and R are the voltage, current, and resistance of the load. The power is in Watts if you use Volts, Amps and Ohms. Our resistors have a power rating of ¼ watt.
  o Calculate the total power out of the battery for part A.1 for just the 9V battery measurements.
  o Calculate the power per resistor. Ask for help if it isn’t now clear as to why we used 2 resistors rather than one for this measurement.
  o Calculate the total power out of the battery for part A.1 for the second battery

- Impedance matching: Impedance matching is important with weak signals, not with batteries. Even so, the concept can be demonstrated using our circuits. Don’t wire this circuit; it would cause excessive heating and a rapid discharge of the battery.
For this part assume that you have a 9V battery with an internal resistance of 30Ω. Using Figure A-3, calculate the voltage that would be measured across the load if the load resistance is 100Ω, 60Ω, 30Ω, 20Ω, and 15Ω. For each load resistance, determine the power that would be dissipated in the load resistor. Plot the power dissipated vs. the load resistance.

If you did this correctly, you will see that the maximum power in the load occurs when the load resistance is equal to the internal battery resistance. This is call impedance matching.

Summary

You should now understand how to calculate the effective resistance of resistors in series and/or in parallel. You have an appreciation of AC and DC signals, and that the load and/or the equipment affects the voltages and currents in the circuit. Lastly you should be comfortable with using the Analog Discovery, including the function generator, the oscilloscope and the voltmeter.

Report

Be sure you have answered all of the questions in the text. A report format will be provided for you, but it will largely just be a list of the questions.