Class #6: Resistor Combinations and Voltage Dividers

**Purpose:** The objectives of this experiment are to gain some experience with the tools we use (i.e. the electronic test and measuring equipment and the analysis software) and to gain some fundamental understanding of voltage dividers.

**Background:** Before doing this experiment, students should be able to

- Apply Ohm’s Law to determine the current through a resistor
- Determine the values of series and parallel combinations of resistors
- Download and install software on a Windows machine

**Learning Outcomes:** Students will be able to

- Build and test a simple resistive voltage divider.
- Apply circuit reduction concepts to resistive circuits to simplify the analysis to a familiar circuit.
- Be able to build simple resistive circuits driven by constant voltages using a small protoboard (aka breadboard).
- Articulate a series of questions posed about simple circuits and answer the questions using data obtained from physical experiments.

**Equipment Required**

- ADALM1000 (M1K) board with Alice Source-Meter tool or Analog Discovery with Digilent Waveforms
- Several different Resistors and wires from Parts Kit
- Protoboard

**Pre-Lab**

*Required Reading:* Before beginning the lab, at least one team member must read over and be generally acquainted with this document.

*Videos:* Before beginning the lab, all team members must read over and be generally acquainted with the voltage divider, series resistors, and parallel resistors videos.
Learning from previous experiments:

You should now know how to set up DC voltage signals (constant) with the source voltage feature of the M1K Board and how to use the Channel A (and Channel B) inputs to measure voltage using the meter or voltmeter function. You should also be familiar with the Split I/O feature of the channels’ when and how to use it.

Part A – Voltage Dividers and Resistor Characteristics

Background

Series and parallel circuits: A fundamental concept we need to understand in order to analyze the circuits we will build is how to mathematically combine resistances.

If any number of resistances are connected in series, you simply add them to find the total resistance, \( R_{\text{tot}} = \sum_{i=1}^{N} R_i = R_1 + R_2 + \cdots + R_N \). This is summarized in Figure 1 with two resistors in series. If we measure the voltage at nodes A and B (use the two leads of the CH A and CH B/voltmeter) and apply Ohm’s Law to get the current, for the current to be the same in the two circuits they must have the same equivalent resistive. In the figure, this is shown by the relationship \( R_3 = R_1 + R_2 \) where \( R_1 \) and \( R_2 \) are in series and can be added to give a single equivalent value.

\[ R_{\text{eq}} = R_1 + R_2 + \cdots + R_N \]

**Figure 1:** Two Resistors in Series

We can extend this to any number of resistors in series, as seen in Figure 2. The equivalent resistance is the sum of all the resistors in series.

**Figure 2:** N Resistors in Series

**Important note:** Resistors are in series if only the two resistors are connected at one of the ends. There cannot be anything else connected.
If any number of resistances are connected in parallel, we need to take the inverse of the sum of the inverses,

\[ R_{\text{tot}} = \frac{1}{\sum_{i=1}^{N} \frac{1}{R_i}} = \left[ \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \right]^{-1} \].

This is summarized in Figure 3 with two resistors in parallel. If we measure the voltage at nodes A and B (use the two leads of CH A and CHB/voltmeter) and apply Ohm’s Law to get the total current, for the total current to be the same in the two circuits they must have the same equivalent resistive.

In the figure, this is shown by the relationship \( R_3 = \left[ \frac{1}{R_1} + \frac{1}{R_2} \right]^{-1} \), where R1 and R2 are in parallel.

![Two Resistors in Parallel](image)

We can extend this to any number of resistors in parallel, as seen in Figure 4. The equivalent resistance is the inverse of the sum of the inverses.

![N Resistors in Parallel](image)

**Important note: Resistors are only in parallel if both ‘sides/ends’ are connected together.**
Voltage dividers: In order to analyze the effect of the equipment, we need to understand a fundamental concept of circuit analysis called a voltage divider. Basically, when a voltage in a circuit is applied across two or more resistances, it divides up in a manner proportional to the resistances. That is, a larger resistance will have a larger voltage drop and that voltage drop will be proportional to the size of the resistance divided by the total resistance of a circuit.

In Figure A-2 above, Vin is divided between R1 and R2. Mathematically, this can be expressed:

\[
Vin = V_{R1} + V_{R2} \quad V_{R1} = \frac{R1}{R1 + R2} Vin \quad V_{R2} = \frac{R2}{R1 + R2} Vin
\]

In the voltage divider circuit, R1 and R2 are in series. Using the series expressions on the previous page, we can determine the total resistance ‘seen’ by the source, \( R_{tot} = R1 + R2 \).
Experiment

A.1) Series and parallel resistors.

In the above circuit, two of the resistors are in series and two of the resistors are in parallel.

- Using the notes on the previous pages, determine which two resistors are in series and which two resistors are in parallel. Answer on template
- For the resistors in series, use the appropriate expression to determine an equivalent single resistor. Answer on template
- For the resistors in parallel, use the appropriate expression to determine an equivalent single resistor. Answer on template
- Redraw the circuit with only two resistors, using the equivalent resistances you just calculated. (You can take a picture with your phone of the circuit and copy it into the template.)

A.2) Some DC Measurements

In Part C of the previous experiment, we analyzed a voltage divider circuit and measured the voltage across each resistor. One of the observations we can make about the voltage divider circuit with two resistors is that the sum of the two resistor voltages must be equal to the source voltage.

- Refer to your Part C results when the CH A Voltage was set to 3V. Verify that the sum of the two resistor voltages is very close to 3V. (There may be a small difference, that is normal.)
  \[ V_S = V_{R1} + V_{R2} \]
- For the same 3V CH A voltage, using the voltage divider expressions on the previous pages to calculate the expected voltages across the resistors. Are your calculations close to your measured values (they should be close)?
  \[ V_{R1} = \frac{R1}{R1+R2} V_S \]
  \[ V_{R2} = \frac{R2}{R1+R2} V_S \]
B.1) Circuit Simplification and Voltage Dividers

The circuit shown in Figure B-1 has a similar configuration to that on the previous page. You should be able to quickly recognize the two resistors in series and the two resistors in parallel. Build the circuit on your protoboard and measure the voltage across the two resistors in series and the across the two resistors in parallel. Figure B-2 is the same circuit with the M1K Board connections included.

- CH A should be in Split I/O mode. We are using it as a source and measuring voltage unit.
- Set the CH A voltage to 4V and record the two voltage measurements, CA-V and CB-V meter using the Alice meter-source tool. Review experiment 5 video as needed.
- Determine the equivalent resistance of the two resistors in series and the equivalent resistance of the two resistors in parallel.
- Redraw the simplified circuit with the equivalent resistances. (You can take a picture of the circuit and copy it into the template.)
- Using the voltage divider expressions, calculate the voltage across the two resistors.
  - Do the two resistor voltages sum to 4V as expected?
  - Are the calculated values close to your measured values?
- Considering the two equivalent resistances, find the closest valued resistors in your parts kit and build the simplified equivalent circuit. (Keep your above circuit and build the new circuit somewhere else on your protoboard.). Your new circuit should look similar to the circuit we analyzed in Part C of the previous laboratory. Measure the voltage across the two equivalent resistors.
B.2) Circuit Simplification and Voltage Dividers

Add a third resistor in parallel, R5, and repeat the process on the previous page.

Summary

You should now understand how to calculate the effective resistance of resistors in series and/or in parallel. You should be able to determine the voltage across a resistor in a voltage divider circuit. Lastly you should be comfortable with using the M1K board and Alice Meter-Source tool to source and measure DC voltages, including the Split I/O functionality.