Purpose: The objective of this experiment is to begin to become familiar with the properties and uses of the exponential function in circuits and engineering in general.

Background: Before doing this experiment, students should be able to
- Analyze simple circuits consisting of combinations of resistors.
- Build simple circuits consisting of combinations of resistors on protoboards and measure input and output voltages vs. time.
- Make differential voltage measurements using M1K or Analog Discovery.
- Review the background from the previous experiments.

Learning Outcomes: Students will be able to
- Determine and validate mathematical expressions for the discharging of capacitors through resistors that utilize the exponential function.
- Use series and small argument representations of exponential functions in RC and RL circuit analysis of both experimental and simulated circuits.
- Do a transient (time dependent) simulation of circuits using LTspice

Equipment Required:
- M1K (with Alice tools) or Analog Discovery (with Waveforms Software)
- Parts Kit
- Matlab
- LTspice

Helpful links for this experiment can be found on the course website under Class #19.

Pre-Lab

Required Reading: Before beginning the lab, read over and be generally acquainted with this document and the other required reading materials.
Part A – Arbitrary Waveform Generator and Oscilloscope plots

Watch demo video for Alice Desktop before working on Part A. Click here.

For the first experiment, we need to set up a sinusoidal voltage. Set up Channel A of M1K board to generate a signal from the CH A pin and measure it with Channel B on the Bin. This can either be done by connecting a wire directly from CH A to Bin or using a protoboard.

**AWG Control:**

- Configure Channel A to generate a voltage waveform described by the function below

\[ 2 \sin(2\pi \times 500 \times t) + 2 \]

Where ‘t’ is the time variable. Think about what you would need to choose for min./max. and frequency values of CH A to achieve the above waveform.

- Configure Channel B such that it can measure a voltage waveform on Bin. If you aren’t sure about how to do any of this, please watch the above video.

**Oscilloscope:**

- Configure the Alice desktop oscilloscope to plot channel A and Channel B voltage waveforms.
- Adjust the time scale such that two to three time periods of the waveforms are displayed.
- Adjust the vertical scale to make efficient use of plot area.
- To distinguish between source and measured waveforms, adjust the vertical scale of Channel B such that it appears half as big as Channel A.
- Explore other options such as trace width, background color.
- Mark any four points using t or v cursors.
- Provide a screenshot of the oscilloscope plots window in your template.
Part B – Discharging a Capacitor Through a Known Resistor

Background

The exponential function $e^x$ is the inverse function of the natural logarithm, $\ln(x)$. The exponential function is also one of the most amazing mathematical functions in that it is its own derivative! There is a lot to know about this function, so you should begin by at least glancing through the Wikipedia entry https://en.wikipedia.org/wiki/Exponential_function.

You will encounter it over and over in your engineering, science and math courses because it characterizes the response of damped systems (and what real system is not damped due to friction or resistance?) and can be used to characterize oscillating systems (a topic of the next class).

Experiment (Use Alice Data Logger Tool for this)

We begin with a very simple experiment with an RC circuit. Connect two identical capacitors (C1 & C2 = 47µF) and a resistor $R_{\text{discharge}}$ in parallel on a protoboard. Also connect your M1K channel A and Ain pins as shown below so you can charge the capacitors and monitor the voltage as a function of time when it discharges. Add the ground wire to the bottom of the circuit. Choose the resistor so that the decay constant RC is about 10 seconds. Note that you must include the total capacitance and the total resistance in your evaluation of the time constant RC. That is, you must include the input resistance of Channel A in addition to both capacitors.

![Figure B-1](image)

What are the values of the capacitors you used and what is the value of $R_{\text{discharge}}$ so that the time constant is about 10 seconds?

Start the Alice Data Logger tool. Configure CH A to source 5 V DC. Put this channel in split I/O mode. We will not use Channel B for this part. Using the Alice Data Logger tool and the steps shown in this video, record about 40 seconds of capacitor discharge. Turn channel A off after the initial charge up to 5 V is complete. This should be very quick. Make sure you log data to file. Select 50 samples per second.
Then, using the csv file, determine the measured voltage at 11 evenly distributed points from 0 to 30 seconds and enter each value in the table below.

<table>
<thead>
<tr>
<th>Time</th>
<th>Measured Voltage</th>
<th>Calculated Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
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<td>3</td>
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<td>6</td>
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<tr>
<td>30</td>
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</tbody>
</table>

Next, we wish to compare the measured voltages to the mathematical function $V(t) = V_o e^{-t/\tau}$, where $\tau = RC$ is the decay constant. Use your measured voltage $V(0) = V_o$ for the amplitude of the exponential function. Your previously determined values for $R$ and $C$ give the time constant.

Calculate the values for the exponential function at each measurement time and enter them into an Excel table. Once you have both columns filled in, load the data into Matlab and plot both the measured and the calculated curve. You can do this using the `plot(data)` command, where data is the name of your imported column in Excel. If you have done this correctly, the two curves should look very much alike.

**Returning to Capacitors in Parallel (Use Alice Data Logger Tool for this)**

Before moving on to the next part of this experiment, we will use the set-up as is to explore how capacitors add in parallel. Start the experiment as before by charging the capacitors with the 5V source. Start the data logger and observe the voltage for about 3 or 4 seconds. Then, remove one of the capacitors, being careful not to disturb any of the other connections. The total capacitance should now be about half what it was. While the capacitor in the circuit is discharging slowly through the resistor, short out the capacitor you removed from the circuit (connect a wire between the two ends) so that it no longer has any voltage or charge. Now connect the uncharged capacitor back in parallel with the charged capacitor. What do you observe? Why do you think the voltage changed when you reconnected the discharged capacitor?

Repeat the process of removing, discharging and reconnecting the second capacitor. Once you reach the end of the 40 second scan, stop the logger and save the plot to your report. Fully annotate the plot explaining the features that you see. Start the process over again by charging the two capacitors. After each 3-4 seconds, remove the second capacitor and then replace it after another 3-4 seconds, but do not discharge it when it is disconnected. Do this for the full 30 seconds, save your plot and fully annotate it. What is different this time and why? To help understand what you see in the second process, start it over with both capacitors and then, after 5 seconds, remove the resistor from the circuit. Wait another 3 seconds and then disconnect the Ain pin. Leave it disconnected for about 15 seconds and then reconnect the Ain wire. Save the plot, annotate it and explain what you see.
Next, we will change the circuit so it will do something similar but much faster. Replace the capacitors with 0.1µF (very small light brownish two terminal devices with 104 written on them, use flashlight to read the numbers) and find a new value for the resistance that gives a time constant of about 1ms.

Setup the above circuit on your Protoboard. Open the Alice desktop application. Configure CH A to produce a square wave that varies between 0V and 2V at a frequency of 100 Hz. Setup CH A in Split I/O mode so that Ain can be used to measure CH A voltage waveform. Setup CH B in Hi-Z, Split I/O mode and use Bin to measure voltage across the two capacitors. Set up the oscilloscope to display 50ms of data and use as much of the vertical range as possible. The time and volts per division are your choice but your goal is to see as much detail as possible. Run the experiment and save an image of the data.