Class #25: Experiment 22
Power in DC and AC Circuits

Purpose: In this experiment, we will investigate the characteristics of power in DC and AC circuits.

Background: Before doing this experiment, students should be able to
- Review online background materials.
- Build and operate simple circuits on a Protoboard.
- Measure the voltages and determine the currents using a math channel in simple Protoboard circuits using M1K board
- Analyze simple circuits consisting of combinations of resistors, especially voltage dividers.
- Do a transient (time dependent) simulation of circuits using LTspice
- Calculate DC power produced and consumed.
- Review the background for the previous experiments.

Learning Outcomes: Students will be able to
- Determine the power consumed by a resistor.
- Identify the maximum power consumed by a load in a voltage divider circuit.
- Determine the power consumed and produced in RL, RC and RLC circuits.

Resources Required:
- M1K board
- Alice tools
- Parts Kit (resistors, inductors and capacitors)

Helpful links for this experiment can be found on the course website under Class #25.
Part A – DC Power

We have spent a large amount of time measuring voltage and current in this course. Those parameters are important for characterizing how a circuit behaves. However, in terms of what it ‘costs’ to operate a circuit, we must look at the power. For any circuit component, we can define power using the relationship

\[ P = VI \]

This equation is valid for all components. For DC circuits (constant sources), it becomes a relatively simple matter of identifying the voltage across and current through the component to determine power. For AC circuits (sinusoidal sources), we will need to add some additional mathematics to calculate average power. From physics, we also know that any power produced must be equal to the power that is consumed. Typically, in circuits, sources produced power and passive components like resistors, capacitors and inductors consume (or store) power. If there are multiple sources and multiple components, this can be represented mathematically as

\[ P_{produced} = \sum_{n=1}^{\text{# of sources}} P_n = \sum_{i=1}^{\text{# of components}} P_i = P_{consumed} \]

(Note, a source is not always a power producer and can sometimes consume power. For example, consider a rechargeable battery while it is charging.)

We will analyze our familiar voltage divider circuit.

![Figure A-1: Voltage divider](image)

1. Set RL = 1kΩ, measure the voltage across Rs and RL.
2. Use Ohm’s Law to calculate the current through each resistor (they should be the same since they are in series).
3. Calculate the power consumed by each resistor.
4. Calculate the power produced by the source (which has the same current as the resistors). Does the power produced equal the power consumed?
5. Repeat your power calculations for R = 68Ω, 100Ω, 470Ω, 2.2kkΩ, 4.7kkΩ, 10kΩ.
6. Using Matlab, plot the power consumed by the load resistor (y-axis), \( P_{RL} \), against the load resistance (x-axis), RL.
7. A constant source in series with a constant source resistance, Rs, with a variable load, RL, is a classic circuit when trying to maximize power to a load. Based on your part 6 plot, what relationship exists between the load resistor and the source resistor such that power to the load is maximized?
In experiment 10, part C (Class11), we implemented the above circuit for several source voltages. For $V_1 = V_2 = 2V$, find the voltage across each resistor and the current through each resistor. Calculate the power consumed by the resistors in the circuit.

Using series relationships, determine the current through each source and the power produced by the sources.

Are the power produced and the power consumed equal?
Part B – AC power

The power expression is valid, $P = VI$, for AC signals, however, both the voltage and current vary with time.

$$V(t) = V_o \sin(\omega t + \theta_v) \rightarrow V_o \angle \theta_v$$

$$I(t) = I_o \sin(\omega t + \theta_i) \rightarrow I_o \angle \theta_i$$

where $V_0$ and $I_0$ are the amplitudes of the voltage and current sinusoids. The amplitude of a sinusoid can be computed by first computing the peak-to-peak amplitude and then dividing it by 2 (from last experiment). $\theta_v$ and $\theta_i$ are the phases of the voltage and current relative to some reference (perhaps the source).

Rather than consider a time varying power expression, we look at the RMS power, with a slightly different expression for power,

$$S = \frac{1}{2} (V_o \angle \theta_v)(I_o \angle \theta_i)^*$$

This expression includes the effects of a phase difference between voltage and current, which is a significant distinction when studying AC power. Resistors are always power consumers. In practical experience, they get hot, converting electrical energy to heat. On the other hand inductors and capacitors are power storage devices, which means they can store power during part of the AC cycle and provide power during part of the AC cycle. We separate these concepts into real power and reactive power. Real power is power used to do work, liking heating, driving a motor, etc. Reactive power does not do any work, but is part of the power system. In terms of the above power expression, we can use a simple array to represent the real and reactive components of power. The voltage and current can be used to determine the power as

$$S = \begin{bmatrix} P_{\text{real}} \quad Q_{\text{reactive}} \end{bmatrix} = \begin{bmatrix} V_o I_o \cos(\theta_v - \theta_i) & V_o I_o \sin(\theta_v - \theta_i) \end{bmatrix}$$

(There is some mathematics here that will be discussed in later courses.)

**Important note**, in the above expression the phase argument for the cos and sin terms are the phase difference between the voltage and current in a given component. This provides a ‘short cut’ to determining the power in a given component.

Figure B-1: RC Circuit

- Implement the above circuit on your protoboard with a 2V amplitude, 1.6kHz sinusoidal source, with 2V DC offset.
- Measure the voltage across the resistor and the source voltage. Determine the amplitude of resistor voltage and the phase of the resistor voltage (using the source voltage as a reference).
- Use the math channel determine the amplitude of the resistor current and phase of the resistor current (using the source voltage as a reference). Note, this current will be the same for all components since the circuit is a series circuit.
- Using the power expression for AC signals, calculate the real and reactive power consumed by the resistor. What do you notice about reactive power in a resistor?
- Measure the voltage across the capacitor and determine the amplitude and the phase.
- Using the power expression for AC signals, calculate the real and reactive power consumed by the capacitor. What do you notice about real power in a capacitor?
- Calculate the power produced by the source. Does the real power produced equal the real power consumed.? Does the reactive power produced equal the reactive power consumed?
Implement the above circuit on your protoboard with a 2V amplitude, 8kHz sinusoidal source, with 2V DC offset. You can make a 20mH inductor by connecting two 10mH inductors in series.

Measure the voltage across the resistor and the source voltage. Determine the amplitude of the resistor voltage and the phase of the resistor voltage (using the source voltage as a reference).

Use the math channel to determine the amplitude of the resistor current and phase of the resistor current (using the source voltage as a reference). Note, this current will be the same for all components since the circuit is a series circuit.

Using the power expression for AC signals, calculate the real and reactive power consumed by the resistor. Again, what do you notice about reactive power in a resistor?

Measure the voltage across the inductor and determine the amplitude and the phase. Use the math channel to determine the amplitude and phase of the inductor voltage.

Using the power expression for AC signals, calculate the real and reactive power consumed by the inductor. What do you notice about real power in a inductor?

Calculate the power produced by the source. Does the real power produced equal the real power consumed.? Does the reactive power produced equal the reactive power consumed?

Replace R1 in Figure B-2 with a 10Ω resistor and change the frequency to 80Hz. Repeat the above calculations. Based on past experience, what is causing the resistor to consume real power?

Implement the above circuit on your protoboard with a 2V amplitude, 3.6kHz sinusoidal source, with 2V DC offset.

Using the same process as in the previous circuits, calculate the power consumed by the resistor, inductor and capacitor and the power produced by the source.

Change the source to a 700Hz signal and repeat the above calculations.

Change the source to an 18kHz signal and repeat the above calculations.

With those three data points, what do you notice about power produced in context with the resonant frequency, \( \omega_0 \). (reference early experiments to review resonant frequency).