Maximum Power Transfer Theorem
Calculus-Based Proof

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Intro to ECSE
Power Delivered to Load Resistor

\[ I_S = \frac{V_S}{R_s + R_L} \]

\[ P_L = V_L \cdot I_L = I_L^2 R_L = I_S^2 R_L \]

\[ = \left( \frac{V_S}{R_s + R_L} \right)^2 R_L = \frac{V_S^2}{\frac{R_s^2}{R_L} + R_L + 2R_s} \]
Maximize Power Delivered to Load Resistor

\[ P_L = \frac{V_S^2}{\frac{R_s^2}{R_L} + R_L + 2R_s} \]

Maximize \[ \sum P_L \] subject to \[ \min \left\{ \frac{R_s^2}{R_L} + R_L + 2R_s \right\} \]

Set \[ \frac{dDY}{dR_L} = 0 \]

\[ \Rightarrow \quad \frac{-R_s^2}{R_L^2} + 1 = 0 \quad \Rightarrow \quad R_L = R_s \quad \Rightarrow \quad R_L = \pm R_s \]

\[ P_{L,\text{max}} = \frac{V_S^2}{\frac{R_s^2}{R_L} + R_L + 2R_s} = \frac{V_S^2}{R_s + R_s + 2R_s} \]

\[ = \left( \frac{V_S}{2} \right)^2 \times \frac{1}{R_s} \]
%% Initialize circuit parameters
Vs = 9; % Constant Source Voltage
Rs = 35; % Constant Source Resistance
RL = 1:100; % Variable Load Resistor

%% Determine Current through each resistor (all same)
I = Vs./(Rs+RL); % Application of Ohm's Law

%% Determine power absorbed by load resistor
PL = (I.^2).*(RL); % P = VI = I^2R = V^2/R

%% Plot power absorbed by resistor as a function of load resistance
plot(RL,PL,'LineWidth',3,'color','k');
hold on;
xlabel('Load Resistance, RL, [ohms]');
ylabel('Power absorbed by load resistor, PL, [watts]');
title('Maximum Power Transfer Theorem');
grid on;