

ENGR-4300

Fall 2008

Test 1

Name **SOLUTION**

Section: 1(MR 8:00) 2(TF 2:00)
(circle one)

Question I (20 points) _____

Question II (20 points) _____

Question III (20 points) _____

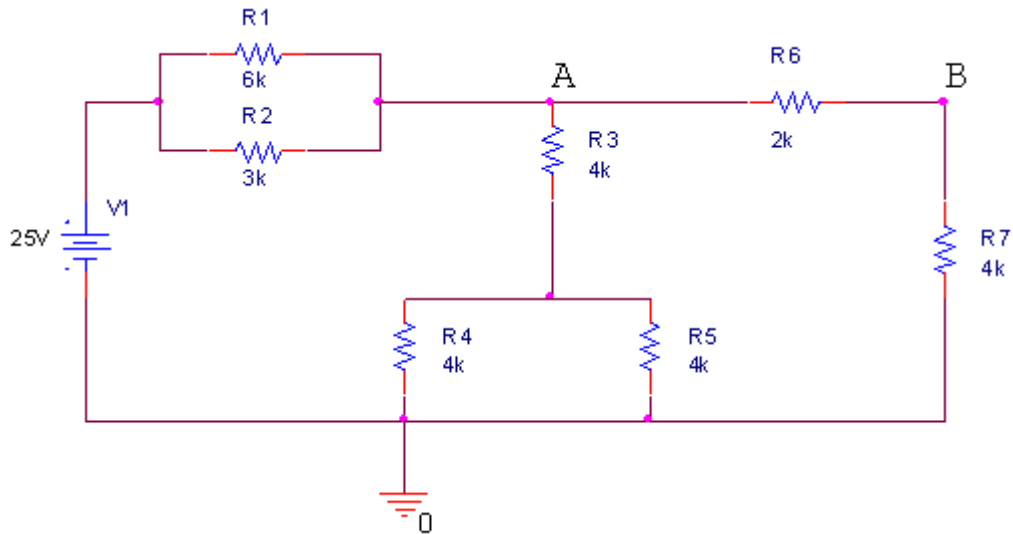
Question IV (20 points) _____

Question V (20 points) _____

Total (100 points) _____

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I – Circuit Analysis (20 points)



1) Find the equation for total resistance. (6 pts)

$$R_1 := 6k\Omega \quad R_2 := 3k\Omega \quad R_3 := 4k\Omega \quad R_4 := 4k\Omega$$

$$R_5 := 4k\Omega \quad R_6 := 2k\Omega \quad R_7 := 4k\Omega$$

$$R_T := \frac{(R_7 + R_6) \cdot \left(R_3 + \frac{R_4 \cdot R_5}{R_4 + R_5} \right) + \frac{R_1 \cdot R_2}{R_1 + R_2}}{R_7 + R_6 + \left(R_3 + \frac{R_4 \cdot R_5}{R_4 + R_5} \right)}$$

2) Find the total resistance and total current. (4 pts)

$$V_1 := 25V$$

$$R_T = 5k\Omega \quad I_T := \frac{V_1}{R_T} \quad I_T = 5mA$$

3) Find voltage at points A and B using the voltage divider equation. (5 pts)

$$R_{12} := \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$R_{34567} := \frac{(R_7 + R_6) \cdot \left(R_3 + \frac{R_4 \cdot R_5}{R_4 + R_5} \right)}{R_7 + R_6 + \left(R_3 + \frac{R_4 \cdot R_5}{R_4 + R_5} \right)}$$

$$V_A := \frac{V_1 \cdot R_{34567}}{R_{12} + R_{34567}} \quad V_A = 15V$$

$$V_B := \frac{V_A \cdot R_7}{R_6 + R_7} \quad V_B = 10V$$

4) Find the current through R4 *and* R6. (5 pts)

$$R_{345} := R_3 + \frac{R_4 \cdot R_5}{R_4 + R_5} \quad \text{or you know } I_c = \text{half of } I_{\text{total}}$$

$$I_{345} := \frac{V_A}{R_{345}}$$

$$I_{345} = 2.5 \text{ mA}$$

$$15V - 10V = 5V$$

Voltage drop across R3

$$2.5 \text{ mA} \cdot 4 \text{ k}\Omega = 10V$$

$$\frac{5V}{4 \text{ k}\Omega} = 1.25 \text{ mA}$$

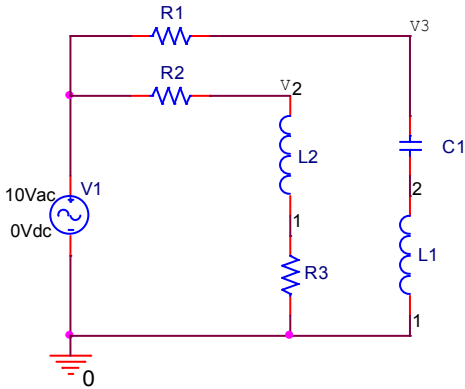
or the current through R4 is half the current of I345

$$I_4 := 1.25 \text{ mA}$$

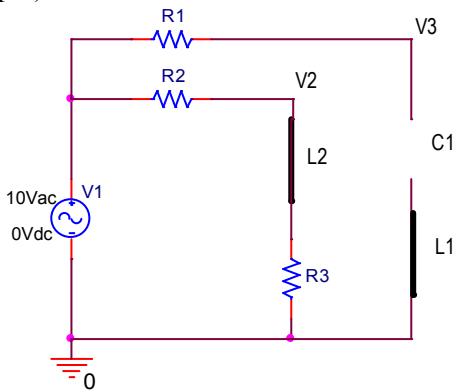
$$I_6 := \frac{V_A}{R_5 + R_6} \quad I_6 = 2.5 \text{ mA}$$

Question II – Filters (20 points)

You are given the following circuit. The input at V1 is a 10V AC signal. Leave all answers for 1) – 5) in terms of R1, R2, R3, C1, L1, L2 and the value of V1. V2 and V3 are measured with respect to ground.

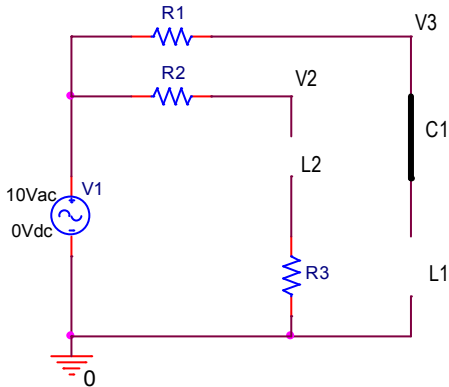


1) Redraw the circuit for $\omega \rightarrow 0$ with appropriate simplifications for the impedances and find V2 and V3. (5 pts)



$V2 = 10R3/(R2+R3)$ $V3 = 10V$

2) Redraw the circuit for $\omega \rightarrow \infty$ (high frequencies) with appropriate simplifications for the impedances and find V2 and V3. (5 pts)



$V2 = 10V$ $V3 = 10V$

3) At what value of ω (greater than 0 and less than ∞) would you expect to find V3 minimized? (3 pts)

At $\omega = \frac{1}{\sqrt{L1C1}}$ the voltage at V3 = 0 when $Z_{L1C1} = 0$.

4) What special name is given to the ω value in 3)? (2 pts)

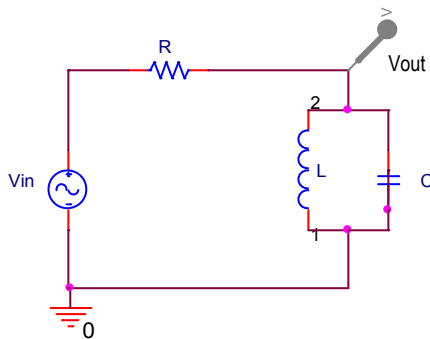
The resonant frequency of the series LC circuit.

5) If Vac is set to 0V and Vdc is set to 5V on V1, what are the values of V2 and V3? (3 pts)

$$V2 = 5R3/(R2+R3) \quad V3 = 5V$$

6) What type of filter is the circuit at V3, Low Pass, High Pass, Band Pass, or Band Reject? (2 pts)

Question III – Transfer Functions (20 points)



1) What is the transfer function $H(j\omega)$ for the circuit above in terms of R , L , and C ? You must reduce it to a simple ratio of polynomials in ω . (6 pts)

$$H(j\omega) = \frac{\frac{1}{j\omega C} j\omega L}{R + \frac{1}{j\omega C} j\omega L} = \frac{\frac{L/C}{R(\frac{1}{j\omega C} + j\omega L) + L/C}}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega LC}{RC(1 - \omega^2 LC) + j\omega LC} = \frac{j\omega LC}{RC - \omega^2 RLC^2 + j\omega LC}$$

2) Substitute the values $R = 2$, $C = 4$, and $L = 1$ into $H(j\omega)$ in 1) and simplify. (4 pts)

$$H(j\omega) = \frac{j\omega(1)(4)}{(2)(4) - \omega^2(2)(1)(4)^2 + j\omega(1)(4)} = \frac{j\omega 4}{8 - \omega^2 32 + j\omega 4} = \frac{j\omega}{2 - \omega^2 8 + j\omega}$$

3) Find the simplified transfer function for low frequencies and high frequencies (4 pts)

$$\text{Low: } H(j\omega) = \frac{j\omega}{2}$$

$$\text{High: } H(j\omega) = \frac{j\omega}{-\omega^2 8} = \frac{-j}{8\omega}$$

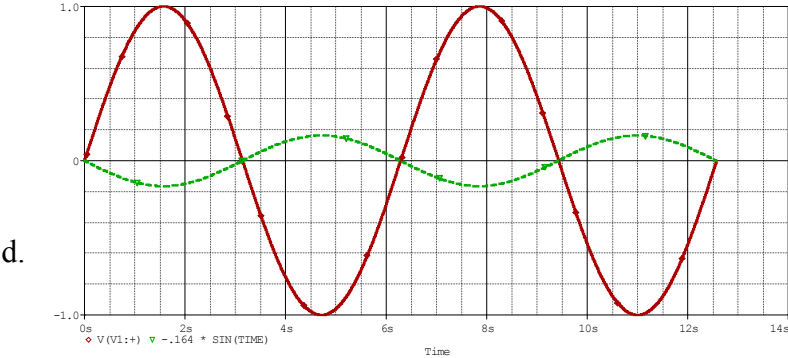
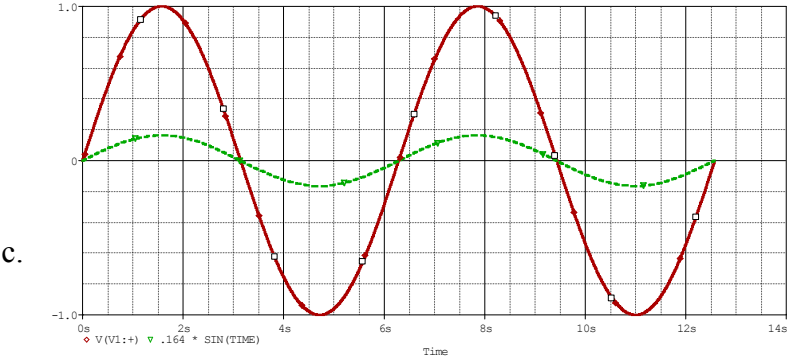
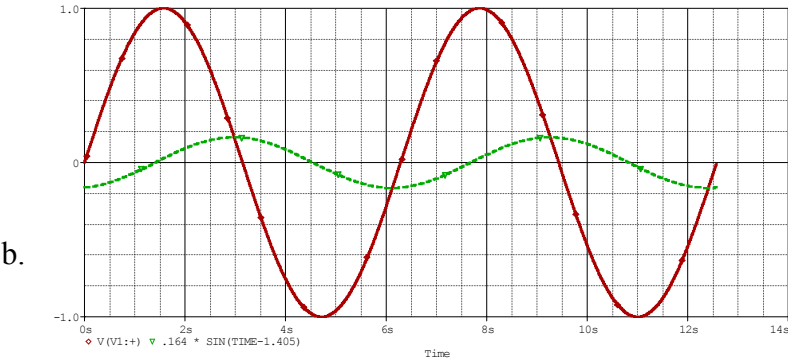
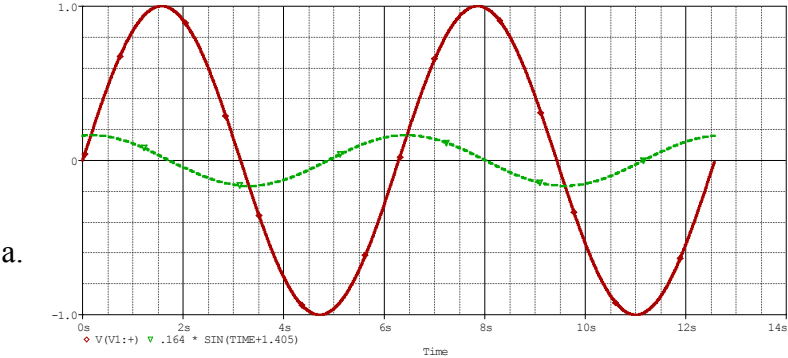
4) For $\omega = 1$ radian/s, determine the magnitude of $H(j\omega)$. (2 pts)

$$|H(j1)| = \left| \frac{j1}{2 - 1^2 8 + j1} \right| = \frac{|j1|}{|-6 + j1|} = \frac{1}{\sqrt{36+1}} = \frac{1}{\sqrt{37}} = 0.164$$

5) For $\omega = 1$ radian/s, determine the phase of $H(j\omega)$. (2 pts)

$$\angle H(j1) = \angle \frac{j1}{2 - 1^2 8 + j1} = \frac{\angle j1}{\angle(-6 + j1)} = \frac{90^\circ}{\arctan(\frac{1}{-6})} = \frac{90^\circ}{170.5^\circ} = 90^\circ - 170.5^\circ = -80.5^\circ$$

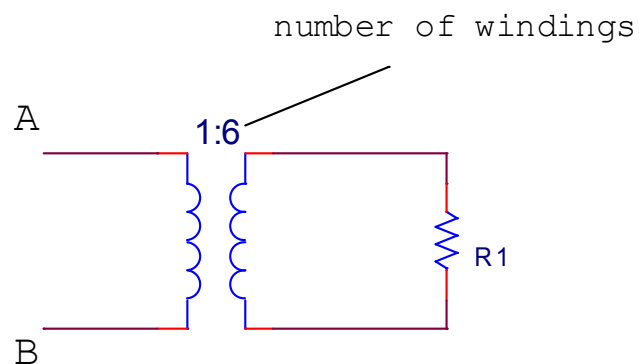
6) Which plot correctly displays the relationship between V_{in} (solid trace) and V_{out} (dashed trace) at $\omega = 1$ radian/s? (2 pts)



e. None of these.

ANSWER IS b), output delayed 80.5°

Question IV: Transformers and Inductors (20 points)



1) In the circuit above, the transformer is ideal. If $R_1 = 6\text{k}\Omega$, find the equivalent impedance, Z_{AB} , seen from points A and B. (5 pts)

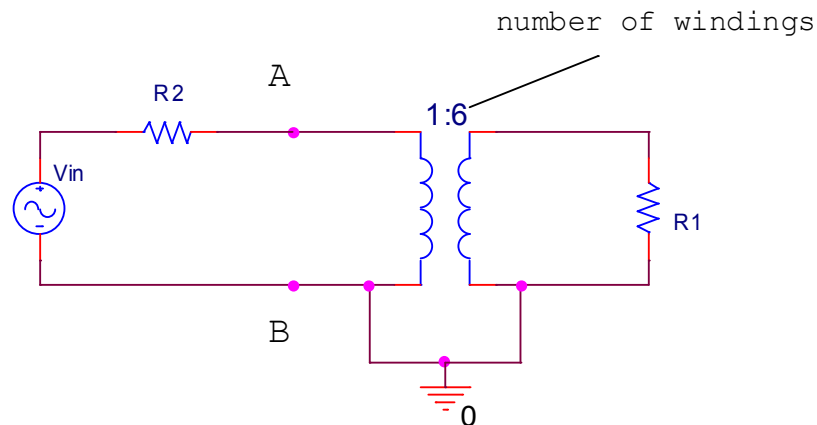
$$N_2 := 6 \quad N_1 := 1$$

$$a := \frac{N_2}{N_1}$$

$$Z_{AB} := \frac{R_1}{a^2}$$

$$Z_{AB} = 166.667\Omega$$

2) We have connected the above circuit to an AC source with a resistor R2.



If the input voltage has an amplitude of 10V, and the voltage at point A is 526mV, what is the value of R2? (8 pts)

$$.526V = \frac{Z_{AB}}{R_2 + Z_{AB}} \cdot 10V$$

$$.526V \cdot (R_2 + Z_{AB}) = Z_{AB} \cdot 10V$$

$$\frac{Z_{AB} \cdot 10V - Z_{AB} \cdot .526V}{.526V} = 3.002 \times 10^3 \Omega$$

3) What is the value of the voltage across R1? (7 pts)

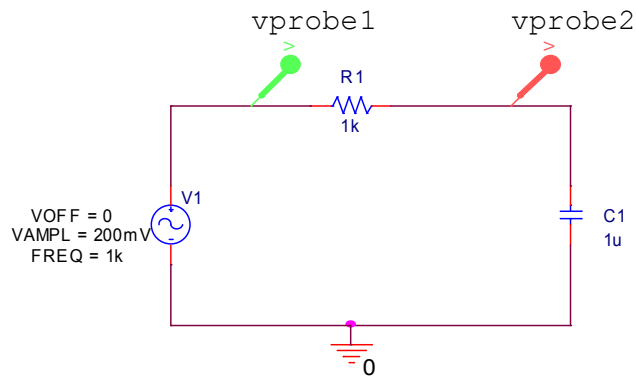
$$V_a = 0.526V$$

$$V_2 := V_a \cdot a \quad V_s := V_a$$

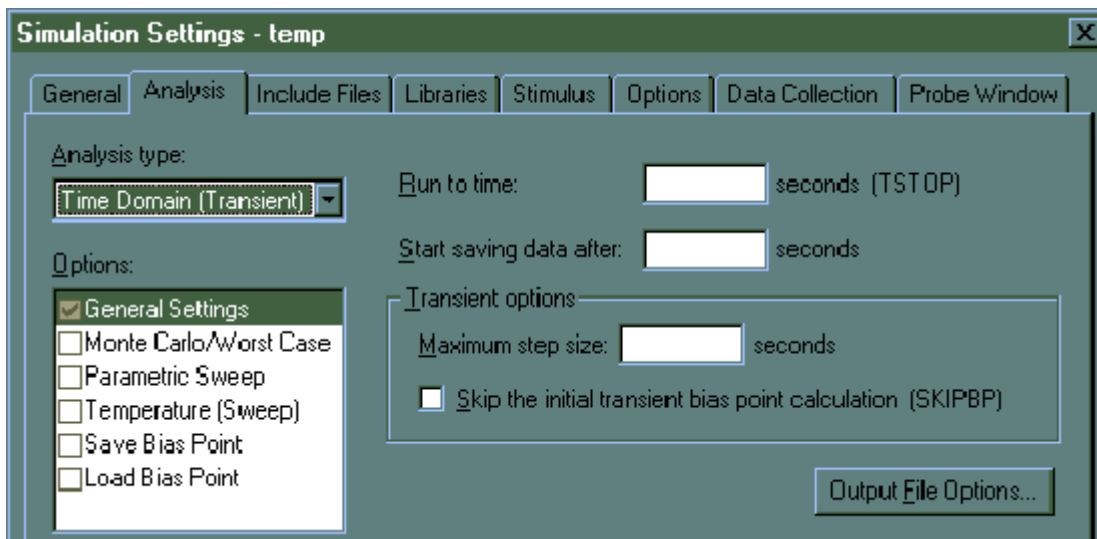
$$V_2 = 3.158V$$

Question V: PSpice (20 points)

The following circuit is setup in PSpice



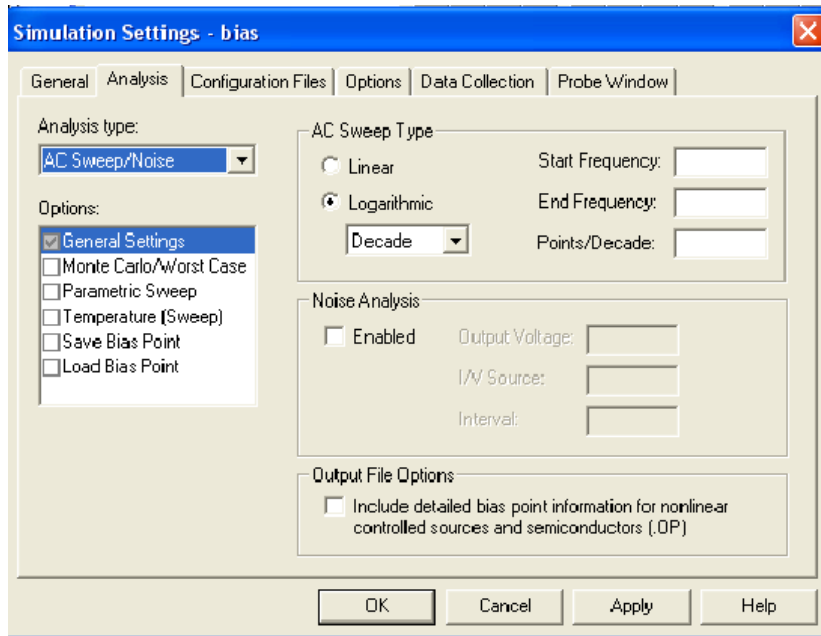
1) Setup a transient analysis in the simulation settings window below that will show 4 cycles of the signal, (the "start saving data after:" box can be neglected) (3 pts)



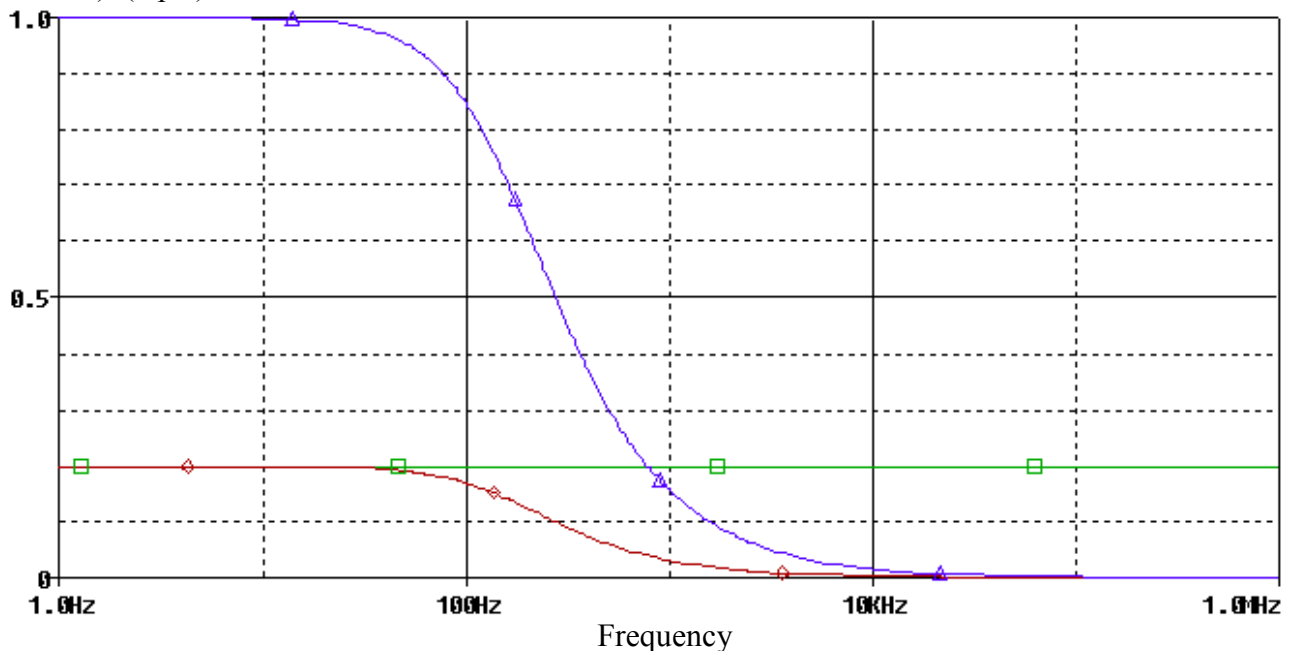
1kHz => 1ms for 1 complete cycle. 4ms => 4 cycles.
Max step size is 4us or something close to this.

2) What steps should be included (to avoid errors) in creating an AC Sweep simulation over a range to 1 Megahertz for the circuit above? (circle all that apply) (6 pts)

- a. Place ground in the circuit
- b. Choose AC Sweep/Noise from the drop down box
- c. Set parameter for VSIN called AC (double click AC source)
- d. Set start frequency to 0 Hz
- e. Set start frequency to 1 Hz
- f. Set end frequency to 1MHz
- g. Set end frequency to 1Meg
- h. Set Points/Decade to 0
- i. Set Points/Decade to 100



3) On the simulation output, label on the plot vprobe1 (Vin), vprobe2 (Vout), and the transfer function (Vout/Vin). (3 pts)



4) What kind of filter is this? (3 pts)

Low pass filter

5) How do you determine the corner frequency from the simulation output? How do you calculate the corner frequency from the circuit component values? *Compare the two results.* (5 pts)

The corner frequency of the circuit is the frequency at which the transfer function is 0.707. Use the cursors to mark this location on your plot.

o Click on the cursor button on the PSpice toolbar.

o Click on the colored symbol for the trace you added (H) at the bottom of your plot. This places the cursor on this trace.

o Drag the cursor along the trace until the y coordinate is as close as possible to 0.707

$$f = \sim 160\text{Hz} \quad \omega = \sim 1000\text{rad/s}$$

In class you were given an expression for the corner frequency of an RC circuit: $\omega_c = 1/RC$.

Calculate the corner frequency using the equation you derived. Don't forget that f (in hertz) is $\omega/2\pi$.

$$\omega = 1/(RC) = 1/(1\text{k} \times 1\mu) = 1000\text{rad/s}$$