ENGR-4300
Fall 2008
Test 2

Name: _______SOLUTION__________

Section: 1(MR 8:00) 2(TF 2:00)
(circle one)

Question I (20 points): ___________

Question II (20 points): ___________

Question III (20 points): ___________

Question IV (20 points): ___________

Question V (20 points): ___________

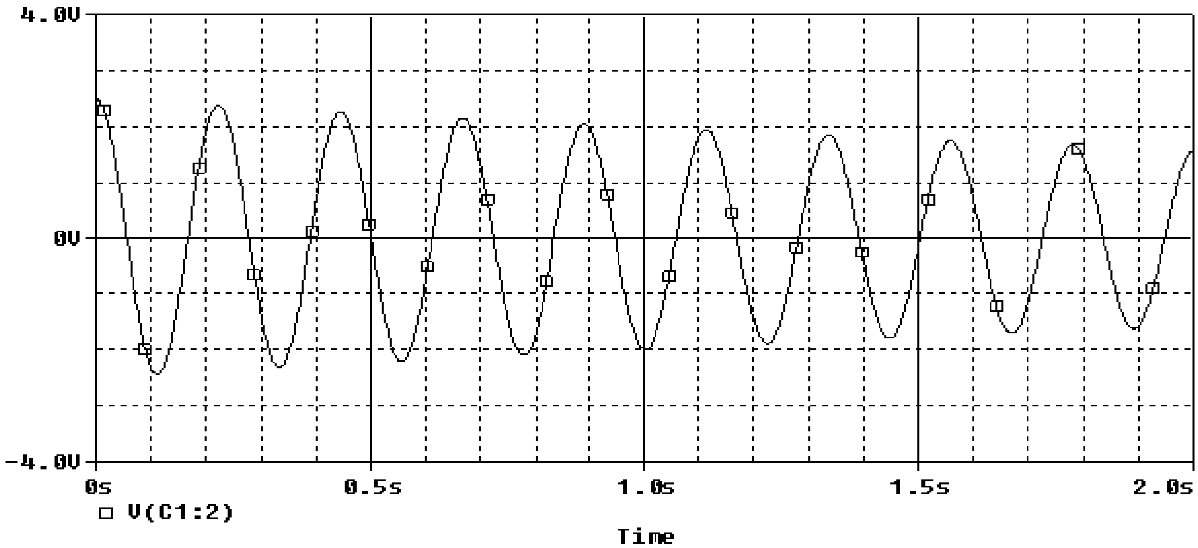
Total (100 points): ______________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

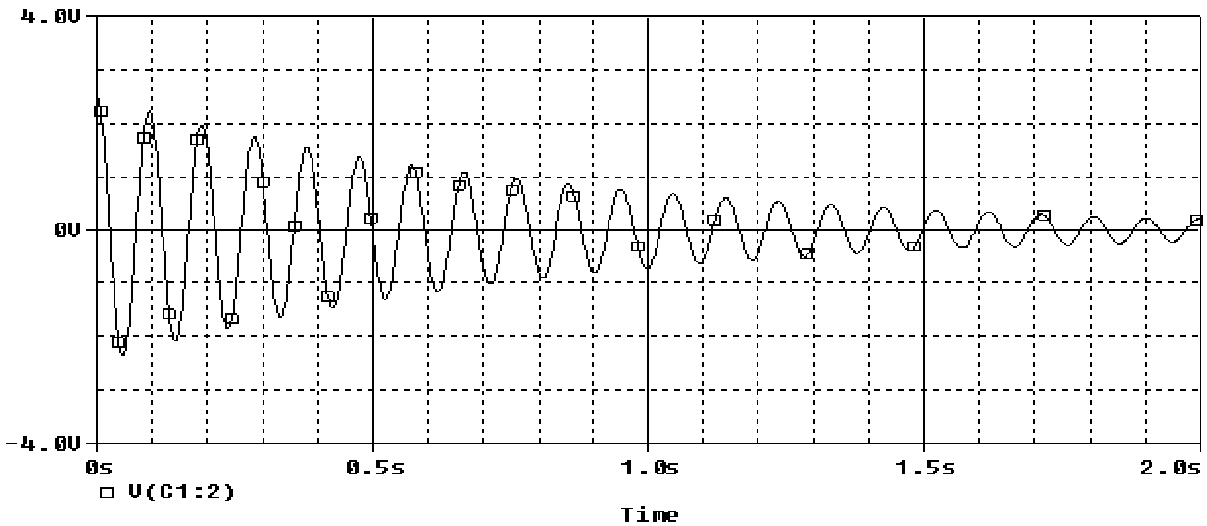
Question I – Bridges and Damped Sinusoids (20 points)

You are given a cantilever beam similar to the one you used in experiment 4. You place two weights on the end of the beam one at a time (0.1 kg and 0.7 kg) and you get the following two plots (may not be in order of mass listed).

Plot 1



Plot 2



1) (2pt) What is the frequency of plot 1? (Use at least 2 significant figures)

2 cyles in .5-.05 ms $\quad time_1 := .5s - .05s \quad cylcles_1 := 2$

$$f_1 := \frac{cylcles_1}{time_1} \qquad f_1 = 4.444\,\text{Hz} \qquad f_1 = 4.4Hz$$

Question I – Bridges and Damped Sinusoids (continued)

2) (2pt) What is the frequency of plot 2? (Use at least 3 significant figures)

5 cylces in .5-.02ms $t_2 := .5s - .02s$ $cycles_2 := 5$

$$f_2 := \frac{cycles_2}{t_2} \qquad f_2 = 10.417\,Hz \qquad f_2 = 10.4Hz$$

3) (6pt) What is the damping constant for plot 1, mark the points on the plot? (Use at least 3 significant figures)

$$(t_0, v_0) = (.22s, 2.4V) \qquad (t_1, v_1) = (1.78s, 1.7V)$$

$t_0 := .22s$

$v_0 := 2.4V$

$t_1 := 1.78s$

$v_1 := 1.7V$

$$v_1 = v_0 \cdot e^{-\alpha(t_1 - t_0)}$$

$$-\frac{\ln\left(\frac{v_1}{v_0}\right)}{(t_1 - t_0)} = 0.221\frac{1}{s} \qquad \alpha = .22\frac{1}{s}$$

4) (6pt) Given the following formula, $k = (m + m_n)\cdot(2\cdot\pi\cdot f_n)^2$, and assuming that the two data points that you found are ideal, find values for k and m.

$$k = (m + 0.1)\cdot[2\cdot\pi\cdot(10.4)]^2 \qquad k = (m + 0.7)\cdot[2\cdot\pi\cdot(4.44)]^2$$

$$k = 4269\cdot m + 426.9 \qquad \text{if rounded up} \qquad k = 779.67m + 544.78$$

$$4270m + 427 = 778.67m + 544.8$$

$$4270 - 778 = 3.492 \times 10^3$$

$$544.8 - 427 = 117.8$$

$$m = \frac{117.8}{3492} = 0.034\ kg$$

$$k := (0.034kg + 0.1kg)\cdot(2\cdot\pi\cdot10.4Hz)^2 \qquad k = 572.178\frac{kg}{s^2}$$

Question I – Bridges and Damped Sinusoids (continued)

5) (2pt) What is the mass of the beam?

$$(0.034) = 0.23\cdot(m_b)$$

$$m_b := \frac{.034}{0.23}kg$$

$$m_b = 0.148\text{kg}$$

6) (2pt) Using the chart for Young's Modulus, determine the probable material that the beam is made out of given that the dimensions of the beam are: width = 1.5 cm, length=15 cm, and thickness = 2 mm.

TABLE 9.1

Young's Modulus Table of Values

Metal	Elastic modulus (N/m2)	Metal	Elastic modulus (N/m2)
aluminum, 99.3%, rolled	6.96×10^{10}	lead, rolled	1.57×10^{10}
brass	9.02×10^{10}	platinum, pure, drawn	16.7×10^{10}
copper, wire, hard drawn	11.6×10^{10}	silver, hard drawn	7.75×10^{10}
gold, pure, hard drawn	7.85×10^{10}	steel, 0.38% C, annealed	20.0×10^{10}
iron, wrought	19.3×10^{10}	tungsten, drawn	35.5×10^{10}

$$k = 572.178\frac{\text{kg}}{\text{s}^2}$$

$$w := 1.5cm$$

$$l := 15cm$$

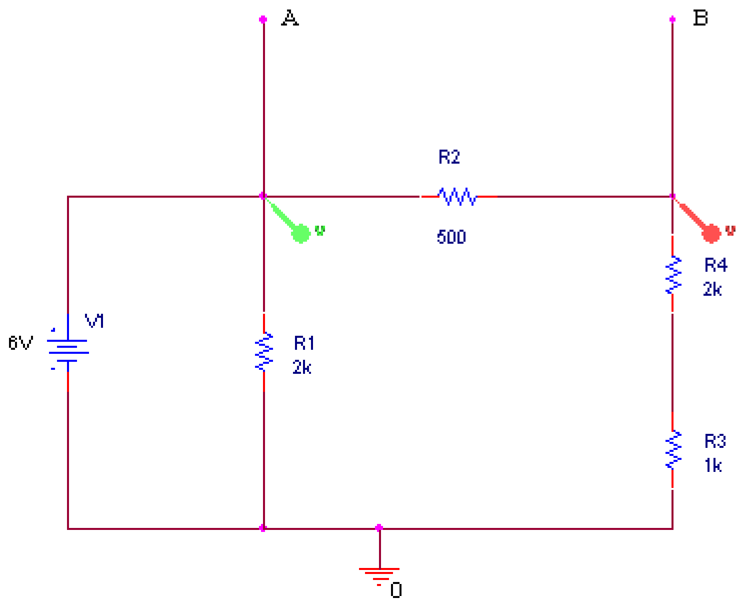
$$t := 2mm$$

$$E := \frac{k\cdot 4\cdot l^3}{w\cdot t^3}$$

$$E = 6.437\times 10^{10}\frac{N}{m^2}$$

aluminum

Question II – Thevenin Equivalents (20 points)



1) (7pt) Find the Thevenin equivalent voltage with respect to A and B for the circuit shown above)
Hint: Vth=V_A-V_B *so find* V_A *then* V_B

$V_1 := 6V \quad R_2 := 500\Omega \quad R_4 := 2k\Omega$

Note: forgot to put R5 at probe B negate R5

$R_1 := 2k\Omega \quad R_3 := 1k\Omega \quad R_5 := 3k\Omega$

$V_A := V_1$

$$V_B := V_1 \cdot \frac{R_4 + R_3}{R_2 + R_4 + R_3}$$

$V_B = 5.143\ V$

$V_{th} := V_A - V_B$

$V_{th} = 0.857\ V$

Question II – Thevenin Equivalents (continued)

2) (6pt) Find the Thevenin equivalent resistance with respect to A and B for the circuit shown above.

Short across a resistor means R1 is negligible

R3 and R4 are is series

R34 is in parallel with R2

R234 and R5 are in series

$$R_{34} := R_3 + R_4$$

$$R_{34} = 3 \times 10^3\ \Omega$$

$$R_{234} := \frac{R_{34} \cdot R_2}{R_{34} + R_2}$$

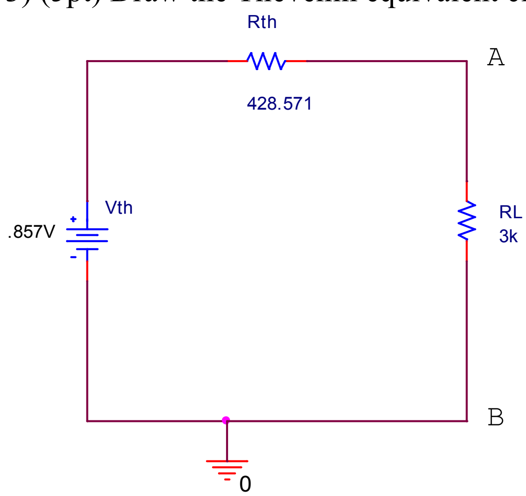
$$R_{234} = 428.571\ \Omega$$

$$R_{2345} := R_{234}$$

$$R_{234} = 428.571\ \Omega$$

$$R_{th} := R_{234}$$

3) (5pt) Draw the Thevenin equivalent circuit with a load resistor RL of 3K between points A and B



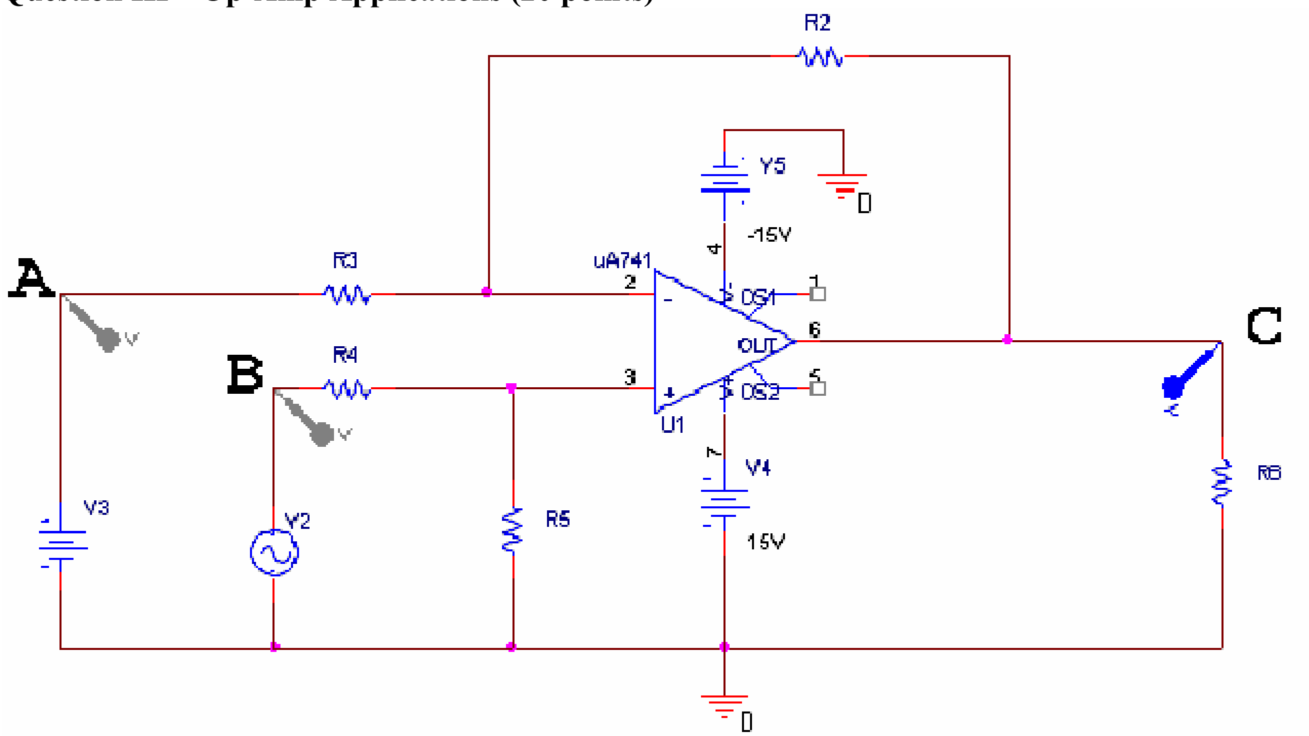
4) (2pt) What is the voltage across R_L? *Hint: Also the voltage at point V_A*

$$R_L := 3k\Omega$$

$$V_{RL} := V_{th} \cdot \frac{R_L}{R_{th} + R_L}$$

$$V_{RL} = 0.75\text{V}$$

Question III – Op-Amp Applications (20 points)



Assume the following components in the above circuit:

V_2 : Voff=2V, Vamp=2V, Freq=1k

$V3$:Vdc=2V

$R2 := 16k\Omega$ $R3 := 2k\Omega$ $R4 := 2k\Omega$

$R5 := 16k\Omega$ $R6 := 1k\Omega$

1) (1pt) The circuit above is an amplifier you've seen. What type of amplifier is it?

difference amplifier or differentia

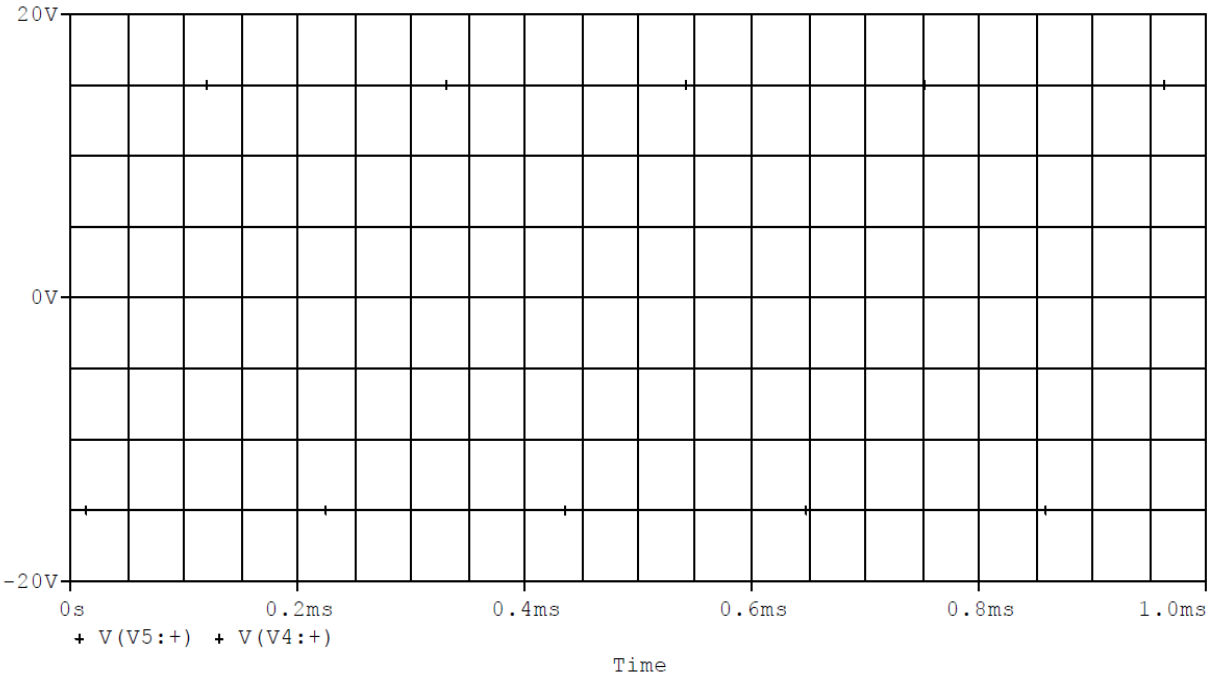
2) (3pt) Write an equation for the output C (Vc) in terms of the input voltages V2 and V3. Simplify (Do not have to enter voltage values)

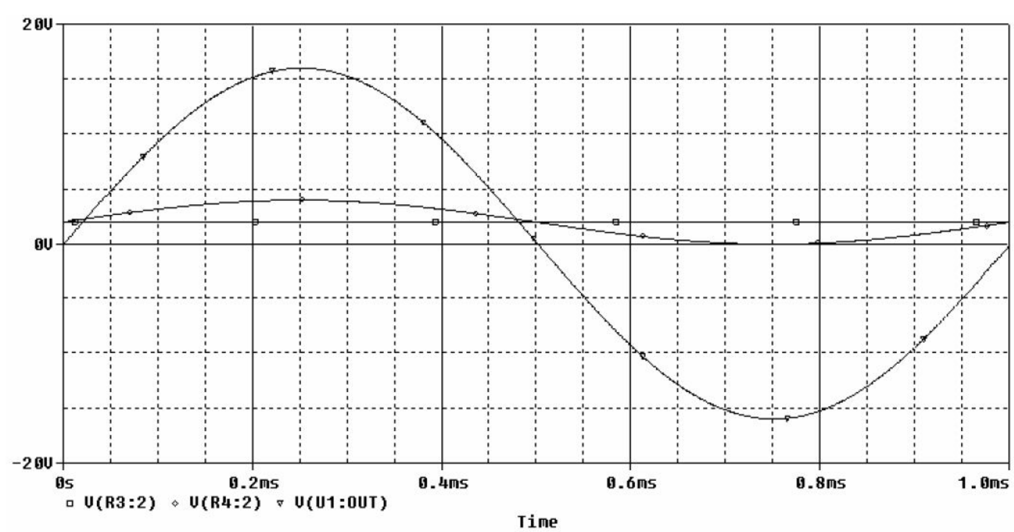
$$V_{out} = \frac{R_f}{R_{in}} \cdot (V_2 - V_3)$$

$$V_{out} = 8 \cdot (V_2 - V_3)$$

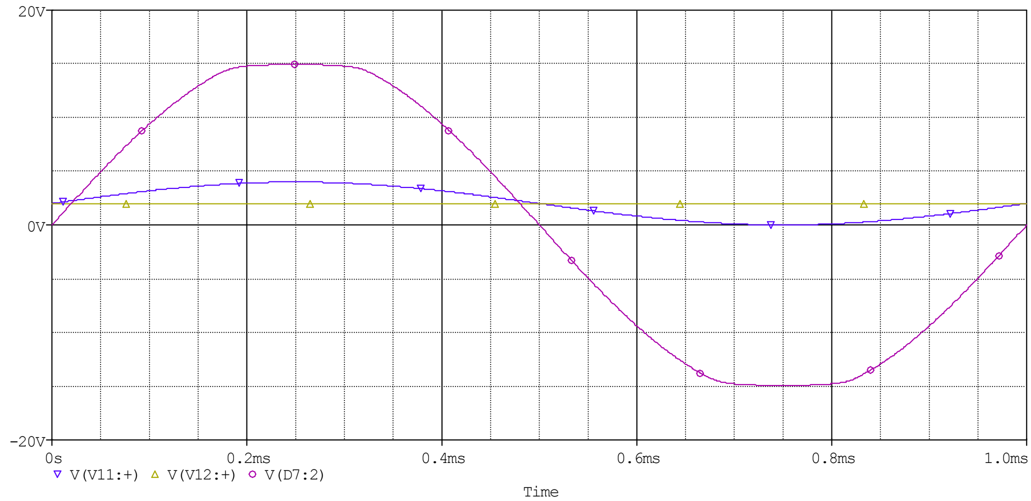
Question III – Op-Amp Applications (continued)

3) (16pt) Sketch and label one cycle of the input at V2 (point B), the input at V3 (point A) and the output at C (Vc) on the plot below



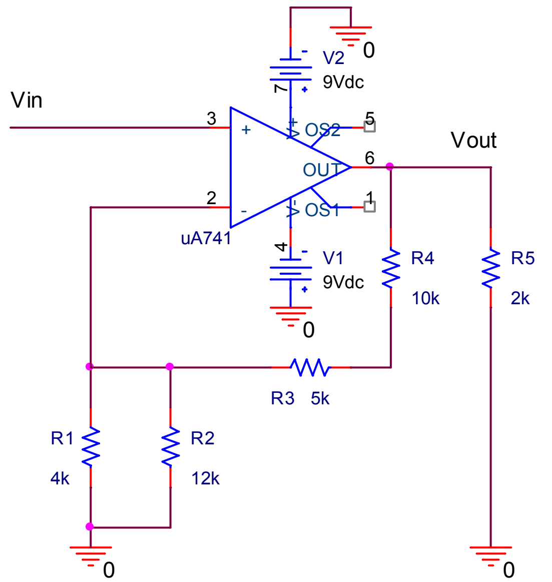


or



(extra credit +2 if plotted second graph with ±15V supplies clipping output)

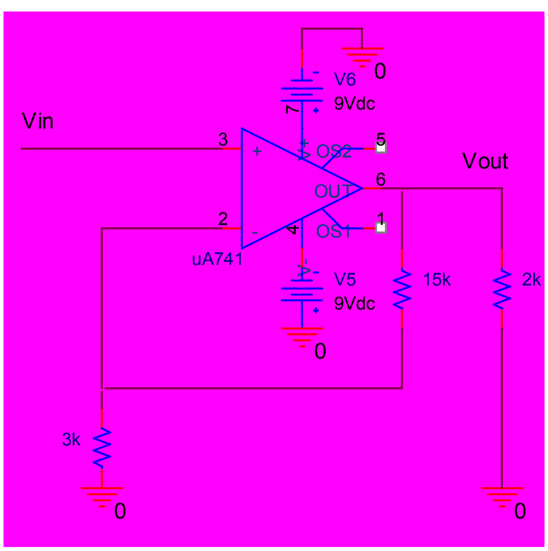
Question IV – Op-Amp Analysis (20 points)



1) (2pt) What op-amp circuit given on your crib sheet does this circuit most closely represent?
a. Inverting Amplifier b. Non-inverting Amplifier c. Adder d. Differential Amplifier
e. Practical Active Differentiator

Non-inverting Amplifier

2) (2pt) Redraw the circuit combining and resistors that are in parallel or series and find the combined values.



15k =10k + 5k 3k = (4k x 12k)/(4k + 12k)

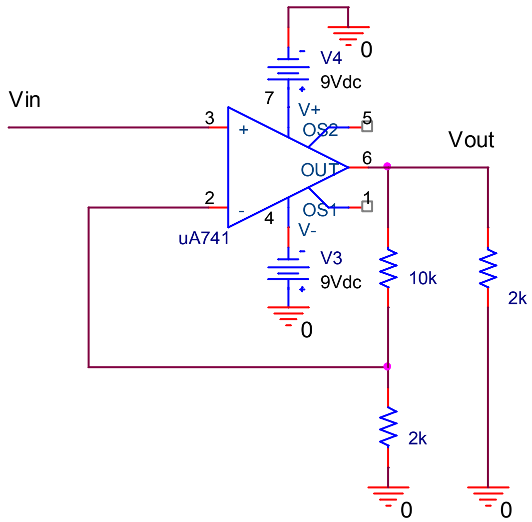
3) (2pt) What are the two golden rules of op-amp analysis?

1. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero (the + and – terminals will have the same voltage).

2. The inputs (+ and – terminals) draw no current.

Question IV – Op-Amp Analysis (continued)

4) (2pt) Using a different circuit below, if Vin = 1V on the '+' input of the op-amp, what is the voltage on the '–' input?



$V^- = V^+ = Vin = 1V$ (Golden Rule)

5) (3pt) From 4), how much current is flowing through the 2k resistor to ground?

$I_{2k} = V^-/R = 1V/2k = 0.5mA$

6) (3pt) By the Golden Rules, how much current in 4) is flowing through the 10k resistor from Vout to V^- (the connection point between the 2 resistors)?

By the Golden Rule, all the current in the 2k must come from the 10k; $I_{10k} = I_{2k} = 0.5mA$

7) (3pt) What is Vout for Vin = 1V?

$$Vout = V^- + I_{10k} \times R_{10k} = 1 + 0.5m \times 10k = 6V$$

or

$$Vout = \left(1 + \frac{10k}{2k}\right)Vin = (1+5)1 = 6V$$

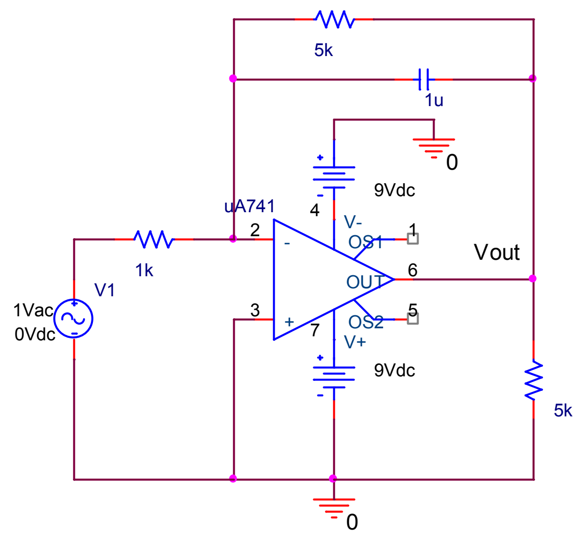
8) (2pt) What is the gain of this op-amp circuit?

$$Gain = \left(1 + \frac{10k}{2k}\right) = (1+5) = 6$$

9) (1pt) For an ideal op-amp in 4), what is the maximum value the input voltage Vin can have before the output will not be able to exhibit the full amplification from 8)?

Max output = 9V (±9V batteries) Vin = 9/Gain = 9/6 = 3/2V = 1.5V

Question V – Op-Amp Integrators and Differentiators (20 points)



1) (2pt) What function is this circuit designed to perform?

Practical Miller Integration

2) (4pt) Write the transfer function Vout/V1 for this circuit. (Substitute in the values provided for the components.)

$$\frac{V_{out}}{V_1} = -\frac{R_f}{R_{in}(1 + j\omega R_f C_f)} = -\frac{5k}{1k(1 + j\omega 5k \cdot 1\mu)} = -\frac{5}{1 + j\omega(0.005)} = -\frac{1000}{200 + j\omega}$$

3) (3pt) For which frequencies will the circuit perform the desired function?

a. Low frequencies below ω_c b. Only a mid band of frequencies around ω_c

c. High frequencies above ω_c

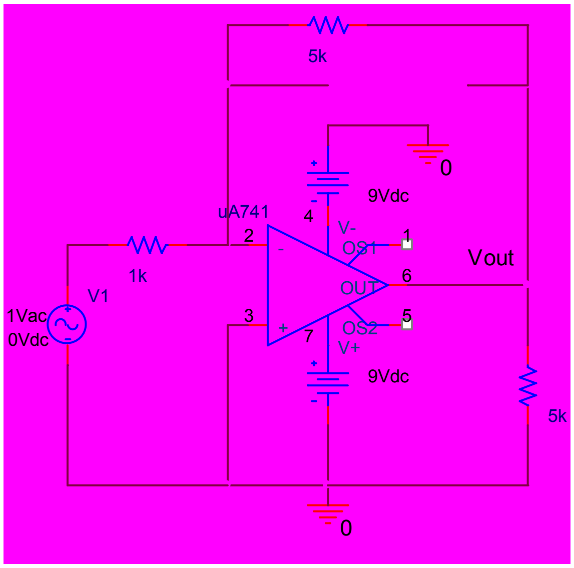
c. High frequencies above ω_c

4) (3pt) Find the corner frequency for the circuit in Hz.

$f_c = 1/(2\pi R_f C_f) = 1/(2\pi \text{ x } 5k \text{ x } 1\mu) = 31.83\text{Hz}$

Question V – Op-Amp Integrators and Differentiators (continued)

5) (2pt) Redraw the circuit with an appropriate substitution for the capacitor as f → 0 for when the input V1 has a very low frequency.



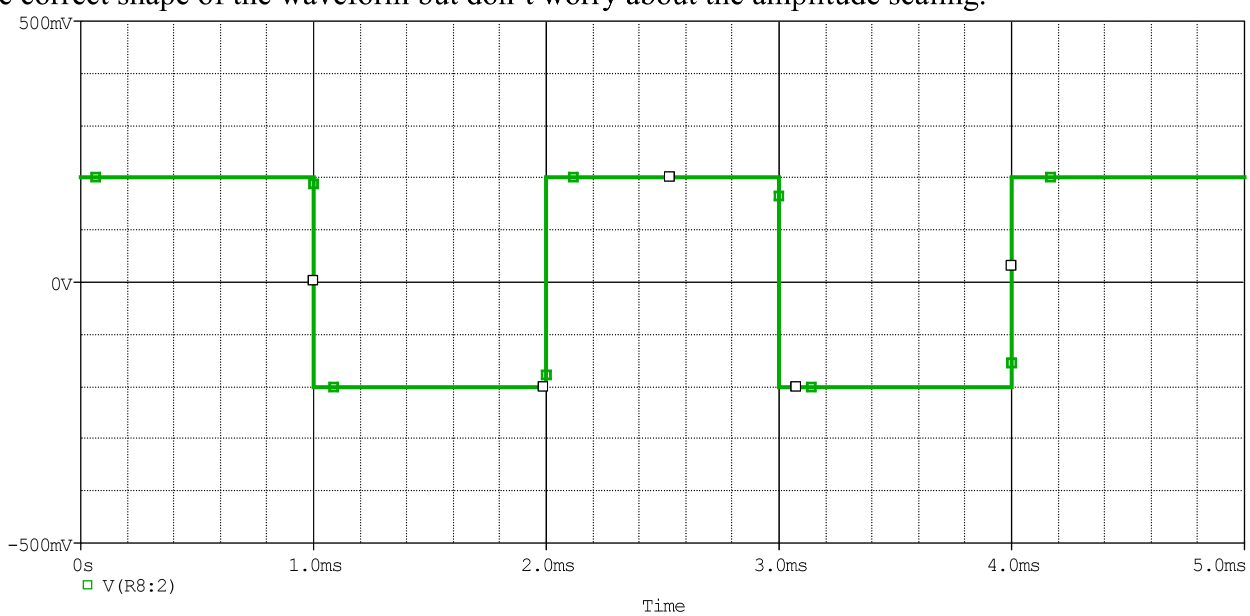
Replace C with open circuit

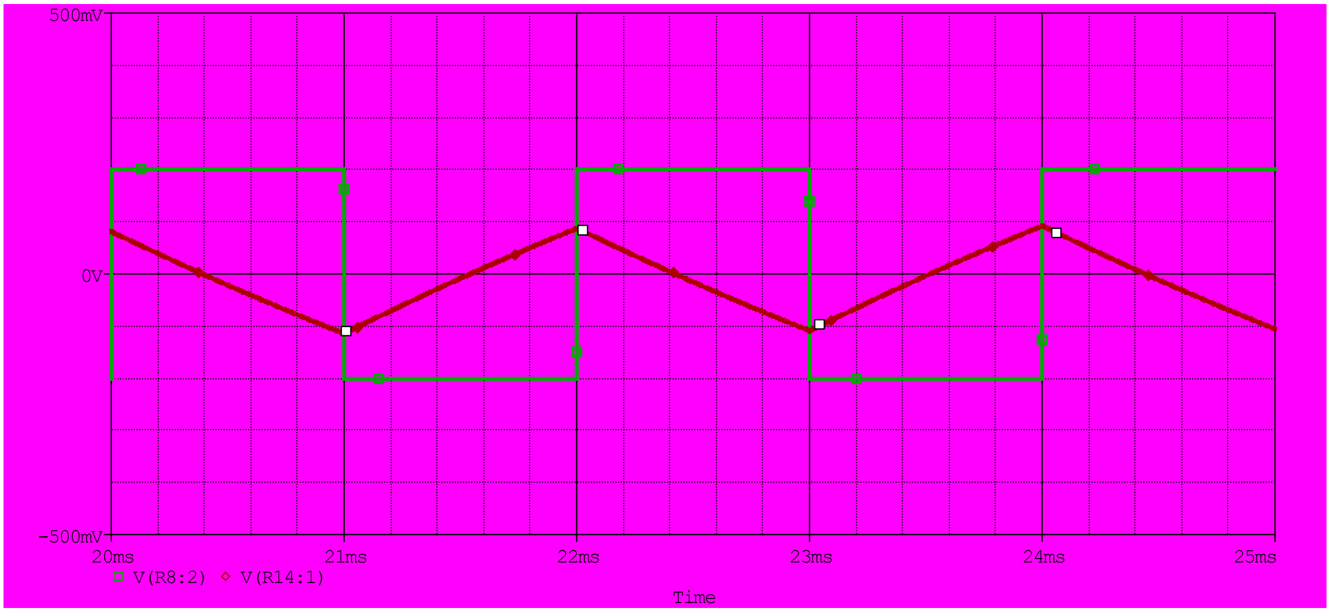
6) (4pt) Show that simplification of the transfer function from 2) for small ω gives the same results as the analysis of the redrawn circuit in 5).

From 2): $\frac{V_{out}}{V_1} = -\frac{R_f}{R_{in}(1 + j\omega R_f C_f)} = -\frac{1000}{200 + j\omega} \rightarrow -\frac{1000}{200} = -5 \quad \text{for } \omega \rightarrow 0$

From the inverting op-amp in 5): $A_V = -Rf/Ri = -5k/1k = -5$

7) (2pt) Sketch the output of the original circuit in 1) to a square wave input on the axis below. Show the correct shape of the waveform but don't worry about the amplitude scaling.




500mV
0V
-500mV
20ms
21ms
22ms
23ms
24ms
25ms
V(R8:2)
V(R14:1)
Time