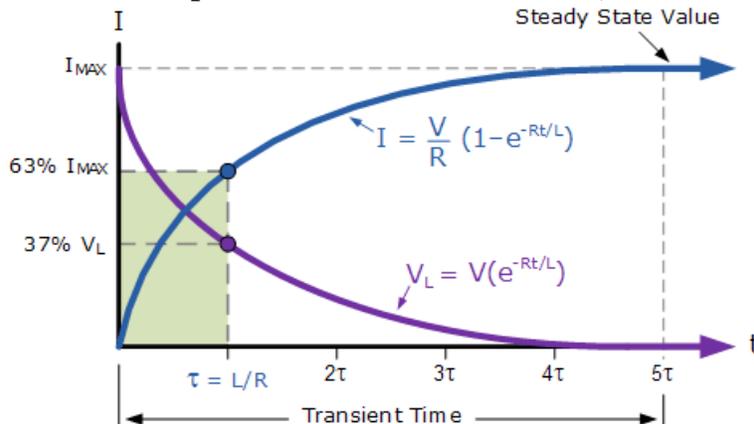


## Class #12: Experiment The Exponential Function in Circuits, Pt 1



**Purpose:** The objective of this experiment is to begin to become familiar with the properties and uses of the exponential function in circuits and engineering in general.

Background: Before doing this experiment, students should be able to

- Analyze simple circuits consisting of combinations of resistors.
- Measure resistance using a Multimeter.
- Do a transient (time dependent) simulation of circuits using LTspice
- Build simple circuits consisting of combinations of resistors, inductors, capacitors, and op-amps on protoboards and measure input and output voltages vs. time.
- Make differential voltage measurements using Analog Discovery and Waveforms.
- Review the background for the previous experiments.

Learning Outcomes: Students will be able to

- Determine and validate mathematical expressions for the charging and discharging of capacitors and inductors through resistors that utilize the exponential function.
- Use series and small argument representations of exponential functions in RC and RL circuit analysis of both experimental and simulated circuits.

Equipment Required:

- **Analog Discovery** (with Waveforms Software)
- **Oscilloscope** (Analog Discovery)
- **Function Generator** (Analog Discovery)
- **Protoboard**
- Resistors, Capacitors, Diodes, 9V battery and connector
- LTspice

Helpful links for this experiment can be found on the course website under Class #12.

### Pre-Lab

*Required Reading:* Before beginning the lab, read over and be generally acquainted with this document and the other **required reading** materials.

*Hand-Drawn Circuit Diagrams:* Before beginning the lab, hand-drawn circuit diagrams must be prepared for all circuits either to be analyzed using LTspice or physically built and characterized using Analog Discovery.

**Due:** At the beginning of Class #14

## Part A – Discharging a Capacitor Through a Known Resistor

### Background

The exponential function  $e^x$  is the inverse function of the natural logarithm  $\ln x$ . It is also one of the most amazing mathematical functions in that it is its own derivative! There is a lot to know about this function, so you should begin by at least glancing through the Wikipedia entry [https://en.wikipedia.org/wiki/Exponential\\_function](https://en.wikipedia.org/wiki/Exponential_function). You will encounter it over and over in your engineering, science and math courses because it characterizes the response of damped systems (and what real system is not damped due to friction or resistance?) and can be used to characterize oscillating systems (a topic of the next class).

### Experiment

We begin with a very simple experiment with an RC circuit and a 9V battery. Connect two identical capacitors ( $C1$  &  $C2 \geq 100\mu\text{F}$ ) and a resistor  $R_{\text{discharge}}$  in parallel on a protoboard. Also connect your Analog Discovery Channel 1 across the resistor and capacitors so you can monitor the voltage as a function of time. Add the ground wire to the bottom of the circuits, for a total of 3 connections to Analog Discovery. Choose the resistor so that the decay constant  $RC$  is about 40 seconds. Note that you must include the total capacitance and the total resistance in your evaluation of the time constant  $RC$ . That is, you must include the input resistance of Channel 1 in addition to both capacitors. You may have to work with a partner so that you have two large capacitors.

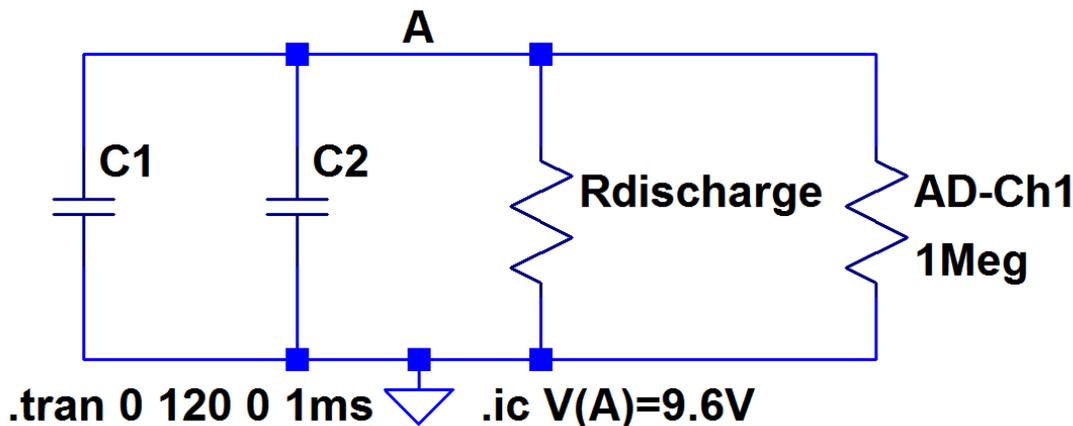
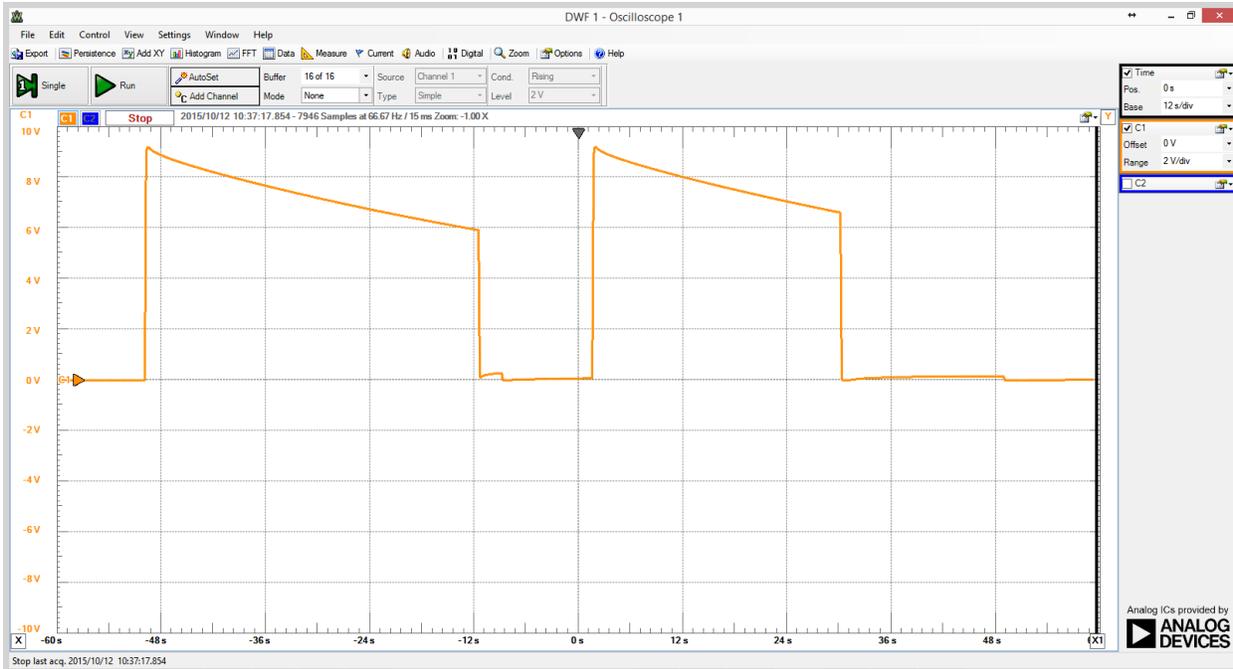


Figure A-1

1. What are the values of the capacitors you used and what is the value of  $R_{\text{discharge}}$  so that the time constant is about 40 seconds?

Open the Analog Discovery Oscilloscope. Set the horizontal scale to show two full minutes of time. Remember that there are ten divisions in both the horizontal and vertical directions on the scope display, so that should tell you how much time you need per division. For the vertical scale, we would like to see a total of 10V, so choose 2V/Div because we want to see both voltages between  $\pm 10\text{V}$ . When you are taking data, you will see the plot will be created very slowly. Turn the triggering off because you will only be looking at one scan through the total time of 2 minutes.

For the power supply in this experiment, we will use a 9V battery, which was discussed in Class #2. Obtain a 9V battery and connector from the TA or instructor. As discussed in the slides and video from Class #2, the black wire on the connector is negative and the red wire is positive. Connect the black battery wire to the ground side of the circuit. Do not connect the red wire to anything yet. Start the oscilloscope. Now momentarily connect & then disconnect the red battery wire to the ungrounded side of the capacitors (node A in the circuit shown in Figure A-1). You should see the voltage rise to around 9V and then slowly start to decay. Watch it decay for about 30-40 seconds so you are sure that the circuit is connected correctly. Then short out the capacitors with another wire by, again, only momentarily connecting it. You should see the voltage drop to zero. The charging and discharging process with the wire should be very fast. An example with a randomly chosen resistor is shown in Figure A-2. Your decay rate should be different. Repeat the process to be sure everything is working.



**Figure A-2**

The data to be collected should nearly cover the full two minutes of the horizontal scale. Make sure that the scope is not running. Then, momentarily touch the battery red wire to Node A and, once the wire is no longer in contact with your circuit, start the scope. Allow the scope to run until the plot reaches the far right end of the scan then then stop the scope. If you have done this well, you should have an exponentially decaying voltage across the entire time. Save a copy of your plot in your report and fully annotate it.

2. Then, using the cursor, determine the measured voltage at 11 evenly distributed points from 0 to 120 seconds and enter each value in the table below.

Time	Measured Voltage	Calculated Voltage
0		
12		
24		
36		
48		
60		
72		
84		
96		
108		
120		

Next, we wish to compare the measured voltages to the mathematical function  $V(t) = V_o e^{-t/\tau}$  where  $\tau = RC$  is the decay constant. Use your measured voltage  $V(0) = V_o$  for the amplitude of the exponential function. Your previously determined values for R and C give the time constant.

3. Calculate the values for the exponential function at each measurement time and enter them in the table. Do this using Excel. Once you have both columns filled in, plot the values vs time using Excel and include the plot in your report. If you have done this correctly, the two curves should look very much alike. Be sure to fully annotate the curves and comment on any differences you observe.

### Returning to Capacitors in Parallel

Before moving on to the next part of this experiment, we will use the set-up as is to explore how capacitors add in parallel and maybe in series. Start the experiment as before by charging the capacitors with the 9V battery. Start the scope and observe the voltage for about 10 or 12 seconds. Then, remove one of the capacitors, being careful not to disturb any of the other connections. The total capacitance should now be about half what it was. While the capacitor in the circuit is discharging slowly through the resistor, short out the capacitor you removed from the circuit so that it no longer has any voltage or charge. Recall the relationship between voltage and charge for a capacitor. Now connect the uncharged capacitor back in parallel with the charged capacitor. What do you observe? Explain what you see using the expression relating capacitor voltage and charge. Repeat the process of removing, discharging and reconnecting the second capacitor. Once you reach the end of the 2 minute scan, stop the scope and save the plot to your report. Fully annotate the plot explaining the features that you see. Start the process over again by charging the two capacitors. After each 10-20 seconds, remove the second capacitor and then replace it after another 10-20 seconds, but do not discharge it when it is disconnected. Do this for the full 120 seconds, save your plot and fully annotate it. What is different this time and why? To help understand what you see is the second process, start it over with both capacitors and then, after 20 seconds, remove the resistor from the circuit. Wait another 10 seconds and then disconnect the orange positive wire for channel 1 of your scope (1+). Leave it disconnected for about 60 seconds and then reconnect the scope wire. Save the plot, annotate it and explain what you see. Show your results to a TA or instructor and discuss your key findings. TA/Instructor: \_\_\_\_\_

### Back to the Study of Charging and Discharging Capacitors

Next, we will change the circuit so it will do something similar but much faster. Replace the capacitors with  $0.1\mu\text{F}$  and find a new value for the resistance that gives a time constant of about 1ms. We will use the function generator (AWG1) rather than a battery. Open the function generator window and set it up to produce a sequence of square pulses that vary between 0V and 2V at a frequency of 100Hz. Recall that you will have to use both the amplitude and offset to do this. Note that Figure A-3 shows the function generator and resistor are now in series. Both scope channels are now to be used because, as usual, we want to monitor both the input and output voltages. Set up the scope to display 30-50ms of data and use as much of the vertical range as possible. The time and volts per division are your choice but your goal is to see as much detail as possible. Run the experiment and save both an image of the data and as a file you can display and analyze in Excel. Be sure to annotate your plot in your report. You will use this same experiment in Part B, so do not disassemble it.

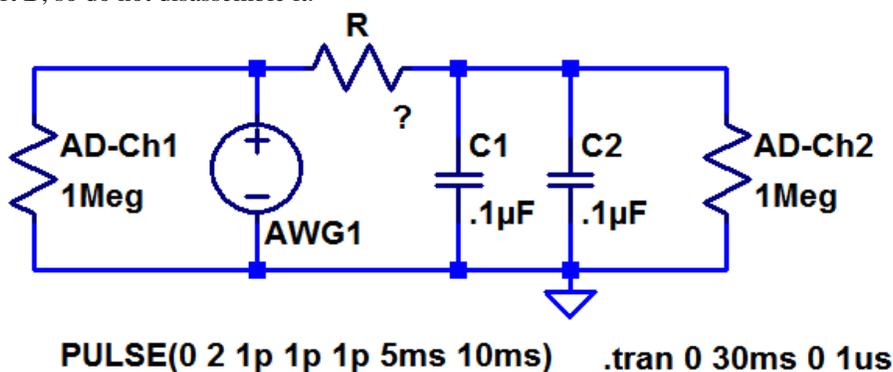


Figure A-3

- In Excel, display only a single half-cycle of the input and output signals. Evaluate the exponential decay function and fill the corresponding column next to the measured output data. Plot and compare both exponentially decaying functions (one measured and one calculated). Annotate and discuss any features you observe.

### Summary

The exponential function is seen to characterize the discharging and charging of capacitors through resistors. This is also the case for inductors, although one has to take into account the inductor resistance. Fortunately, it is usually much smaller than resistors in circuits.

### Part B – Exponential Functions and IV Formulas for L & C



The experiments you just completed can help us understand the relationship between current  $I$  and voltage  $V$  for a capacitor. Before revisiting the experiment, you will need to review the properties of the exponential function again and evaluate the time derivative of the function you plotted in Excel. That is

5. Evaluate the time derivative of  $V(t)$ :  $\frac{dV(t)}{dt} = \frac{d}{dt} \left( V_o e^{-t/\tau} \right)$  Be sure you confirm your result before proceeding. TA/Instructor: \_\_\_\_\_

### Experiment

Repeat the last experiment from Part A, but monitor the voltage across the capacitor with Channel 1 and across the resistor with Channel 2. Use your measurement of the voltage across the resistor to determine the current through the capacitor combination (treat the capacitors as one total capacitance). Save and annotate your plot. Export your data to Excel. You should have a column for time, one for current and one for voltage. Again, limit yourself to a single half-cycle of the capacitor voltage in which is it decaying. Find the mathematical expression for the voltage as a function of time and plot it in the column next to the measured data. If you review the Wikipedia entry for capacitors (<https://en.wikipedia.org/wiki/Capacitor>), you will find that the relationship between current and voltage should be

$$I_c(t) = C \frac{dV_c(t)}{dt}$$

Use your expression above for the time derivative of the capacitor voltage, multiply it by your value for  $C$  and compare it with your result for current  $I(t)$ . This should be done by adding another column in Excel and plotting the two columns vs time.

The exponential function can be written as a series expansion. This information can be found many, many places, but a good first resource is (<http://people.math.sc.edu/girardi/m142/handouts/10sTaylorPolySeries.pdf>). If the argument of the exponential is very small, then we can quite accurately represent the function with only the first two terms:

$e^x \approx 1 + x$ . This can be very handy because it is much easier to deal with this very simple algebraic expression.

6. Evaluate this expression for the exponential voltage function  $V(t)$ . Note that, like the derivative, you will have to take the time constant and sign into account. Add this as another column in your Excel spreadsheet and plot it, along with the other two functions, to see where the approximation is reasonably good.

### Summary

We have seen that the exponential function is very, very useful in RC circuits. It plays the same role in RL circuits, and any dynamic system with damping. You should be able to summarize its importance at this point.

### Part C – Task Checklist

- R for  $\tau = 40s$
- Plot of capacitor voltage vs time for 120s
- Export and analyze measured and calculated capacitor voltages vs time
- Explore effects of capacitors in parallel
- R for  $\tau = 1ms$
- Plot of capacitor voltage vs time for several cycles
- Export and analyze measured and calculated capacitor voltages vs time
- Plot of  $I$  and  $V$  vs time for square wave driving capacitor through resistor.
- Compare measured current with current from derivative of voltage
- Export and analyze  $I$  and  $V$  vs time for square wave,  $R$  and  $C$
- Evaluate small argument expression for capacitor voltage
- Add approximate expression to Excel analysis and plot