Experiment 3
Inductors and Transformers

Purpose: Partly as preparation for the next project and partly to help develop a more complete picture of voltage sources, we will return to considering inductors. The extension we are primarily concerned with is the mutual inductor or transformer. The transformer has three uses: stepping up or down voltages, stepping up or down currents, and transforming impedances. Like other devices we have considered, the transformer does not work in an ideal manner for all circumstances.

Background: Before doing this experiment, students should be able to
- Measure inductance using a commercial impedance bridge.
- Do a transient (time dependent) simulation of RC, RL and RLC circuits using Capture/PSpice.
- Do an AC sweep (frequency dependent) simulation of RC, RL and RLC circuits using Capture/PSpice, determining both the magnitude and the phase of input and output voltages.
- Determine the complex transfer function for RC, RL and RLC circuits and simplify for high and low freq.
- Be able to define what is meant by high and low frequencies in the context of RC, RL and RLC circuits.
- Identify whether an RC, RL or RLC circuit is a low-pass, a high-pass, a band-pass or a band-reject filter.
- Find the corner frequency for RC and RL circuits and the resonant frequency for RLC circuits.
- Find a practical model for a real inductor and determine the range of frequencies in which the real inductor behaves nearly like an ideal inductor.
- Review the background for the previous experiment.

Learning Outcomes: Students will be able to
- Estimate the inductance of simple magnetic toroid using well-established analytic and empirical formulas.
- Build simple cylindrical (solenoid) and toroidal inductors using enameled magnet wire and a plastic tube or a ferrite magnetic core.
- Estimate the inductance of a toroid inductor by building a resonant RLC circuit and finding the resonant frequency.
- Do a transient (time dependent) simulation of a resistor loaded transformer circuit using Capture/PSpice.
- Do an AC sweep (frequency dependent) simulation of a resistor loaded transformer circuit using Capture/PSpice.
- Analyze a simple transformer using Capture/PSpice and demonstrate that it works as designed for some range of frequencies.
- Build a simple transformer from two windings on a toroidal core and demonstrate that it works as designed for some range of frequencies.

Equipment Required:
- DMM (Digital Multi-meter)
- Analog Discovery (with Waveforms)
- Oscilloscope (Analog Discovery)
- Function Generator (Analog Discovery)
- Breadboard
- Enameled Magnet Wire
- Ferrite Core & Its Spec Sheet
- Electrical Tape
- Sandpaper
- Misc Resistors & Capacitors

Helpful links for this experiment can be found on the links page for this course. Be sure to check all of the links provided for Exp 3.

Pre-Lab

Required Reading: Before beginning the lab, at least one team member must read over and be generally acquainted with this document and the other required reading materials listed under Experiment 3 on the EILinks page.

Hand-Drawn Circuit Diagrams: Before beginning the lab, hand-drawn circuit diagrams must be prepared for all circuits either to be analyzed using PSpice or physically built and characterized using your Analog Discovery board.
Part A - Making an Inductor

Background

Calculating inductance: An inductor consists of a wire of conductive material wound around a (usually) solid object called a core. The inductance of an inductor depends on the material and geometry of both the coil and the core. Inductors have larger values when the core material is a magnetic material like iron. The value of the inductance will also depend on the geometry of the core material. Each physical coil geometry has a unique equation to calculate its inductance. Just for simplicity, we will address only two standard geometries, cylindrical and toroidal cores. This produces the kind of inductors most often used in practice. The ones we have been using are cylinders potted in plastic, so you cannot see what the coil looks like. Search online for solenoid or solenoidal inductor and you will get a better idea of what such coils look like in practice. In some classroom demonstrations in physics, a simple open structure is used so that it is easier to see the geometry. An example of such a coil from Pasco is shown below along with a couple of other inductors like the ones we are using.

If the core cylinder has a radius equal to \( r_c \) and we wind a coil \( N \) times around the cylinder to cover a length \( d \), the inductor will, ideally, have an inductance equal to:

\[
L = \frac{\mu_0 N^2 \pi r_c^2}{d} \text{ Henries}
\]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ Henries/meter} \). If the core is not air, but rather some magnetic material, replace \( \mu_0 \) with \( \mu \) which is usually many times larger than \( \mu_0 \). By many times we can mean as much as \( 10^5 \) times larger. You should know that this formula only works well when the length \( d \) is much, much larger than the radius \( r_c \).

Note that the ideal formula above will produce a result that agrees exactly with the actual inductance of the coil. They are useful to find ballpark values for inductance only.

A semi-empirical formula was developed by H. A. Wheeler in the 1920s. He was a distinguished electrical engineer who worked at the National Bureau of Standards (now NIST) and Hazeltine Corporation (headquartered in Greenlawn, Long Island and now part of BAE Systems). His formula, accurate to within 1% as long as \( d > 0.8r_c \), gives the inductance in \( \mu\text{H} \), if the dimensions are in inches (first expression) or meters (second expression).

\[
L \approx \frac{r_c^2 N^2}{9r_c + 10d} \mu\text{H} \quad \text{or} \quad L \approx \frac{4r_c^2 N^2}{0.9r_c + d} \mu\text{H}
\]

This formula can be found in many places on the internet because it is useful for analyzing RF coils. There are even many calculators for evaluating the expression.

Toroidal inductors have some distinct advantages over cylinders. The magnetic field they produce is better confined to inside the magnetic material which cuts down on interaction between components and also makes transformers
work better. A variety of toroidal core inductors are shown below, along with a few cylindrical inductors for reference.

Ideally, \( L = \frac{\mu_0 N^2 h_c}{2\pi} \ln\left(\frac{b}{a}\right) \)亨利s when the core is air or plastic. Otherwise replace \( \mu_0 \) with \( \mu \) for the core material. \( h_c \) is the height of the core material. \( a \) and \( b \) are inner and outer radii and are shown in the figure at the right.

It is generally not necessary to apply this formula for commercial cores. The manufacturer knows everything about the core except for the number of windings of wire we choose to add to it. Thus, in the spec sheet for most cores, manufacturers usually include the inductance factor with typical values like a few \( \mu \)H per \( N^2 \). Thus, for roughly 30 turns of wire, the inductance will be a few mH. Since the spec sheet for the core used in class has this information, you will use the simpler approach to find the inductance.

**Calculating Resistance:** When one makes an inductor, the wires used can have a large variety of cross sectional areas. There are some inductors made with very thick wires, while others are made with very thin wires. Thin wires permit one to wind many more turns of wire around a core and thus increase the inductance. Thick wires have lower resistance for any given length. All wires have a resistance given by the expression \( R = \frac{l}{\sigma A_w} \Omega \), where \( l \) is the length of the wire, \( A_w \) is the cross sectional area of the wire (thickness), and \( \sigma \) is the conductivity of the wire material. For copper, the conductivity is about \( 6 \times 10^7 \) Siemens/meter. The unit of Siemens is 1/\( \Omega \). Sea water has a conductivity of 5 Siemens/meter. There are many handbooks like the CRC Handbook of Chemistry and Physics that have the resistance of different diameter wires per mile (or another unit of distance). It is also quite easy to calculate the resistance of a piece of wire using the formula above. The links page contains several links to tables with conductivity information. One of the more useful tools for finding information like resistance per unit length of copper wire is the MegaConverter (http://www.megaconverter.com/mega2/).
Other Losses: Both air and magnetic core inductors have losses due to finite wire resistance. Magnetic cores also have hysteresis loss that occurs because they are magnetized and demagnetized over and over when they experience a sinusoidal current. As a result, higher frequency losses tend to be larger than DC losses. However, we will only be concerned with wire losses in this experiment, except to note when they are insufficient to account for all losses.

Experiment

Build an Inductor
In this part of our experiment, we will build two inductors and compare their calculated properties to measured properties. One inductor will be a cylinder, wound on a piece of plastic tubing, and the other a toroid, wound on a commercial ferrite core.

Inductor #1:
- Build an inductor from enameled wire using the following procedure.
  - Use the piece of plastic tubing you will be given for your coil winding. Measure the tube O.D.
  - Use 30 – 40” (0.75 – 1m) of enameled wire. Note the gauge of the wire you are using. Measure and record the wire length. There are rulers on the side of the center table for this purpose.
  - Leave about 2 inches (5 cm) of wire as leads for making connections, and wrap it tightly around your tube. Carefully keep track of the number of times you wind the wire around the tube. This is the number of turns of your inductor, N. If you have good eyesight, you should also count the number of turns.
  - Secure the windings with electrical tape. A tape dispenser is located by the wire reels near the two entrances to the classroom. Also stabilize the leads by twisting them as close to the cylinder as you can.
  - Remove some of the enamel from the ends of the wire (0.25 to 0.5 inch). The enamel is the insulation for this type of wire, so you cannot make electrical contact unless it is removed. There should be some sandpaper for this purpose or you can use a knife. You will be provided with wood or plastic blocks to do your sanding on. Please do not sand the table tops when you remove the enamel!
- Calculate the properties of the inductor
  - Calculate an estimate for the resistance of the coil. Look up the dimensions of the wire in a table of wire properties listed by gauge. Calculate the resistance using the equation given above. You can also confirm your answer by checking it against any online resistance calculator.
  - Calculate an estimate for the inductance using Wheeler’s formula. Be careful of your units.
- Measure the properties of the inductor
  - Measure the resistance using the digital multimeter (DMM). When you measure small resistances, it is important to first measure the resistance of the wires you are using to connect the coil to the meter. Then, add the coil and measure again. The resistance of the inductor will be the difference between the resistance of the wires alone and the resistance of the wires with the inductor. Does your measured value agree at least roughly with the calculated value?
  - Measure the inductance of your coil directly with one of the impedance bridges on the table in the center of the classroom. Do not use the alligator clip leads that may be attached to the impedance bridge. Rather, slide your wires into the slots to obtain the best possible electrical connection. Which equation gave you a better estimate of the measured value? The impedance bridges may also be able to measure resistance, but the DMM is more accurate.

Inductor #2:
- Build an inductor from enameled wire using the following procedure.
  - Use a commercial toroidal ferrite core you will be given for your coil winding. Note the type of core and find its spec or data sheet.
  - Use 30 – 40” (0.75 – 1m) of enameled wire. Note the gauge of the wire you are using. Measure and record the wire length. There are rulers on the side of the center table. (You should use exactly the same length of wire as above, so all questions about resistance will be a duplicate of the cylindrical coil.)
Leave about 2 inches (5 cm) of wire as leads for making connections, and wrap it tightly around your core. Carefully keep track of the number of times you wind the wire around the core. This is the number of turns of your inductor, N. If you have good eyesight, you should also count the number of turns.

Secure the windings with electrical tape. A tape dispenser is located by the wire reels near the two entrances to the classroom. Also twist the leads as close to the toroidal core as you can.

Remove some of the enamel from the ends of the wire (0.25 to 0.5 inch). The enamel is the insulation for this type of wire, so you cannot make electrical contact unless it is removed. There should be some sandpaper for this purpose or you can use a knife. You will be provided with wood or plastic blocks to do your sanding on. Please do not sand the table tops when you remove the enamel!

- Calculate the properties of the inductor
  - Calculate the inductance of your inductor using the inductance parameter from the ferrite core spec sheet.

- Measure the properties of the inductor
  - Measure the inductance of your coil directly with one of the impedance bridges on the table in the center of the classroom. Compare the measured and calculated values? The impedance bridges may also be able to measure resistance, but the DMM is more accurate.

Summary

We can get an approximate expression for the inductance using an equation for an ideal model based on the geometry of the inductor and the materials from which it is made. The ideal model of the cylindrical inductor will generally over-estimate the inductance. We can get a larger inductance and also a more predictable inductance by winding the coil around a piece of iron or other magnetic material rather than a plastic tube. The permeability of iron is many times larger than that of air. We can also use an equation to estimate resistance.

Part B - Measurement of Inductance

Background

Using circuits to estimate inductance: Now we will use an experimental method to estimate the coil inductance. In the circuit of Figure B-1, V is the Analog Discovery function generator. L is the inductance of the coil to be measured. C is a known capacitor chosen to make L easy to determine. D is a diode used to direct current toward the inductor and to shut off current when the source voltage goes negative so that the inductor can send its energy to the capacitor.
Capacitors and inductors are both energy storage devices so that when they are connected together, they will trade energy back and forth at a characteristic frequency. They form an harmonic oscillator just like a spring-mass system that trades kinetic for potential energy (and vice versa) at a characteristic frequency. For an LC circuit, this frequency is given by the expression \( f_o = \frac{1}{2\pi\sqrt{LC}} \). Thus, if we can observe the decaying sinusoidal voltage across the capacitor and inductor and determine its frequency, we can determine \( L \) from our knowledge of \( C \). To do this measurement and model the behavior of the inductor using PSpice, we have to modify the ideal circuit as shown below. Inductors have finite resistance, so we have to add a resistor \( R_L \) to our model of the inductor. The Analog Discovery function generator (Wavegen) can only deliver a limited amount of current or power to a load. Since the resistance of our coil is small, we have to add a resistor \( R \) to limit the current from the function generator. Circuit components (\( R_c \) and \( C_c \)) representing the two analog input channels on Analog Discovery are also included for completeness. For this circuit, you are to use \( C = 0.1\mu F \) because it is one of the larger capacitors you have that is not electrolytic. Electrolytic capacitors usually have a relatively large ESR = equivalent series resistance, which can damp the decaying sinusoid out so fast you cannot easily read the oscillation frequency. For \( R \), you are to choose \( R = 1\Omega, \ 10\Omega \) or \( 100\Omega \), whichever makes the current through the diode around half the upper limit Analog Discovery can provide. The limit is about 50mA. Which of the three resistance values works best? Note: we will not study diodes until near the end of the semester. However, you will need to know one thing about diodes to answer this question. For the diode to turn on and conduct current, the voltage applied must exceed a value called the Forward Voltage. For the 1N4148, \( V_f \) is about 0.7V. Thus, when the diode is ON, the forward voltage applied to the circuit (here equal to +IV) will not all appear across the remaining circuit components. Rather the voltage available will be \( V - 0.7V \). \( R \) must be chosen to handle the possibility that the load (here \( L \) in parallel with \( C \)) may go to zero, because the impedance of \( C \) goes to zero at high frequencies and the impedance of \( L \) goes to \( R_L \) (usually very small) at low frequencies. You now have enough information to answer this question.

**Figure B-1.**

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The inductor will have a positive imaginary impedance given by \( j\omega L \) while the capacitor will have a negative imaginary impedance given by \( \frac{1}{j\omega C} \) or equivalently \( -\frac{j}{\omega C} \). If we redraw our circuit at low and high frequencies, we can see that at both extremes, the output voltage will be small. At low frequencies, the inductor is nearly a short to ground. At high frequencies, the capacitor is a low impedance path to ground. At the resonant frequency, 
\[
  f_0 = \frac{1}{2\pi \sqrt{LC}}
\]
the parallel combination of an ideal inductor and a capacitor has an infinite impedance. Real L’s and C’s have losses so the actual impedance is finite, but the magnitude of the impedance peaks at or near the resonant frequency. We can measure the resonant frequency and use this equation to solve for L.

**Experiment**

*The Resonant Frequency of a Circuit can be used to Find Inductance*

In this experiment, we will build an RLC circuit, find its resonant frequency, and use it to solve for an unknown inductance.

- Find the values for the actual components in your circuit.
  - Measure the capacitance of the 0.1\( \mu \text{F} \) capacitor using the impedance bridge. It is the actual value of C that will determine the resonant frequency, not the labeled value.
  - Make sure the air core inductor is well secured to the plastic tube. Any changes in the geometry of the inductor will change its inductance.
  - Write down the mathematical values you calculated for the inductances in part A. We will call this inductance \( L_c \). (Remember that we are treating the inductances as unknown, so we cannot use the inductance measured with the impedance bridge. However, you should use the better of the two cylindrical inductor formulas.)
  - Calculate an estimate for the two resonant frequencies, 
    \[
    f_{initial} = \frac{1}{2\pi \sqrt{L_c C}}
    \]
    using the capacitance you measured and the inductances you calculated.
• Find the experimental resonant frequency for both inductors you have built. For each one,
  o Set up the circuit in Figure B-1 on your Protoboard, but be sure that you include the current limiting
    resistor R. Use the capacitor whose value you measured and your DIY inductor. Be careful to make sure
    that the inductor is making contact with the protoboard. Bad connections can be a common problem.
  o Set your function generator output to a square wave with a 1.5V amplitude and an offset of -0.5V. Set the
    frequency to be 100Hz. Observe the voltage across resistor R on scope channel 1 and the voltage across the
    capacitor and inductor on scope channel 2. Always display both input and output voltages. Note that your
    channel 1 measurement is differential and not referenced to ground.
  o When you observe a clear decaying sinusoidal voltage, determine the frequency of the signal.
  o Create a math channel for the current through the resistor R.
  o Save a picture of the measured current through R and the voltage across L & C.

• Calculate an estimate for the unknown inductance, \( L_{est} \), using the equation \( f_o = \frac{1}{2\pi \sqrt{L_{est}C}} \). Use the exact
  resonant frequency you identified and the capacitance you measured with the bridge. What value did you
  obtain for your unknown inductance? How close is this to the one you measured on the impedance bridge? Is it
  closer than the estimate you found using the inductance equations?

Be sure to do this for both of the inductors you built.

Use a Simulation to get the Best Estimate of the Unknown Inductances
In this part of the experiment we will use PSpice to simulate the circuit we built and get the closest estimate we can
  to the actual inductance and resistance for the inductors. For both inductors,

• Simulate the circuit in Figure B-2 in PSpice. This is the same circuit as the one pictured in B-1, but the inductor
  (because it does not have an insignificant internal resistance) is more accurately simulated by two components,
  an inductor and a resistor. Components representing the scope are also included.
  o Use the measured value for the capacitor and for the resistor R. Recall that inductors do not have
    negligible resistance. Therefore, in the PSpice model, the inductor you built looks like an inductor and a
    resistor in series. Use the value you measured for the resistance of the inductor for RL and the inductance
    that you just calculated (\( L_{est} \)) for L.
  o Create a transient simulation that will duplicate what you measured with Analog Discovery. Adjust RL and
    possibly also L to obtain the best possible agreement.
• When you have found good values for RL and L, print out the plot of the simulation. Fully annotate your plot.

Summary
In this experiment, you have found an estimate for the impedance of the inductor you built by placing it into a
circuit. By finding the resonant frequency of the circuit, you were able to find an estimate for L in two ways: by
calculating it using the equation for resonant frequency and by simulating the circuit in PSpice and choosing the
inductance and resistance values such that the resonant frequency and sinusoidal voltage decay matched observed
experimentally. How do these values compare to the resistance and inductance for the two inductance you measured
using the impedance bridge and multimeter?

Part C - Transformers

Background

Inducing a current: Inductors work by creating a magnetic field. When you run a current through an inductor it
becomes an electromagnet. The direction of the magnetic field can be found by placing your right hand around
the core in the direction of the coil. Your thumb will point in the direction of the magnetic field, as shown in Figure C-1.
If one inductor is placed near another inductor, then the magnetic fields of the two inductors will interact with one another. If you have ever built an electromagnet, you will know that the magnets will attract or repel one another if the current in the coil is large enough. Even when we cannot sense that they are interacting, a current in one coil will induce a current in a nearby coil.

**Transformers:** A transformer is a device that takes advantage of the fact that one inductor can induce a current in another inductor. We use it to transform one voltage level into another. A “step up” transformer will make a small voltage larger and a “step down” transformer will make a large voltage smaller. We make transformers by winding coils of wire around some kind of a core material. Sometimes the core material is just air, as when we wind the wire around a plastic tube, for example. Most of the time, the core material is iron or some other magnetic material. To maximize this interaction, we usually wind the coils onto the same core and make them look as similar as possible.

In the transformer circuit in Figure C-2, the voltage source ($V_S$) and the 50Ω resistor ($R_S$) represent a sinusoidal voltage source like a standard function generator. (The internal resistance of the Analog Discovery function generator is zero so we have added the 50Ω resistor to see how a standard and much more expensive function generator behaves.) $R_L$ is the load on the transformer represented by a single resistance and TX1 is the transformer. The primary (or source) inductor in the transformer, $L_S$, draws a current from the source. The magnetic field created by $L_S$ induces a current in the secondary (or load) inductor, $L_L$. This creates a new voltage to power the load, $R_L$.

Transformers only work for time varying currents and voltages. Note that when the coils are as similar as possible, all the geometric terms in the formulas for the inductance will be the same for the primary and the secondary coils. Only the number of turns will be different. We can demonstrate this using our equation for the inductance of a long, thin coil.

\[
\frac{L_S}{L_L} = \frac{L_1}{L_2} = \frac{\left(\mu_0N_1^2\pi r_e^2\right)}{\left(\mu_0N_2^2\pi r_e^2\right)} = \frac{N_1^2}{N_2^2}
\]
Note that we will use \( L_1 = L_S \) and \( L_2 = L_L \) interchangeably. Since we try to make \( \mu, r, \) and \( d \) the same, these terms cancel out and the ratio of the inductances depends only upon the ratio of the number of turns squared.

**Transformer Analysis:** To analyze just how a transformer works, we have to add an additional kind of inductance, called mutual inductance. If a coil of wire of inductance \( L_1 \) is very near another coil of inductance \( L_2 \), there will be a mutual inductance \( M \) between the two coils, where \( M^2 = k^2 L_1 L_2 \). The constant \( k \) is the coupling coefficient. If the coils are perfectly coupled, \( k = 1 \). Usually \( k \) is a little less than 1 in a good transformer.

Referring to Figure C-2, the two loop or mesh equations that apply to the two current loops in the transformer circuit are

Primary Loop:
\[
V_S = I_S (R_S + j\omega L_S) - I_L (j\omega M)
\]

Secondary Loop:
\[
0 = -I_S (j\omega M) + I_L (R_L + j\omega L_L)
\]

where \( L_S \) is the inductance of the primary (source) coil and \( L_L \) is the inductance of the secondary (load) coil. By convention, the coil connected to the source is called the primary coil and the coil connected to the load is called the secondary coil. The \( L \) and \( M \) terms have opposite signs because the loop currents go in opposite directions. Thus, the voltage generated by one coil will influence the other in the opposite direction. The input impedance is the impedance across the primary coil, \( L_S \). Since \( V = IZ \) and the current through all the elements in the primary coil is \( I_S \)

\[
V_S = I_S R_S + I_S Z_{in}
\]

\[
Z_{in} = \frac{V_S}{I_S} - R_S = j\omega L_S - \frac{I_L}{I_S} (j\omega M)
\]

(Note that \( R_S \) and \( Z_{in} \) form a voltage divider.) We can solve the primary loop equation for \( Z_{in} \) by solving for \((V_S/I_S) - R_S\)

\[
Z_{in} = \frac{V_S}{I_S} - R_S = j\omega L_S - \frac{I_L}{I_S} (j\omega M)
\]

If we solve the secondary loop equation for \( I_L/I_S \), we can then substitute for this ratio in the equation above:

\[
I_L = \frac{j\omega M}{j\omega L_L + R_L}
\]

\[
Z_{in} = j\omega L_S - \frac{(j\omega M)^2}{j\omega L_L + R_L} = j\omega L_S + \frac{\omega^2 M^2}{j\omega L_L + R_L}
\]

We know that by definition, \( M^2 = k^2 L_1 L_2 \) and for an ideal transformer, the coupling constant \( k \) is 1, so

\[
Z_{in} = j\omega L_S + \frac{\omega^2 k^2 L_S L_L}{j\omega L_L + R_L} = j\omega L_S + \frac{\omega^2 L_S L_L}{j\omega L_L + R_L}
\]

\[
Z_{in} = \frac{L_S R_L}{L_L}
\]

Now, let both \( L_S \) and \( L_L \) or \( \omega \) become very large (tend to infinity). The \( R_L \) term in the denominator will drop out and

\[
Z_{in} = L_S R_L
\]

If we define a constant, \( a \), that has the following property: \( L_L = a^2 L_S \), then

\[
Z_{in} = R_L/a^2
\]

Thus, the transformer transforms the load resistance \( R_L \) by the square of the turns ratio, \( a = N_L/N_S \) where \( N_S \) is the number of turns in the primary coil and \( N_L \) is the number of turns in the secondary coil. This is the first, and most
stringent, relationship for an ideal transformer. Remember that $Z_{in}$ is the ratio of the input voltage to the input current. In any circuit that we build or simulate, we can determine $Z_{in}$ by finding this ratio.

The relationship between the primary and secondary currents in the transformer can be found from the second loop equation. Solving again for the case where the inductances become very large, we find that

$$a^2 = \frac{L_L}{L_S} \quad a = \sqrt{\frac{L_L}{L_S}} = \frac{N_L}{N_S} = \frac{V_L}{V_S} = \frac{I_S}{I_L}$$

Note that both the voltage and current relationships shown contain no sign information. Depending on how the transformer is wired, it is possible for minus signs to appear in these expressions. Thus, you should consider that they hold only for magnitudes. As with $Z_{in}$, we can determine these ratios by measuring the voltages and currents separately and then taking their ratios. When we design a particular transformer, we usually have a turns ratio in mind. For example, in the DC power supplies that come with consumer electronics (also called wall warts), there is a step-down transformer that takes the 120Volt line voltage and steps it down to a smaller voltage, like 6 or 12Volts. In such devices, $N_S/N_L$ is chosen to be 10 or 20, depending upon the desired output voltage. This voltage is then rectified with a full-wave rectifier and sometimes regulated with a Zener diode to produce a DC voltage. You will learn more about rectifiers and Zener diodes later in this course.

**Working range of a transformer:** Although the equations for transformer behavior are quite simple, many assumptions need to be made in order to satisfy them. A transformer will not behave in a circuit according to the equations for all frequencies or load resistances. **In order for a transformer to be working properly, the following expressions must all be satisfied.** When $a^2 = L_L/L_S$:

$$Z_{in} = \frac{R_L}{a^2} \quad a = \frac{V_L}{V_S} \quad a = \frac{I_S}{I_L}$$

**Experiment**

**Simulation of a Transformer**

In this section, we will use PSpice to create a transformer circuit and find the input frequencies where it behaves according to the transformer equations.

- Set up the circuit pictured in Figure C-3 in Capture.
  - For the voltage source $V_S$, use VSIN and assume an amplitude of 1Vp-p, a frequency of 1kHz, and a DC offset of 0V. Also set the AC voltage to 1Vp-p.
  - Rs must be included for the simulation to work. Set it to a small value, such as 1Ω, it represents wire resistance.
  - Set the load resistance, RL, to 50Ω.
For the transformer (XFRM_LINEAR in the ANALOG library), you need to set the coupling coefficient. Choose 1 for perfect coupling. You also need to set the primary and secondary inductances, L1 and L2. For your first simulation, set L1 to 4mH and L2 to 1mH.

Perform an AC sweep from 1Hz to 1MegHz.

In order to determine when the transformer is working correctly, we need to determine the frequencies where the relationships for \( Z_{in} \), \( V \) and \( I \) are satisfied. Recall this means that \( V_2/V_1 \) and \( I_1/I_2 \) must be equal to a constant, \( a \). To find where our transformer works, we can plot the three relationships and determine where all three are satisfied. To avoid dividing by zero current, we will plot \( 1/a \).

Find the constant, \( a \), using the ratio of \( \sqrt{L_2} \) to \( \sqrt{L_1} \). Remember that the ratio of N2 to N1 is determined from the ratio of the square roots of L2 and L1.

Add a trace of \( V_1/V_2 \), where \( V_1 \) is the voltage across the primary coil and \( V_2 \) is the voltage across the secondary coil.

Add a trace of \( I_2/I_1 \), where \( I_2 \) is the current through the secondary load resistor and \( I_1 \) is the current through the resistor in the primary loop. Because PSpice is picky about polarities, this ratio may be negative. Multiply it by -1 or change the polarity of one of the resistors to make it positive.

For the third criteria, we must solve \( Z_{in} = RL/a^2 \) for \( 1/a \). This means \( 1/a \) must be equal to \( \sqrt{Z_{in}/RL} \). \( Z_{in} \) is the impedance of the primary inductor, \( L_1 \). It is equal to the voltage across \( L_1 \) \( (V(TX1:1)) \) divided by the current in the source loop (I1). Therefore, to plot \( 1/a \), we must plot \( \sqrt{V(TX1:1)/I(TX1:1)} \). Add a trace of \( 1/a = \sqrt{V(TX1:1)/I(TX1:1)} \).

Save one plot with all three ratios. Mark the frequency range where all three criteria are satisfied on the plot.

By trying a variety of values for L1 and L2, find a transformer for which the three relationships are satisfied and \( a=10 \) \( (1/a=1/10) \). In this case, the voltage across the primary coil will be 1/10 the voltage across the secondary coil and the other two expressions will also be satisfied. Use the smallest possible inductances you need to make the transformer work properly at 1kHz and above. (To change the effective frequency, you must increase or decrease both L1 and L2, while keeping the ratio of the square roots the same \( (1/10) \).) Write down the values for L1 and L2 that you have selected and save your PROBE plot with the three ratios that demonstrate that the transformer works as specified.

Now repeat the task of the last paragraph for frequencies of 100Hz and up. Write down the values for L1 and L2 that you have selected and save your PROBE plot with the three ratios that demonstrate that the transformer works as specified. What design advantages exist for circuits that are to work at higher frequencies rather than lower frequencies?

Summary

In this section you learned that the behavior of transformers are governed by simple proportions. However, real transformers only conform to those simple ratios under certain conditions.

Part D - Making a Transformer

Background

Designing a transformer: To design a transformer for which the output voltage is the same as the input voltage, we must have the same number of turns on our output coil as on our input coil. Here we will design a transformer where the output voltage is half the input voltage. We must also find a range of frequencies for which our transformer actually works as it should. Since you already have one coil, the addition of a second coil will result in a configuration that can be used as a transformer.

Experiment

Build a Transformer

Make a transformer by winding a second coil directly over the ferrite core inductor you built.
• Wind a second coil on your ferrite toroid with about \( \frac{1}{2} \) as many turns as your first coil, using a wire that is about half as long. If we had used the coil wound on the plastic tube, we would only get decent coupling if the additional (secondary) coil was as similar as possible to original (primary) coil. Because it has an air core (with a very low permeability), it is necessary to wind the second coil directly on top of the first coil and the lengths of the primary and secondary coils have to be the same. The good news for us is that, for the ferrite toroid, the primary and secondary coils can be located anywhere. Once you get your transformer to work, you should try to move the coils around a bit to see if anything significant happens.

• As with your first coils, twist the leads on your secondary coil, so that you know how to hook the transformer up.

• Remove enamel from the ends of your second coil so you can make good electrical contact.

• You have now built a transformer where one of your coils is the primary and one is the secondary.

• Calculate the value of \( a \) using the number of turns in the two coils. How should this value affect the magnitude of your output voltage relative to your input voltage?

• Measure the inductance of your secondary coil with the impedance bridge.

Find a Frequency Where your Transformer Works

We know from our PSpice simulation that this transformer will only work at certain frequencies.

• Hook one of your two inductors to the function generator, W1. Put a 1Ω, current-limiting resistor in series with the function generator because this circuit draws too much current at low frequencies. Set the amplitude of the function generator to 0.2V. Connect the other inductor to a load resistance of 47Ω. (50Ω isn’t a standard value.) Note that the transformer wires don’t always contact well with the protoboard. The most reliable way to hook up the transformer is with mini-grabbers and alligator clips. Note that 1Ω is not really small enough for all possible frequencies. Thus, you may find that the function generator will output very strange looking signals if it is asked to source too much current. The resistor should, however, be sufficient for answering all questions.

• Find a frequency for which the voltage ratio of your transformer works more-or-less as expected. This may be very high because your transformer does not have perfect coupling. Note that you will observe a change in amplitude of both the input and the output voltages as you increase the frequency. This is consistent with the behavior of the transformer you used in PSpice. The ratio of the voltages should be about 2:1 or 1:2 depending on which coil is your primary and which is the secondary.

• Your turns ratio won’t be exact and the coupling isn’t perfect, so don’t expect to see ideal results.

• Obtain an Analog Discovery plot of the output of your transformer at a frequency for which the voltage ratio equation is satisfied. Be sure that you display the input to the transformer on one scope channel and the output on the other. Save this plot and include it with your report.

When you are done with the transformer, save the parts. You will be reusing the wire in Project 1 to wind coils for the Beakman’s motor. The left over copper wire will be recycled and the ferrite cores must be returned.

Summary

In a transformer, the time-varying field is produced by an electromagnet with a time-varying current in it (coil #1) and sensed by a similar coil (coil #2). As far as a stationary coil is concerned, it is not possible to tell whether the time-varying magnetic field is produced by a stationary electromagnet or a moving permanent magnet, as long as the field produced oscillates in time. The time-varying magnetic field generates a voltage and current in the second coil. This is the basic principle of electrical generators. Electrical motors are mostly just generators run backwards. In the next project, we will build a motor using a permanent magnet and a coil like the ones you have just wound. You should recall the analysis done here when you do the project so you will be able to figure out the resistance and inductance of your motor coil.
Checklist and Conclusions

The following should be included in your experimental checklist. Everything should be labeled and easy to find. Partial credit will be deducted for poor labeling or unclear presentation. ALL PLOTS SHOULD INDICATE WHICH TRACE CORRESPONDS TO THE SIGNAL AT WHICH POINT.

Hand-Drawn Circuit Diagrams for all circuits that are to be analyzed using PSpice or physically built and characterized using your Analog Discovery board.

Part A (12 points)

Answer the following questions (both coils):
1. What value did you calculate for the resistance of the inductors? How did this compare to the measured resistance? (4 pt)
2. What values did you calculate for the inductance of the inductors? How did these compare to the measured inductance? (8 pt)

Part B (30 points)

Include the following plots (both coils):
1. Analog Discovery plots of the input voltage, the current through resistor R and LC voltage of the RLC circuit. (4 pt)
2. PSpice plot of the transient response of your RLC circuit with the values of L and R that produce results close to those observed in the experiment. Plot the same parameters as above. (4 pt)

Answer the following questions (for both coils):
1. Which of the three possible values for the current limiting resistor R, works best? (2 pt)
2. What are the measured values for the capacitance of your capacitor C and the resistance of your resistor R? (2 pt)
3. What are your best estimates for L_{est} for both coils from part A? That is, what are the values closest to what you measured with the impedance bridge? (2 pt)
4. What did you calculate for the expected resonant frequencies of your two circuits? (Please show what values you substituted into the equation.) (2 pt)
5. At what frequencies did you actually measure the resonances of your circuits? (2 pt)
6. What values did you get for L_{est} using the measured resonant frequency? (Please show what values you substituted into the equation.) (4 pt)
7. What values of L did you get when you adjusted your PSpice circuits to match the experimental response of the circuit you built? (4 pt)
8. Which of the inductance values you found was closest to the one measured with the impedance bridge? By what percentage was it off? Why do you think this gave you the best estimate? (4 pt)
Part C (16 points)

Include the following plots:
1. *PSpice* plot of the initial 3 traces of the ratios for the transformer when L1 = 4mH and L2 = 1mH. (2 pt)
2. *PSpice* plot of the three ratios that prove the transformer works at 1kHz and up. (2 pt)
3. *PSpice* plot of the three ratios that prove the transformer works at 100Hz and up. (2 pt)

Answer the following questions:
1. In what frequency range did the original transformer function as it should? (2 pt)
2. Use your output traces and the three design criteria equations to describe how you know these values satisfy the criteria for an ideal transformer. (2 pt)
3. What are the values of L1 and L2 you chose to obtain correct transformer operation for the range of 1kHz and up? (2 pt)
4. What are the values of L1 and L2 you chose to obtain correct transformer operation for the range of 100Hz and up? (2 pt)
5. What design advantages exist for circuits that are to work at higher frequencies rather than at lower frequencies? (2 pt)

Part D (14 points)

Include the following plot:
1. Analog Discovery plot with the input and output voltages of your transformers. (4 pt)

Answer the following questions:
1. What is the inductance of your secondary coil? (2 pt)
2. What is the value of \( a \) for your transformer? (2 pt)
3. At what frequency did your transformers work as expected? How do you know this? To answer this question, you will need to analyze the voltages observed and show how closely they satisfy the basic formulas for the transformer. (4 pt)
4. List at least 2 things that you could do to the design of your coil to improve your results? (2 pt)

Other (8 points)

1. Are all plots and figures included, labeled and are they placed in a logical order. Can they be fully understood without reading the associated text? (6 pt)
2. List member responsibilities. (2 pt)

List group member responsibilities. Note that this is a list of responsibilities, not a list of what each partner did. It is very important that you divide the responsibility for each aspect of the experiment so that it is clear who will make sure that it is completed. Responsibilities include, but are not limited to, reading the full write up before the first class; collecting all information and writing the report; building circuits and collecting data (i.e. doing the experiment); setting up and running the simulations; comparing the theory, experiment and simulation to develop the practical model of whatever system is being addressed, etc.
Summary/Overview (0 to -10 pts) There are two parts to this section, both of which require revisiting everything done on this experiment and addressing broad issues. Grading for this section works a bit differently in that the overall report grade will be reduced if the responses are not satisfactory.

1. Application: Identify at least one application of the content addressed in this experiment. That is, find an engineered system, device, process that is based, at least in part, on what you have learned. You must identify the fundamental system and then describe at least one practical application.

2. Engineering Design Process: Describe the fundamental math and science (ideal) picture of the system, device, and process you address in part 1 and the key information you obtained from experiment and simulation. Compare and contrast the results from each of the task areas (math and science, experiment, simulation) and then generate one or two conclusions for the practical application. That is, how does the practical system model differ from the original ideal?

Total: 80 points for experiment packet
0 to -10 points for Summary/Overview
20 points for attendance
100 points

Attendance (20 possible points)
2 classes (20 points), 1 class (10 points), 0 class (0 points)
Minus 5 points for each late.
No attendance at all = No grade for this experiment.
Experiment 3
Section: ______
Report Grade: ______
__________________________ Name
__________________________ Name

**Checklist w/ Signatures for Main Concepts**

For all plots that require a signature below, you must explain to the TA or instructor:
- the purpose of the data (using your hand-drawn circuit diagram),
- what information is contained in the plot and
- why you believe that the plot is correct.
Any member of your group can be asked for the explanation.

PART A: Making an Inductor
Questions 1-2

PART B: Measurement of Inductance

1. Analog Discovery plot of RLC at resonant frequency
2. *PSpice* of the transient RLC with value of L
Question 1 on the value of the current limiting resistor
Questions 2-8

PART C: Transformers

1. *PSpice* plot initial 3 traces of ratios when L1 = 4mH and L2 = 1mH
2. *PSpice* of three ratios at 1kHz and up
3. *PSpice* of three ratios at 100Hz and up
Questions 1-5

PART D: Making a Transformer

1. Analog Discovery plot of your transformer
Questions 1-4

Group Responsibilities

Summary/Overview