Class #16: Experiment
RC Circuits

Purpose: The objective of this experiment is to begin to become familiar with the properties and uses of the exponential function in circuits and engineering in general.

Background: Before doing this experiment, students should be able to
- Analyze simple circuits consisting of combinations of resistors.
- Build simple circuits consisting of combinations of resistors, inductors, capacitors, and op-amps on protoboards and measure input and output voltages vs. time.
- Make differential voltage measurements using Analog Discovery and Waveforms.
- Review the background from the previous experiments.

Learning Outcomes: Students will be able to
- Determine and validate mathematical expressions for the charging and discharging of capacitors and inductors through resistors that utilize the exponential function.
- Use series and small argument representations of exponential functions in RC and RL circuit analysis of both experimental and simulated circuits.
- Do a transient (time dependent) simulation of circuits using LTspice

Equipment Required:
- **Analog Discovery** (with Waveforms Software)
- **Oscilloscope** (Analog Discovery)
- **Function Generator** (Analog Discovery)
- **Protoboard**
- Resistors, Capacitors
- LTspice

Helpful links for this experiment can be found on the course website under Class #12.

Pre-Lab

Required Reading: Before beginning the lab, read over and be generally acquainted with this document and the other required reading materials.
Part A – Function Generator and Oscilloscope plots

For the first experiment, we need to set up a sinusoidal voltage. Set up Analog Discovery to generate a signal from function generator 1 (W1, GND) and measure it with scope channel 1 (1+, 1−). This can either be done by connecting the wires directly (Orange to Yellow and Orange/White to Black) or using a protoboard. Both options are shown in Figure A-1. Note that the connection pins that come with Analog Discovery can be broken apart, but it is helpful often to have some remain in pairs or other combinations.

Select the WaveGen (for the function generator) and the Scope (for the oscilloscope) from the Digilent Waveforms window. You should see the window similar to that shown in Figure A-2 (with ‘Basic’ selected) for the function generator and the window shown in Figure A-3 for the scope. The values for various parameters shown in both windows will likely be different. We will set those up next.

Function Generator:

- First we will set the frequency. The frequency of the function generator is adjusted as follows:
  - Make sure that you choose the channel or channels you are using in the “Channels” menu. The default choice when Waveforms starts is usually Channel 1 (AWG1). If this is the case, you do not need to use the Channels drop down menu. We only need one Function Generator in this experiment. Note that we will use Function Generator and Arbitrary Waveform Generator (AWG) pretty much interchangeably. This system can produce any time-varying signal, so it is an AWG, but we mostly use it to produce sine, triangular and square waves like a function generator.
  - Make sure that the “Frequency” box is checked and AWG configuration mode is “Basic”. Select the “Frequency” box or drag the “frequency bar” for Ch.1. Set it to display 1kHz.
  - Amplitude: Make sure the Amplitude is checked. Set the voltage amplitude to 200mV.
  - Your WaveGen window should look like Figure A-5. Make sure is showing.
Scope:
- First we set up the vertical and horizontal scales for the display. On scope channel C1, select the Volts/div to 100mV, the offset to 0 V. Uncheck C2, since we will not be using channel 2. The Time/div should be set at 200µs/div. The voltage and time scale settings are found on the right hand side of the scope window.
- To make a measurement, connect the source (W1) to scope input (1+) and scope input (1-) to ground. Make sure the connections you made above are good.
- When you are ready, press the “Run AWG1” button on the WaveGen and the “Run” button on the Scope. These are the ‘On’ buttons for the function generator and scope, respectively. If you cannot see a signal on the scope, double-check to make sure all of the settings are correct.
- It is useful to know how to copy plots and paste them into word. In the upper left hand corner, under the Files menu, select Export. Select the Images tab and you can save the Oscilloscope screen to a file or use the Copy to Clipboard option to put the image directly into your laboratory report. Save a copy of the Oscilloscope plot for your report.
- Change the frequency up or down as desired. How does this change the signal on the scope? The purpose of this step is to see what kind of signals this setup can produce. You should play around a little with different frequencies, voltage amplitudes, signal shapes, etc.
- Set WaveGen again so the display reads 1kHz and the amplitude is 200mV with no offset. Use the ‘Copy Window as Image’ option in the Edit drop down menu on the Scope and paste the image in your report document. Clearly label both the amplitude and period of the signal you have measured.
Connect your speaker and a 220μF capacitor between the function generator output and ground (similar to the LED circuit we have done previously). Can you hear tone (sound)? Change the amplitude of the sine wave to adjust the volume.

Let us investigate how our perception of loudness changes as the frequency of the sine wave is varied. With the sine wave amplitude fixed at your comfortable level, vary the frequency over the range from 100Hz to 10,000Hz. Try cycling through the following frequencies, without changing the signal amplitude: 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and 10,000Hz. Which frequency do you hear the loudest? Is there any variation among the members of your team?

**Experiment with the Equipment**

At this point, you will have put the function generator and scope through some basic tasks. Experiment with the other features of the function generator and see what happens. Some very interesting and annoying waves can be produced. Play around a little and then find a particular set of operating conditions that you find the most interesting. Under what circumstances might you experience some of the signals that you can generate with the function generator?
Part B – Discharging a Capacitor Through a Known Resistor

Background

The exponential function $e^x$ is the inverse function of the natural logarithm, $\ln(x)$. The exponential function is also one of the most amazing mathematical functions in that it is its own derivative! There is a lot to know about this function, so you should begin by at least glancing through the Wikipedia entry https://en.wikipedia.org/wiki/Exponential_function.

You will encounter it over and over in your engineering, science and math courses because it characterizes the response of damped systems (and what real system is not damped due to friction or resistance?) and can be used to characterize oscillating systems (a topic of the next class).

Experiment

We begin with a very simple experiment with an RC circuit. Connect two identical capacitors ($C_1$ & $C_2 = 47\mu\text{F}$) and a resistor $R_{\text{discharge}}$ in parallel on a protoboard. Also connect your Analog Discovery Channel 1 across the resistor and capacitors so you can monitor the voltage as a function of time. Add the ground wire to the bottom of the circuits, for a total of 3 connections to Analog Discovery. Choose the resistor so that the decay constant $RC$ is about 10 seconds.

Note that you must include the total capacitance and the total resistance in your evaluation of the time constant $RC$. That is, you must include the input resistance of Channel 1 in addition to both capacitors. You may have to work with a partner so that you have two large capacitors.

![Diagram](image1.png)

**Figure B-1**

1. What are the values of the capacitors you used and what is the value of $R_{\text{discharge}}$ so that the time constant is about 10 seconds?

Open the Analog Discovery Oscilloscope. Set the horizontal scale to show two full minutes of time. Remember that there are ten divisions in both the horizontal and vertical directions on the scope display, so that should tell you how much time you need per division. For the vertical scale, we would like to see a total of 5V, so choose 1V/Div because we want to see both voltages between ±5V. When you are taking data, you will see the plot will be created very slowly. Turn the triggering off because you will only be looking at one scan through the total time of 2 minutes.

Connect the black (ground) Analog Discovery wire to the ground side of the circuit. Do not connect the red (Power Supply) wire to anything yet. Start the oscilloscope. Now momentarily connect & then disconnect the red Power Supply to the ungrounded side of the capacitors (node A in the circuit shown in Figure A-1). You should see the voltage rise to around 5V and then slowly start to decay. Watch it decay for about 10-20 seconds so you are sure that the circuit is connected correctly. Then short out the capacitors with another wire by, again, only momentarily connecting it. You should see the voltage drop to zero. The charging and discharging process with the wire should be very fast. An example with a randomly chosen resistor is shown in Figure A-2. Your decay rate should be different. Repeat the process to be sure everything is working.
The data to be collected should nearly cover the full two minutes of the horizontal scale. Make sure that the scope is not running. Then, momentarily touch the battery red wire to Node A and, once the wire is no longer in contact with your circuit, start the scope. Allow the scope to run until the plot reaches the far right end of the scan then stop the scope. If you have done this well, you should have an exponentially decaying voltage across the entire time. Save a copy of your plot in your report and fully annotate it.

2. Then, using the cursor, determine the measured voltage at 11 evenly distributed points from 0 to 120 seconds and enter each value in the table below.

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<th>Time</th>
<th>Measured Voltage</th>
<th>Calculated Voltage</th>
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<tbody>
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<td>30</td>
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</table>

Next, we wish to compare the measured voltages to the mathematical function \( V(t) = V_o e^{-\frac{t}{\tau}} \) where \( \tau = RC \) is the decay constant. Use your measured voltage \( V(0) = V_o \) for the amplitude of the exponential function. Your previously determined values for R and C give the time constant.

3. Calculate the values for the exponential function at each measurement time and enter them into an Excel table. Once you have both columns filled in, load the data into Matlab and plot both the measured and the calculated curve. You can do this using the `plot(data)` command, where data is the name of your imported column in Excel. If you have done this correctly, the two curves should look very much alike.
Returning to Capacitors in Parallel
Before moving on to the next part of this experiment, we will use the set-up as is to explore how capacitors add in parallel and maybe in series. Start the experiment as before by charging the capacitors with the 5V source. Start the scope and observe the voltage for about 3 or 4 seconds. Then, remove one of the capacitors, being careful not to disturb any of the other connections. The total capacitance should now be about half what it was. While the capacitor in the circuit is discharging slowly through the resistor, short out the capacitor you removed from the circuit so that it no longer has any voltage or charge. Now connect the uncharged capacitor back in parallel with the charged capacitor. What do you observe? Why do you think the voltage changed when you reconnected the discharged capacitor? Repeat the process of removing, discharging and reconnecting the second capacitor. Once you reach the end of the 2 minute scan, stop the scope and save the plot to your report. Fully annotate the plot explaining the features that you see. Start the process over again by charging the two capacitors. After each 3-4 seconds, remove the second capacitor and then replace it after another 3-4 seconds, but do not discharge it when it is disconnected. Do this for the full 30 seconds, save your plot and fully annotate it. What is different this time and why? To help understand what you see is the second process, start it over with both capacitors and then, after 5 seconds, remove the resistor from the circuit. Wait another 3 seconds and then disconnect the orange positive wire for channel 1 of your scope (1+). Leave it disconnected for about 15 seconds and then reconnect the scope wire. Save the plot, annotate it and explain what you see.

Back to the Study of Charging and Discharging Capacitors
Next, we will change the circuit so it will do something similar but much faster. Replace the capacitors with 0.1µF and find a new value for the resistance that gives a time constant of about 1ms.). Open the function generator window and set it up to produce a sequence of square pulses (Square on the Wavegen link) that varies between 0V and 2V at a frequency of 100Hz. You will have to use both the amplitude and offset to do this. Note that Figure B-3 shows the function generator and resistor are now in series. Both scope channels are now to be used because, as usual, we want to monitor both the input and output voltages. Set up the scope to display 30-50ms of data and use as much of the vertical range as possible. The time and volts per division are your choice but your goal is to see as much detail as possible. Run the experiment and save an image of the data.

Figure B-3