On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Unless otherwise stated in a problem, provide 3 significant digits in answers. Read the entire quiz before answering any questions. Also it may be easier to answer parts of questions out of order.
Analog Discovery 2 partial set of Specifications -

**Analog Inputs**
- Channels: 2
- Channel type: differential
- Resolution: 14-bit
- Input impedance: $1\text{M}\Omega || 24\text{pF}$
- Scope scales: $500\text{uV}$ to $5\text{V/div}$
- Analog bandwidth with included flywires: $9\text{ MHz} @ 3\text{dB}$, $2.9\text{ MHz} @ 0.5\text{dB}$, $0.8\text{ MHz} @ 0.1\text{dB}$
- Input range: $\pm25\text{V}$ ($\pm50\text{V}$ diff)
- Input protected to: $\pm50\text{V}$
- Cursors with advanced data measurements
- Captured data files can be exported in standard formats
- Scope configurations can be saved, exported, and imported

**Arbitrary Waveform Generator**
- Channels: 2
- Channel type: single ended
- Resolution: 14-bit
- AC amplitude (max): $\pm5\text{ V}$
- DC Offset (max): $\pm5\text{ V}$
- Analog bandwidth with included flywires: $9\text{ MHz} @ 3\text{dB}$, $2.9\text{ MHz} @ 0.5\text{dB}$, $0.8\text{ MHz} @ 0.1\text{dB}$
- Slew rate (10V step): $400\text{V/µs}$
- Standard waveforms: sine, triangle, sawtooth, etc.
- Advanced waveforms: Sweeps, AM, FM.
- User-defined arbitrary waveforms: defined within WaveForms software user interface or using standard tools (e.g. Excel)

**Power Supplies**
- Voltage range: $0.5\text{V}…5\text{V}$ and $0.5\text{V}…-5\text{V}$
- $P_{\text{max}}$ (USB powered): $500\text{mW}$ total
- $I_{\text{max}}$ (USB powered): $700\text{mA}$ for each supply
- $P_{\text{max}}$ (AUX powered): $2.1\text{W}$ for each supply
- $I_{\text{max}}$ (AUX powered): $700\text{mA}$ for each supply
- Accuracy (no load): $\pm10\text{mV}$
- Output impedance: $50\text{mΩ}$ (typical)

**Voltmeters**
- Channels (shared with scope): 2
- Channel type: differential
- Measurements: DC, AC, True RMS
- Resolution: 14-bit
- Accuracy (scale $\leq 0.5\text{V/div}$): $\pm5\text{mV}$
- Accuracy (scale $\geq1\text{V/div}$): $\pm50\text{mV}$
- Input impedance: $1\text{MΩ} \parallel 24\text{pF}$
- Input range: $\pm25\text{V}$ ($\pm50\text{V}$ div)
- Input protected to: $\pm50\text{V}$
I. Voltage Dividers (20 points) As stated on the cover page: Round answers to 3 significant digits. Show formulas first and show your work. No credit will be given for numbers that appear without justification.

Consider the circuit diagram shown below. Rm represents the resistance of an ammeter (a current measuring device). The circuit with the dashed line is the model of an ohmmeter, which can be used to measure the value of an unknown resistance Rx.

Case 1: Given that when Rx = 0 Ω, the current through resistor Rm is 2 mA, i.e. \( i_m = 2 \text{ mA} \).

Case 2: Given that when Rx = 2000 Ω, the current through resistor Rm is 1 mA, i.e. \( i_m = 1 \text{ mA} \).

a. (2 pts) Find voltage between points A and B for each of the cases described above.

\[
\text{Case 1: } V_{AB} = i_m R_m = 20 \text{ mV} \\
\text{Case 2: } V_{AB} = i_m R_m = 10 \text{ mV}
\]

b. (1 pt) What is the equivalent resistance between points A and B, \( R_{AB} \)? (Express in terms of \( R_2 \))

\[
R_{AB} = R_2 \parallel R_m = \frac{10 R_2}{10 + R_2}
\]
c. (5 pts) Using voltage divider and your answer to parts a and b, develop a relationship between R1 and R2 for case 1.

\[ V_{AB} = 20 \text{mV} = V_1 \left( \frac{R_{AB}}{R_1 + R_{AB} + R_x} \right) \]

\[ \Rightarrow (R_1 + R_{AB})2 \text{mV} = R_{AB} \]

\[ R_1 + \frac{10R_2}{10+R_2} = \frac{5000R_2}{10 + R_2} \Rightarrow 10R_1 - 4990R_2 + R_1R_2 = 0 \]

**equation 1**


d. (8 pts) Using voltage divider and your answer to parts a and b, develop a relationship between R1 and R2 for case 2.

\[ V_{AB} = 10 \text{mV} = V_1 \left( \frac{R_{AB}}{R_1 + R_{AB} + R_x} \right) \]

\[ \Rightarrow \left( R_1 + \frac{10R_2}{10+R_2} + 2000 \right)1 \text{mV} = \frac{10R_2}{10 + R_2} \]

\[ 10R_1 - 7990R_2 + R_1R_2 = -20000 \]

**equation 2**
e. (4 pts) Solve the linear relationships derived in the previous parts, to determine the values of resistors R1 and R2 such that both case 1 and 2 are satisfied.

\[
\text{equation 1} - \text{equation 2} \\
3000R_2 = 20000 \Rightarrow R_2 = 6.667 \, \Omega
\]

Substitute in equation 1.

\[
10R_1 - 33268.33 + 6.667 R_1 = 0
\]

\[
\Rightarrow R_1 = 1996 \, \Omega
\]
II. Resistor Combinations, concepts and miscellaneous (20 points) Note: Page 2 of this quiz has background information.

The following circuit consists of 7 resistors, 1 DC voltage source and has 4 voltage markers placed at points A, B, C, and D. Note that the following questions are generally independent of each other.

a. (1 pt) Given that resistors have 5% tolerance, what is the 4-band color code for resistor R6?

Brown - Green - Red - Gold

b. (6 pts) Given that voltage at point A, \( V_A = 9 \text{V} \), find the voltages at point B and the source voltage \( V_1 \).

\[
V_A = V_1 \left( \frac{R_A}{R_A + R_1} \right) \Rightarrow \quad 9 = V_1 \left( \frac{4 \text{K}}{4 \text{K} + 8 \text{K}} \right)
\]

\[
\Rightarrow \quad V_1 = 27 \text{V}
\]

Voltage @ point B = \( V_B = V_A \left( \frac{4 \text{K}}{8 \text{K} + 4 \text{K}} \right) \)

\[
= 9 \left( \frac{4 \text{K}}{12 \text{K}} \right) = 3 \text{V}
\]

\[ V_B = 3 \text{V} \]
c. (2 pts) Given that voltage at point B, \( V_B = 4V \), find the current through resistor \( R_6 \).

\[
I_{R6} = \frac{V_B}{R_5 + R_6 + R_7} = \frac{4}{12K} = 0.333 \text{ mA}
\]

\[
\text{d. (2 pts) Given that the current through } R_6 \text{ is } 0.25 \text{ mA, find the power dissipated through resistor } R_7.
\]

\[
P_{R7} = I_{R7}^2 \cdot R_7 = (0.25 \text{ mA})^2 \times 2.5K = 0.15625 \text{ mW}
\]

e. (1 pts) A ceramic capacitor has a code “1 0 3” written on it. What is its capacitance?

\[0.01 \mu F\]

Now consider that the same resistive circuit is built on a protoboard, \( V_1 \) is set to 5V DC supplied by Analog discovery 2 board, and voltage at point A is being measured using scope channel 2.

\[f. \text{ (2 pts) What voltage would the analog discovery scope channel 2 measure at point A? Hint: Add the input resistance of Analog Discovery channel.}\]

\[
V_{A, \text{meas}} = 5 \left( \frac{4K \parallel 1M}{8K + 4K \parallel 1M} \right) = 1.662 \text{ V}
\]
g. (4 pts) You are now asked to change R1 from $8\,\text{k}\Omega$ to a new value such that the current through resistor R1 is close to $30\,\mu\text{A}$. Voltage source V1 is set to 5V. What is your new choice of R1?

\[ 5\,\text{V} = i \left( R_1 + R_A \right) \]

\[ \Rightarrow R_1 + 4\,\text{k}\Omega = \frac{5}{30\,\mu\text{A}} \]

\[ \Rightarrow R_1 = 162.67\,\text{k}\Omega \]

h. (2 pts) When defining the VSIN component (sinusoidal voltage source), indicate any two (of the four) parameters that are available when you place the part.

- Amplitude
- Frequency
- Offset
- AC
III. Filters & Transfer Functions (20 points) For this problem assume AC Steady State.

a) Use the circuit shown for this part.
1. Find the transfer function of the circuit shown. Simplify such that there are no fractions in the numerator or denominator of the transfer function. \( H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \) (6pts)

\[
\begin{align*}
\frac{1}{Z} &= \frac{1}{Z_1 + Z_2 + Z_3}\\
&= \frac{1}{\frac{1}{j\omega L} + \frac{1}{j\omega C}}\\
&= \frac{j\omega LC}{j\omega L + j\omega C}\\
H(j\omega) &= \frac{V_{out}}{V_{in}} = \frac{Z_3}{Z_1 + Z_2 + Z_3}\\
&= \frac{1}{j\omega L + j\omega C}\\
&= \frac{1}{j\omega L + \frac{1}{j\omega C}}
\end{align*}
\]

\[
H(j\omega) = \frac{1}{R + j\omega L + j^2\omega^2 C}
\]

2. Determine the amplitude and phase of the transfer function for the circuit for very small frequency and for very high frequency. Do not take this to 0 or infinite Hz. (4pts)

\[
\begin{align*}
\omega \to 0 & : H(j\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} \\
&= \frac{R}{R + \frac{1}{j\omega C}} \\
&= \frac{R}{R - j\omega C}
\end{align*}
\]

\[
\omega \to \infty & : H(j\omega) = \frac{1}{R + j\omega L + \frac{1}{j\omega C}} \\
&= \frac{1}{-j\omega C}
\]

3. Redraw the circuit and simplify the circuit for operation at low and high frequency. For this part you take it to extremes, low frequency is dc operation. High is approaching infinity. (2pts)

Low Frequency
\[
L = \text{short}, \quad C = \text{open}
\]

High Frequency
\[
L = \text{open}, \quad C = \text{short}
\]

Note: \( \text{mag} = 1, \quad \text{phase} = 0 \)

You must include units.

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b) In the circuit shown, Z1 and Z2 represent a single component, either a resistor, capacitor or inductor. Complete the table below by entering a Y for yes is that configuration of Z1 and Z2 would be a low pass or a high pass filter. (5pts)

<table>
<thead>
<tr>
<th>Z1</th>
<th>Z2</th>
<th>Low Pass (Y or N)</th>
<th>High Pass (Y or N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>C</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>C</td>
<td>R</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>R</td>
<td>R</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>R</td>
<td>L</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>L</td>
<td>R</td>
<td>Y</td>
<td>N</td>
</tr>
</tbody>
</table>

c) Draw a low pass filter using just a 1kΩ resistor and a 0.2uF capacitor. Label the input and output.
And calculate the corner frequency for this circuit. (3pts)

\[
V_{in} \quad \text{to} \quad 0.2\mu F \quad V_{out} \\
\omega_c = \sqrt{ \frac{1}{2} } \omega_0 \\
0 = \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^3 \times 2 \times 10^{-6}}} \\
\omega = 5 \times 10^3 \\
f = \frac{\omega}{2\pi} = 796 Hz
\]
IV – Phasors and Transformers (20 points)

1) Assume an ideal transformer with full coupling.
   a. For the given information, determine the turns ratio, \( a \). And determine the ratios
      \( \frac{V_{out}}{V_{in}}, \frac{I_{out}}{I_{in}} \) and the transformer input impedance \( R_{in} \). (\( R_{in} = \frac{V_{in}}{I_{in}} \)) (6 pts)

      \[
      a = \frac{L_2}{L_1} = \sqrt{\frac{160}{10}} = 4
      \]

      \[
      \frac{V_{in}}{V_{out}} = \frac{a}{1} = 4
      \]

      \[
      R_{in} = \frac{800}{16} = 50 \Omega
      \]

      \[
      \frac{I_{out}}{I_{in}} = \frac{1}{4} = 0.25
      \]

      \[
      \text{a} = 4 \quad (2pt)
      \]

      \[
      \frac{V_{out}}{V_{in}} = 4 \quad (1pt)
      \]

      \[
      \frac{I_{out}}{I_{in}} = 0.25 \quad (1pt)
      \]

      \[
      R_{in} = 500 \quad (2pt)
      \]

   b. Solve for \( V_{in} \) (voltage across the input terminals of the ideal transformer) and \( V_{out} \), the
      voltage across the output terminals and the of the ideal transformer. Assume the phase
      of \( V_4 \) is zero degrees and give the answer in the form of \( v(t) = V_1 \cos(\omega t + \phi) \) (2 pts)

      \[
      R_{in} = 50 \Omega
      \]

      \[
      V_{in} = \frac{500}{600+100} \cdot 12 = 10 V
      \]

      \[
      V_{in} = 10 V \quad (2pt)
      \]

      \[
      V_{out} = V_{in} \cos(2\pi \cdot 10^4 t)
      \]

      \[
      V_{out} = 10 \cos(2\pi \cdot 10^4 t) \quad (2pt)
      \]

   c. Above you were told to assume that the transformer is ideal. For that to be valid, the
      impedance of the primary inductor should be much larger than the source resistance. Is
      that valid in this case? Explain or justify. Would it be valid if the signal source was
      at 60Hz? (3pts)

      \[
      j \omega L = (2\pi)(10^{-1})(10^{-2}) = 6282
      \]

      \[
      6282 \gg 100 \Omega \quad \text{reasonably true}
      \]

      \[
      \text{so yes. valid}
      \]

You must include units.
2) Phasors: This circuit shown has 2 complex impedances, Z1 and Z2, connected as shown.
Given: \( V_{in} = 10V \angle 0^\circ \) and the voltage across Z2 is measured to be \( V_{Z2} = 6V \angle 30^\circ \)

a. Write \( V_{in} \) and \( V_{Z2} \) in Cartesian form. (2pts)
\[
10V \angle 0^\circ = 10 + j0V = 10V \\
6V \angle 30^\circ = 6 \cos \theta + j6 \sin \theta = (5.2 + j3)V
\]

b. Determine \( V_{Z1} \), the voltage across Z1 in Cartesian and polar form (3pts)
\[
\vec{V_{in}} = \vec{V_{Z1}} + \vec{V_{Z2}} \quad \text{use Cartesian} \quad \vec{V_{Z1}} = \vec{V_{in}} - \vec{V_{Z2}} \\
= (10 - (5.2 + j3))V = (4.8 - j3)V
\]
\[
4.8 - j3 = (\sqrt{(4.8^2 + 3^2)}) \angle \theta = \tan^{-1} \frac{-3}{4.8} = 5.66 \angle -32^\circ
\]

c. If Z2 is a 1k\( \Omega \) resistor, and only a resistor, what is the current through Z2 in both polar and Cartesian form? (3pts)
\[
\vec{I} = \frac{\vec{V_{in}}}{Z_{eff}} = \frac{6 \angle 30^\circ}{1k} = \frac{6 \angle 30^\circ}{1000} = (5.2 + j3) \text{mA}
\]

3) Give the names of 2 of the people teaching this course. This can be first names or last names and can be the professors and/or the teaching assistants. (1pt)

See bottom of page - or

Zhihuo, Liang, Ruixuan, Yan, James, Cassidy

You must include units.

Spelling doesn’t matter

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