Traffic Flow Control in Vehicular Communication Networks

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Abstract—Control of conventional transportation networks aims at bringing the state of the network (e.g., the traffic flows in the network) to the system optimal (SO) state. This optimum is characterized by the minimality of the social cost function, i.e., the total cost of travel (e.g., travel time) of all drivers. On the other hand, drivers are assumed to be rational and selfish, and make their travel decisions (e.g., route choices) to optimize their own travel costs, bringing the state of the network to a user equilibrium (UE). In this paper we study the SO and UE of the future connected vehicular transportation network, where users consider the travel cost and the utility from data communication when making their travel decisions. We leverage the data communication aspect of the decision making to influence the user route choices, driving the UE state to the SO. We propose an algorithm for calculating the SO state, and the values of the data communication utility that drive the UE to the SO. This result provides a guideline on how the communication system operator can adjust the parameters of the communication network (e.g., data pricing and bandwidth) to achieve the optimal social cost. We also discuss the insights on a secondary optimization that the operator can conduct to maximize its own utility without deviating the transportation network state from the SO.

I. INTRODUCTION

In transportation systems, the prospect of wide-scale connected autonomous vehicles (CAVs) is approaching its realization, due to the advances in control and communication. In a traditional transportation network, the drivers make travel decisions (e.g., route choices, travel timing) that minimize the transportation related costs, such as travel time, travel distance, etc. With the emergence of CAVs that form vehicular ad-hoc networks (VANET), data communication network connectivity is not only going to be an important factor for enabling vehicular control, but also going to change the CAV users traveling behavior. Some CAV users will expect the type of data communication service they are accustomed to at their homes and offices. Thus, CAV users may choose routes not only depending on travel time and costs, but also based on the quality of data service that will be provided on the route, since this directly affects their productivity and/or quality of life. CAV users may choose to take a route with longer travel time in order to have a better data communication network connectivity (just as travelers may choose a more expensive hotel, or a less convenient hotel location, if it offers a high speed WiFi connection). Evidence of this behavior has been recently reported in [1], where data connectivity affects the route choice of (human) drivers. Hereafter in this paper, we will refer to the travel decision makers (i.e., drivers or CAV users) as “users”.

Travel decision making among users can be analyzed in a game theoretical setting [2], [3]. The travel decision of each user impacts the state of the transportation network, and thereby may also impact the transportation costs of all users. The Nash equilibrium of this game is referred to as the Wardrop equilibrium or the user equilibrium (UE). Thus, UE occurs if no user can be better off by unilaterally changing his travel decision. In a traditional transportation network, the UE state is achieved if every user tries to minimize his travel cost (e.g., travel time).

In contrast to UE, we can also consider the system optimal (SO) state. The system optimal state occurs if the social cost function, i.e., the total of the travel costs of all users, is minimized. In general, assuming that the users are selfish and rational, it is known that UE and SO are not the same. This phenomenon is sometime referred to as the Braess’ paradox [4], [5]. The ratio between the social costs at UE and at SO is called the price of anarchy (PoA) [6].

In this paper, we study the user-equilibrium (UE) and the system-optimum (SO) in the vehicular communication network. In the vehicular communication network, the interdependency between the traffic condition and the communication network and the users valuation of the travel cost and the data communication lead to a different UE. For example, a heavy traffic flow in a road segment leads to a longer travel time, but the large number of cache-enabled vehicles can potentially lower the communication cost due to the increase of the cache hit probability. On the other hand, as more users choose to use the road segment with low travel cost and low communication cost, traffic congestion may occur and the network connectivity may be saturated, which will discourage other users from using this road segment.

The interaction between the transportation network and the users decisions has been thoroughly studied. However, the effect of the additional factor in the decision making, i.e. data communication, remains unknown. In this work, we model the influence of the traffic condition and the data service on users route planning. We leverage the communication network to push the user equilibrium (UE) to the system optimum (SO). Specifically, we make the following
contributions:
- We incorporate the data communication aspect in the modeling of users trip planning, and characterize both the the SO and the UE states of the vehicular communication network.
- We derive the sufficient condition for the SO and UE to coincide (and therefore eliminate the price-of-anarchy). We exploit the communication network to optimize the system with respect to the social cost function.
- We present the insights on how the system operator can manage the communication network via a secondary optimization in order to provide a desired data service.

The remainder of this paper is organized as follows. In Section II we review the related work on the data communication and user behavior in vehicular communication networks. In Section III we present the model of the transportation network and the communication network, and propose a general cost function. In Section IV we discuss the SO and the UE, and derive the sufficient condition for steering the UE to the SO by leveraging the communication cost. A case study of a network in the Capital District is demonstrated in Section V. We conclude our work and discuss future extensions in Section VI.

II. RELATED WORK

In the systems and control community, there are some recent papers that discuss the price of anarchy (PoA) in transportation networks. For example, the recent work by Wang et al [7] seeks to eliminate the PoA (PoA=1) by imposing scaled marginal-cost road pricing on the a transportation network with a single origin-destination pair. The very recent work by Zhang et al [8] seeks to derive the users travel cost functions from city-wide real traffic data. Knowledge of the cost functions’ is key in deriving both the SO and UE.

Incorporating the communication network in the transportation networks enables a wide range of applications [9], for example, interactive entertainment, urban sensing [10], collision avoidance in platoon formation [11], improving the intersection capacity via platoons [12], etc. A wealth of research focuses on vehicle-to-vehicle (V2V) communication [13] in the transportation networks. [14] presents the network-layer V2V connectivity requirements in one-way and two-way street scenarios. In the two-way street scenario, the store-carry-forward routing model is used, where the packets are relayed by the vehicles moving in the opposite direction. A physical-layer perspective is considered in [14] which quantifies the maximum number of hops that ensure a desired bit error rate in the V2V communication. Vehicle-to-infrastructure (V2I) communication is an alternative to the V2V communication, which is also well studied in a variety of context. For example, [1] adopts the cognitive radio technology where the vehicles are the secondary users and the TV base stations are the primary users. Data rate is taken into consideration for route selecting. It provides a guideline on which spectrum sharing mode to use based on the intensity of TV base stations.

It is expected that CAV users’ needs for and valuation of data service vary based on their socioeconomic characteristics and trip-related features. There is wealthy literature on people’s behavior in response to transportation service and data communication service. These studies, however, reside in different research fields. Transportation studies typically focus on traveler behavior including mode choice, route choice, departure time choice, etc. For traveler route choice, the main focus ranges from the effects of road pricing [15], fuel costs [16], congestion level [17], reliability [18], land use [19], to advanced traveler information system [20]. User responses to cost and quality of data communication service have been investigated in a wide spectrum of fields including information system, psychology, and business management. Studies have looked into effects of perceived fee [21], user prior experience and habits [22], social influence [23], perceived monetary value, among others. In a recent literature review, [24] summarized main areas and methods on research related to people’s data communication behavior in the past decade. However, no existing study have explored the user behavior when facing the joint choice of transportation and data service, which is the key feature of CAV users.

III. SYSTEM MODEL

In this section, we first present the transportation network model and the communication network model in section III-A. Then we discuss the costs incurred by traffic and by data communication in section III-B. For ease of reference, related notations are shown in Table I.

A. Network model

The transportation network consists of a number of road segments which we refer to as links. Infrastructure related parameters, such as the free-flow speed, stay the same through out a link. The set of all links in the transportation network is denoted by $A$. Each vehicle in this transportation network travels from a origin to a destination via a set of links. We refer to a set of links that connect an origin and a destination as a route. The set of all possible origin-destination pairs (O-D pairs) is denoted by $N$. There are one or more routes between each O-D pair. The set of routes between the O-D pair $i$ is represented by $K_i$. We assume that the trip rate $q_i$ for every O-D pair $i \in N$ can be drawn from the historical data, and thus is known to the operator a-priori. Without loss of generality, we only consider one-way traffic, i.e. all links are directed, since any two-way or multi-lane link can be equivalently replaced by multiple one-way directed links. The indicator variable $\delta_{i,k}(a)$ is defined such that $\delta_{i,k}(a) = 1$ if link $a$ is passed by the traffic along route $k \in K_i$. Otherwise $\delta_{i,k}(a) = 0$. Fig. 1 shows an example network that consists of two origins and two destinations. Each O-D pair can potentially be traversed using multiple routes. For example, the O-D pair $(r_1, d_1)$ can be traversed using 1-2 or 3-4-2.

The vehicles travel along the links and form the link flow vector $x$, where the entry $x_a$ represents the traffic flow on link $a$. Similarly, the route flow vector $y$ represents the
number of vehicles that choose certain routes, where the entry $y_{i,k}$ denotes the flow on route $k$ that connects the O-D pair $i$.

All routes should satisfy the flow conservation constraints, i.e. the sum of the route flows along all routes that connect an O-D pair equals the O-D trip rate, and the flow on a link equals the sum of the flows that enter the link [3]. The flow conservation constraints are given by:

$$
\sum_{k \in K_i} y_{i,k} = q_i, \quad \forall i \in N,
$$

(1)

$$
y_{i,k} \geq 0, \quad \forall i \in N, k \in K_i,
$$

(2)

$$
x_a = \sum_{i \in N} \sum_{k \in K_i} \delta_{i,k}(a)y_{i,k}, \quad \forall a \in A.
$$

(3)

Note that $x$ is a linear function of $y$.

Each user is a participant both in the transportation network and in the communication network. We envision the use of vehicles as nodes with the network interfaces that support V2V communication. Vehicles can communicate with each other by broadcasting in an ad-hoc manner. Besides network interfaces on the vehicles, there are wireless Road Side Units (RSUs) located along the roads, which enables the vehicles to access the Internet. Both the V2V connection and the V2I connection have a considerable amount of network delay, due to the high mobility and the rapidly changing topology of the vehicular communication network [25]. The bandwidth allocation vector is denoted by $b$ with the entry $b_a$ representing the bandwidth allocated to link $a$. The operator may charge the user in link $a$ for the data service at the price $v_a$. The data prices on all links form the data price vector $v$.

B. Cost Functions

We associate each route with a cost. As aforementioned, users in the vehicular communication network do not only value travel cost, such as travel time and travel distance, they also need data service for a better travel experience. Therefore, the route cost consists of two parts: travel cost and communication cost. We do not constraint the travel cost to be any specific type of disutility. Instead, we represent the travel cost of a link $a$ as a function of the traffic flow vector, i.e. $T_a(x)$. We also define the route travel cost $T_{i,k}(x)$ as the sum of the link travel cost along the route:

$$
T_{i,k}(x) = \sum_{a \in A} \delta_{i,k}(a)T_a(x).
$$

(4)

The communication cost is a measure of communication network performance, and is a function of the traffic flows and other relevant network parameters. The communication cost involves content downloading delay, data price charged, etc. We represent the route communication cost $C_{i,k}$ as a function of the traffic flow vector $x$, the bandwidth allocation vector $b$, and the data price vector $v$. The specific form of the communication cost function is left to our future work.

The route cost is formulated as follows. When a user chooses which route to take, they are presented with the travel cost (e.g. travel time) and the communication cost (e.g. network delay). Without specifying how user preference would affect their trade-off between the travel cost and the communication cost, we denote the route cost of route $k$ that connects the O-D pair $i$ by $J_{i,k}(\{T_{i,k}(x), C_{i,k}(x, b, v)\})$. The specific form of this route cost function is left to our future work after we explore the user behavior and their valuation of data and transportation service.

![Fig. 1: A transportation network consisting of two origins $r_1$ and $r_2$, and two destinations $d_1$ and $d_2$. Links are indexed by the numbers next to them. Node $n_1$ is the intersection of link 3, 4 and 6. Node $n_2$ is the intersection of link 2, 4, and 5.](image)

**TABLE I: Table of Notations**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>set of links (road segments)</td>
</tr>
<tr>
<td>$N$</td>
<td>set of all origin-destination (O-D) pairs</td>
</tr>
<tr>
<td>$K_i$</td>
<td>set of all routes connecting O-D pair $i \in N$</td>
</tr>
<tr>
<td>$q$</td>
<td>trip rate vector with entry $q_i$ denoting the trip rate between O-D pair $i \in N$</td>
</tr>
<tr>
<td>$x$</td>
<td>link flow vector with entry $x_a$ denoting the flow on link $a \in A$</td>
</tr>
<tr>
<td>$y$</td>
<td>route flow vector with entry $y_{i,k}$ denoting the flow on route $k \in K_i$ that connects O-D pair $i \in N$</td>
</tr>
<tr>
<td>$b$</td>
<td>bandwidth allocation vector with entry $b_a$ denoting the bandwidth allocated to link $a \in A$</td>
</tr>
<tr>
<td>$v$</td>
<td>data price vector with entry $v_a$ denoting the data price charged on link $a \in A$</td>
</tr>
<tr>
<td>$\delta_{i,k}(a)$</td>
<td>$=1$, if link $a$ is on route $k$ between O-D pair $i$; $=0$, otherwise</td>
</tr>
<tr>
<td>$T_a(\cdot)$</td>
<td>travel cost of link $a \in A$</td>
</tr>
<tr>
<td>$C_{i,k}(\cdot)$</td>
<td>communication cost of route $k \in K_i$ that connects the O-D pair $i \in N$</td>
</tr>
<tr>
<td>$J_{sys}$</td>
<td>system cost</td>
</tr>
<tr>
<td>$J_{i,k}(\cdot)$</td>
<td>cost of route $k \in K_i$ that connects O-D pair $i \in N$</td>
</tr>
</tbody>
</table>

IV. PROBLEM FORMULATION

In this section, we first formulate the system optimal state and the user equilibrium state. Then we derive the sufficient condition for the overlapping state, a state at which the traffic flow solutions for the SO and the UE are equal. Lastly, a guideline is provided for the system operator on the management of the communication network and on a secondary optimization for a desired data service.

A. System Optimal

From the system’s perspective, a low total travel cost improves the social welfare. For example, a low average travel time or average travel distance can alleviate the traffic congestion, reduce air pollution, and be more energy efficient. The state where the total travel cost is minimized is referred to as the System Optimal state (SO). The link flow at the SO is the solution of the following minimization problem:

$$
\min \ J_{sys} = \sum_{a \in A} x_a T_a(x),
$$

(5)
which is subject to the flow conservation constraints (1) through (3). We adopt the typical assumption that the system cost function \( J_{sys} \) is a convex function of \( x \) (e.g., [3]). This assumption implies that the SO is the unique and local minimum of \( J_{sys} \). Also assume that the SO occurs at an interior point of the positive orthant of \( y \) (i.e., \( y \) is strictly positive), so that all routes are used at the SO. If this is not the case, then routes with zero traveler can be simply removed from \( K_i \) without any loss of generality.

The condition of the local optimality of (5) under the flow conservation constraint (1) through (3) can be derived using Lagrange multiplier. The first-order condition for the solution of the above formulation is, for all \( i \in N, k \in K_i \)

\[
y_{i,k}(T_{i,k}(x) - u_i) = 0
\]

where \( u_i \) is a positive Lagrange multiplier, and \( T_{i,k}(x) \) denotes the marginal travel cost of route \( k \) that connects the O-D pair \( i \), which is the sum of the marginal travel cost of all links on the route. The physical meaning of the marginal travel cost of a link is the marginal contribution of an additional user who uses the link to the total travel cost of the network. So we have

\[
\tilde{T}_a(x) = T_a(x) + \sum_{b \in A} x_b \frac{\partial T_a(x)}{\partial x_b},
\]

\[
\tilde{T}_{i,k}(x) = \sum_{\alpha \in A} \delta_{i,k}(\alpha) \tilde{T}_a(x).
\]

The first-order condition (6) can be interpreted as: at SO, the marginal travel costs on all routes connecting the same O-D pair are the same. Since we assume that all routes are taken at the SO, the first-order condition of the SO can be written as, for all \( i \in N, k \in K_i \),

\[
\sum_{\alpha \in A} \delta_{i,k}(\alpha) \left( T_a(x) + \sum_{b \in A} x_b \frac{\partial T_b(x)}{\partial x_a} \right) = u_i.
\]

Solving for \( x \) from (7) and the flow conservation constraints gives us the traffic flows at the SO.

### B. User Equilibrium

If users behave non-cooperatively, in steady state the system reaches a user equilibrium (UE), where no user can benefit by unilaterally changing routes. We assume that the UE occurs at an interior point of the positive orthant of \( y \) (i.e., \( y \) is strictly positive). From the Wardrop’s first principle, at UE, the costs of all routes that connect the same O-D pair are the same. Therefore, in the traditional transportation network, the necessary condition for UE is given as follows: for any \( k, l \in K_i \),

\[
T_{i,k}(x) = T_{i,l}(x).
\]

In the vehicular communication network where the users also take into consideration the communication cost when planning their trips, the UE deviates from that in the traditional transportation network. As described in Section III-B, the route cost \( J_{i,k} \) is a function of the travel cost and the communication cost. Therefore, (8) becomes: for all \( i \in N, k \in K_i \),

\[
J_{i,k}(T_{i,k}(x), C_{i,k}(x, b, v)) = \lambda_i,
\]

where \( \lambda_i \) is a positive constant for the O-D pair \( i \).

### C. When UE and SO Overlap

The SO is regarded as the ideal state: a closer UE to the SO in terms of the traffic flows results in a higher social welfare. Consider a simple network that consists of a single O-D pair connected by two routes. Each of the routes has only one link with travel cost \( T_1 = 7 + 6x_1 + 4x_2 \) and \( T_2 = 1 + 2x_1 + 10x_2 \) respectively. Fig. 2 shows how the system cost varies with the traffic flow on a link under different trip rates in this network. Given the trip rate \( q \), one can solve for the traffic flows at the SO and at the UE. Connecting the SO points (UE points) under all possible trip rates gives the SO trace (UE trace), which is shown by the black dashed line (red dashed line) in Fig. 2. We note that the UE deviates from the SO as the trip rate increases.

![Fig. 2: System cost v.s. traffic flow under different trip rates.](image-url)
numerical values of the communication costs of every routes are obtained. Then, the operator can monitor the link traffic flows and adjust the route cost by tuning the communication network related parameters in real time, such as bandwidth and data price, so that the USO condition is satisfied. We call such technique as UE-SO Overlapping via Communication cost (USOC). USOC guarantees that the social welfare is maximized at equilibrium even if the users behave non-cooperatively. If the user profile information is known to the operator, the communication cost function will be user-specific. For example, if two users have different profiles and data preferences, they may be presented with different data prices for the same route. However, the USO condition has to be satisfied regardless of the actual form of the communication cost function.

The USO condition decouples the traffic flow control of the transportation network and the management of the communication network. Given the specific forms of the cost functions, the communication cost can be solved for using the USO condition. However, the solution only gives the numerical values of the communication costs under the overlapping state, and does not specify how to tune the communication network related parameters in order to achieve such costs. This provides the system operator with the opportunity to conduct certain secondary optimization according to their interests for a desired data service. For example, the operator can minimize/maximize the total bandwidth allocated to the network subject to the USO condition.

V. CASE STUDY

In this section, we consider a specific case where the cost function takes on a certain form, and we apply the USOC technique on an example network. Specifically, we assume that the route cost is the weighted sum of the travel cost and the communication cost, i.e.

\[ J_{i,k} = \alpha T_{i,k} + (1 - \alpha) C_{i,k}, \alpha \in (0, 1). \]

From (10), it is sufficient that for every route \( k \) that connects the O-D pair \( i \),

\[ C_{i,k} = \sum_{a \in A} \delta_{i,k}(a) \left( T_a(x) + \frac{1}{1 - \alpha} \sum_{b \in A} x_b \frac{\partial T_a(x)}{\partial x_b} \right). \]

(11)

Fig. 3 is obtained from Google Maps, which shows part of the transportation network in the Capital District around Albany, NY. The network under consideration is a grid consisting of the grey and blue links. To simplify the calculation, we assume that there are two O-D pairs in this network: drivers from Latham (node A) either go to Downtown Albany (node C) or Delmar (node E). Therefore, all traffic on link 1 and link 4 is from node A. Also assume that the drivers will only use the links that are indexed in Fig. 3. The links marked as blue (links 1, 2, and 5, denoted by 1-2-5) form a possible route from node A to node E. There are two other routes for the O-D pair (A,E): 4-6, and 4-3-5. Similarly, there are two routes for the O-D pair (A,C): 1-2 and 4-3.

We obtain the traffic flow data from the NYS Traffic Data Viewer [26]. The Traffic Data Viewer (TDV) is a GIS web application for viewing the annual average daily traffic. According to the data that the TDV averages over several weeks in Spring 2005 and 2006, the traffic flow on link 1 and link 4 are 3786/h, and 4827/h respectively. There are theoretical models and practical methods to estimate the trip rate, but for demonstration, we assume that half of the drivers from node A are traveling to node C, and the other half are traveling to node E. So the trip rates for O-D pair (A,E) and (A,C) are both 4306.5/h. Fig. 4 shows a graph representation of the network topology in Fig. 3. We assume that the link travel cost depends on the traffic flow only on that link.

The weight towards the travel cost is assumed to be 0.6, so the route cost function is

\[ J_{i,k} = 0.6T_{i,k} + 0.4C_{i,k}. \]

We combine the first-order condition (7) with the flow conservation constraints to solve for the SO:

\[
\begin{align*}
(0.11 + \frac{2x_4}{10^4}) + (0.08 + \frac{2x_6}{5 \times 10^4}) &= u_{(A,E)} \\
(0.11 + \frac{2x_4}{10^4}) + (0.09 + \frac{2x_5}{5 \times 10^4}) + (0.05 + \frac{2x_6}{5 \times 10^4}) &= u_{(A,E)} \\
(0.05 + \frac{2x_3}{5 \times 10^4}) + (0.09 + \frac{2x_5}{10^4}) + (0.05 + \frac{2x_6}{5 \times 10^4}) &= u_{(A,E)} \\
(0.05 + \frac{2x_5}{5 \times 10^4}) + (0.09 + \frac{2x_2}{10^4}) &= u_{(A,C)} \\
(0.11 + \frac{2x_3}{10^4}) + (0.09 + \frac{2x_6}{5 \times 10^4}) &= u_{(A,C)} \\
x_1 + x_4 &= 8613, \ x_2 + x_6 = 4306.5 \\
x_4 = x_3 + x_6, \ x_1 = x_2
\end{align*}
\]
Solving the above linear system gives

\[
\begin{align*}
x & \approx [3605 \ 3605 \ 1403 \ 5008 \ 702 \ 3605]^T \\
T & \approx [0.12 \ 0.13 \ 0.12 \ 0.16 \ 0.06 \ 0.15]^T
\end{align*}
\]

Substituting the above solution into (11) yields

\[
C \approx [0.52 \ 0.48 \ 0.62 \ 0.57 \ 0.62]^T
\]

where the elements \( C_1 \) through \( C_5 \) are the communication costs of routes 1-2, 4-3, 1-2-5, 4-3-5, and 4-6, respectively. At the overlapping state, the route costs for the O-D pair (A,C) and (A,E) are approximately 0.36 and 0.43 respectively, regardless of what route the users choose. The operator can then use these route communication costs to adjust the bandwidth and data price in real time. For example, the following optimization problem needs to be solved given the solution in (12):

\[
\min_{\mathbf{v}} \max_{\mathbf{b}} f(\mathbf{v}, \mathbf{b}) \quad \text{s.t.} \quad C = [0.52 \ 0.48 \ 0.62 \ 0.57 \ 0.62]^T,
\]

where \( f(\cdot) \) is the objective function.

VI. CONCLUSION AND FUTURE WORK

In this paper, we model the user trip planning when both the traffic condition and the data communication influence user trip decision. A sufficient condition is derived for overlapping the UE with the SO, which provides a guideline on how the system operator can adjust the network parameters to achieve the optimal social welfare even if the users are non-cooperative. There are a number of open problems in this work. The specific form of the cost function is unknown, which is worth deeper analysis after surveying user preference and behavior. Even if the cost function does not have a concrete form, some greedy methods can be applied to adjust the communication cost in order to approximately approach the overlapping state, which is a promising direction of our future work. Moreover, the SO can be reformulated to incorporate not only the travel cost, but also other types of social welfare (e.g. fair distribution of the communication resources). Lastly, the convergence of the traffic flow in real world networks using the USOC technique warrants further investigation.

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