ADAPTIVE: A Dynamic Index Auction for Spectrum Sharing with Time-Evolving Values

Alhussein A. Abouzeid

Rensselaer Polytechnic Institute

abouzeid@ecse.rpi.edu

January 23, 2014
Wireless Spectrum

- Refers to the part of the electromagnetic spectrum corresponding to radio frequencies < around 300 GHz
- Spectrum is a finite and valuable resource
  - only 50 MHz remain un-assigned
  - The Federal Communications Commission (FCC) auctions for about 4% of US spectrum raised $78 billion since 1994.
- Indirect value of radio spectrum: 5-10% of US economy (≈ 1.4 trillion/year)
Spectrum Utilization

- FCC reports that many of the allocated spectrum bands are idle most of the times or not used in some areas.

Dynamic Spectrum Sharing

- A promising approach to improve spectrum utilization

- Realized by cognitive radio technology
  - A radio that can change its transmitter parameters according to the interactions with the environment in which it operates.

- Unlicensed secondary users (SU) are allowed to utilize the radio spectrum owned by a primary owner (PO).
Why Auctions?

- The seller is not assumed to know any prior information about the valuation of items to the buyers.
- Auctions can be designed to maximize buyers’ valuations.
- Requires minimum interaction between seller and buyers.
Spectrum Auctions

- In the simplest form of a spectrum auction
  - There is a set of SUs (buyers) who bid to obtain channel access
  - A PO (auctioneer) who collects these bids and determines the winner (or winners) and payments

- Two components of every auction:
  - the allocation rule
  - the payment rule

- Main objectives:
  - revenue maximization (optimality) for the auctioneer
  - social welfare maximization (efficiency)
Existing spectrum auctions assume that SUs have static and known values for the channels. In reality, however, SUs do not know the exact value of channel access a priori, but they learn it over time.

Here, we study spectrum auctions in a dynamic setting where SUs can change their valuations based on their experiences with the channel.
We propose ADAPTIVE, a dynamic index auction for spectrum sharing with time-evolving values, that maximizes the social welfare and has desired economic properties.
The PO (a base station or an access point) is willing to auction its idle channel to the SUs.
System Model

In the ADAPTIVE mechanism:

- PO is the auctioneer
- SUs are the bidders
- Channel is the auctioned item
- $\theta_i$ denotes the type of SU $i$
  - A real number reflecting monetary value of channel access for SU $i$
  - Captures the urgency for channel access
- $e_{i,t}$ denotes SU $i$’s experience at time $t$
  - We consider SU’s experience as SNR of the channel
  - SU’s experience evolves only when he gets the channel, otherwise its experience does not change
  - An SU’s experience at the instants that it gets the channel evolves in a Markovian model
SU’s valuation for the channel is a stationary function of its type and experience:

\[ v(\theta_i, e_{i,t}) = \theta_i \cdot B \log(1 + e_{i,t}) \]

Where \( B \) is the channel bandwidth.

The function \( v \) takes into account both the channel quality experienced by SUs and SU’s monetary value that reflects urgency for channel access.
The ADAPTIVE Mechanism

- At each time step, SUs report \((\theta_i, e_i, t)\) to the PO who determines two outputs:
  - The channel allocation denoted by \(Q\) that contains \(q_{i,t} \in \{0, 1\}\) determining the winner at time \(t\)
  - The payment of SU \(i\) at time \(t\) denoted by \(p_{i,t}\)

- The expected future social welfare at time \(t\) can be defined as:

\[
S(\theta, e_t) = \max_{Q \in Q} \mathbb{E} \left[ \sum_{t' = t}^{\infty} \sum_{i} \delta^{t' - t} q_{i,t'} v(\theta_i, e_i, t') \middle| \theta, e_t \right]
\]

Where \(0 < \delta < 1\) is the common discount factor.
Efficient Allocation Policy of ADAPTIVE

- We cast the channel allocation problem into a *multi-armed bandit* problem.

- In a multi-armed bandit problem, there is an operator that chooses to operate exactly one of the machines at each time step. The chosen machine generates a reward and updates its state. The operator’s objective is to maximize the sum of rewards.
The channel allocation problem in ADAPTIVE can be transformed into a multi-armed bandit problem.

- an SU $\rightarrow$ an arm in the bandit model
- SUs’ valuations $\rightarrow$ rewards generated by pulling arms
- Allocating the channel to an SU $\rightarrow$ pulling an arm
- experience update of the winning SU $\rightarrow$ State change in the bandit model

Now, we can use the Gittins index policy to solve the efficient allocation problem.
Efficient Allocation Policy of ADAPTIVE (Cont’d)

- The PO gives the channel to the SU with the highest Gittins index:

\[ G_i(\theta_i, e_i, t) = \max_{\tau_i} \mathbb{E} \left[ \frac{\sum_{t'=t}^{\tau_i} \delta^{t'-t} \nu(\theta_i, e_i, t')}{\sum_{t'=t}^{\tau_i} \delta^{t'-t}} \mid \theta_i, e_i, t \right] \]

- Gittins index of each SU can be computed independently in polynomial time.
The Payment Rule

- We specify payments such that each SU’s utility coincides with its marginal contribution to the social welfare.

- The winning SU \( i \) at time \( t \) pays:

\[
p_{i,t} = (1 - \delta) S_{-i}(\theta, e_t)
\]

Where \( S_{-i}(\theta, e_t) \) is the expected future social welfare without SU \( i \).
The ADAPTIVE mechanism has the following economic properties:

- *Periodic Ex Post Incentive Compatibility*; for every bidder and at any time, truth-telling is the best response to the truthfulness of the other bidders.

- *Periodic Ex Post Individual Rationality*; bidders do not suffer as a result of participating in the auction.
Numerical Results

- We compare the performance of ADAPTIVE which is a dynamic valuation auction with the well-known Vickrey auction (also called second price auction) as the representative of static auctions.

- We set the common discount factor, $\delta$, to 0.7 and change the number of SUs from 3 to 21. Each setting is run 500 times in MATLAB.
Numerical Results

Figure: Social welfare Vs the number of SUs.

Figure: Discounted social welfare Vs the number of SUs.
Numerical Results

Figure: Revenue of the PO Vs the number of SUs.

Figure: Average utilities Vs the number of SUs.
Numerical Results

Figure: Revenue of the PO Vs $\delta$, with 12 SUs.
Conclusion

- ADAPTIVE is the first spectrum auction that considers dynamically evolving values.
- ADAPTIVE runs in polynomial time and results in efficient allocation with desired economic properties.

A possible direction for future work:
- Extend ADAPTIVE to a dynamic population model that will be a dynamic population and dynamic valuation mechanism.
Acknowledgements:

- Joint work with Mehrdad Khaledi, PhD Candidate, RPI, khalem@rpi.edu
- Work partially funded by NSF.
Thank You!