Heavy-tailed GARCH Models: Pricing and Risk Management Applications in Power Markets

Shi-Jie Deng
Industrial and Systems Engineering
Georgia Institute of Technology
Agenda

• Background and motivations.
• Quantile-based GARCH Models.
  – Parameter inference
  – Applications
    • Electricity derivatives pricing
    • Risk management measures
• Semi-parametric Estimation of Confidence Intervals of Conditional Quantiles of GARCH Models
  – Normal approximation and data-tilting
  – Applications
• Research Discussion and Conclusion.
Significance and Broader Impacts

• Methodology
  – Non-gaussian fat-tailed GARCH models based on quantile distributions
  – A two-step procedure for parameter inference: quasi-MLE and quantile-based estimation
  – A one-step semi-nonparametric estimation scheme of high/low quantiles and their confidence intervals

• Applications
  – Modeling financial time series data as well as energy (e.g., electricity price and load) data
  – Financial derivatives/contracts pricing
  – Risk management measures (e.g., CVaR)
Background and Motivations

• Rapid developments of power markets in mid-1990s
  – Power Exchange/ISO in California.
  – PJM, NY, NE power pools.

• Market setbacks since 2000
  – Fallen or financially distressed power merchants
  – Dropping liquidity in power exchange/OTC markets

• The nature of incompleteness in energy (power) markets
  – Almost non-storable underlying
  – Limited physical supply and inelastic demand
  – Tremendous price and quantity volatility (e.g., price spikes)
  – Limited ability in hedging quantity risks
Background and Motivations (con’t)

• Redesign of power markets
  – FERC Standard Market Design
  – Development of futures/contract markets

• Risk management needs
  – Independent power producers hedge their production.
  – Power marketers quantify, monitor and control trading risks in wholesale and retail markets.

• Trading, Asset valuation, and project selection and financing
  – Pricing and risk management tools to support trading
  – Evaluation of potential investment opportunities in power generation
  – Support for project financing
Implied Volatility of Call (Sept.) Options at Cinergy
(Inferred from Black-Scholes formula based on broker quotes for calls and forwards.)

Marginal Cost

Resource stack includes:
- Hydro units;
- Nuclear plants;
- Coal units;
- Natural gas units;
- Misc.

Marginal Cost

瞎
Empirical findings

- Extensive research in empirical finance demonstrates that typical financial return data often exhibit the following stylized features:

  - Volatility clustering
  - Various tail behaviors (Fat-tailed, Skew to the left, Unbalance-tailed)
  - The tails are often heavier than Normal, but lighter than student
  - The heaviness of the tails varies greatly with the sampling frequency of the return data.
Literature on Energy Price Modeling and GARCH Modeling

• Energy commodity spot price models.
  – Schwartz (J.of Fin 1997)
  – Miltersen and Schwartz (JFQA 1998)
  – Hilliard and Reis (JFQA 1998)

• Electricity spot price models and electricity derivatives.
  – Kaminski (Risk Book 1997)
  – Barz and Johnson (1998)
  – Deng (PSERC 1998)
  – Mount and Ethier (PSERC 1998)
  – Deng, Sun and Meliopoulos (PSERC 2003)

• ARCH/GARCH modeling and option pricing.
  – Engle (Econometrica 1982), Bollerslev (J. Econ. 1986)
  – Nelson (Econometrica 1991)
  – Hall and Yao (Econometrica, to appear)
  – Duan (Math Fin. 1995), Heston and Nandi (RFS 2002)
Gaussian GARCH models

\[ X_t = \sigma_t Z_t, \]
\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \]

where \( Z_t \) is an iid series with \( N(0,1) \) law.
Quantile-based GARCH Models
(Deng and Jiang, 2003)

• Exhibit heavy, flexible and asymmetric tail behaviors
• Enable maximum likelihood estimation (MLE)/quasi-MLE combined with quantile-based estimation
• Have explicit conditional quantile functions
• Allow efficient computation in pricing and risk management applications
  – Easy and fast simulation
A Two-step Estimation Scheme

- Step 1: Quasi-MLE for estimating GARCH coefficients
  - Hall and Yao (Econometrica, to appear)

\[ L_\nu(a, b, c) = \sum_{t=\nu}^{n} \left\{ \frac{X_t^2}{\tilde{\sigma}_t^2(a, b, c)} + \log \tilde{\sigma}_t^2(a, b, c) \right\}, \]

- Step 2: Quantile-based estimation for obtaining parameters for the innovation term.
Empirical Estimation

- MLE estimation of GARCH(1,1) coefficients:
  - $\alpha_0 = 0.0023$; $\alpha_1 = 0.0587$; $\beta = 0.9252$.
- Q-Q Plot of the innovation term.
  - Daily electricity price: PJM Western Hub

\[\begin{align*}
\alpha &= 0.92191 \\
\beta &= 0.77793 \\
\delta &= 0.56062 \\
\mu &= -0.21724
\end{align*}\]
Empirical Estimation (con’t)

• Q-Q Plot of unconditional marginal distrib.
Option Pricing

Simulation is an effective method for pricing path-dependent options such as Asian options and American-style options.

The key step is to evaluate the following form

\[ E^Q(\Phi(S)) \]

where \( Q \) is a risk-neutral probability measure. By law of large numbers, this is given by

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \Phi(S^{(i)})
\]

where \( S^{i}, i = 1, 2, \ldots \) are iid drawings of the underlying asset process \( S \).
VaR and ES

Let $X$ be the random return of some asset or portfolio, its $VaR^\alpha$ (value at risk) at level $\alpha$ (the minimum loses incurred in the $\alpha\%$ worst cases of our portfolio) is defined as follows.

$$
x^\alpha(X) = \sup\{x | P[X \leq x] \leq \alpha\}
$$

$$
VaR^{(\alpha)}(X) = -x^{(\alpha)}(X)
$$

Note that if the quantile function of $X$ is $q(u)$ then, $VaR^{(\alpha)}(X) = -q(\alpha)$

ES: The expected loss incurred in the in the $\alpha\%$ worst cases of a portfolio. The ES of $X$ at level $\alpha$ can be calculated by

$$
ES^{(\alpha)} = -\frac{1}{\alpha} \int_0^\alpha q(u)du
$$

where $q(u)$ is the quantile function of $X$. 
Application: Value Energy Contracts

• Energy (electricity) derivatives are complex financial instrument
  – Physical characteristics of underlying
  – Path-dependent and American-style (exercisable at any time)

• Examples:
  – Tolling agreements.
    • Independent power producers hedge operational risks.
    • Power merchants implement asset-light operations.
    • Fossil-fueled power producers hedge output risks.
  – Swing contracts
  – Gas storage contracts
Application: Pricing Methodology

- European-style financial contracts
  - Simulation

- American-style path-dependent financial contracts
  - Dynamic programming regression approximation: Longstaff and Schwartz (2001), Tsitsiklis and Van Roy (2001)
  - Simulation
Interval Estimation for the Conditional Quantile of Fat-tailed GARCH Models (Chan, Deng, Peng, and Xia, 2003)

- One-step conditional quantile estimation of heavy-tailed GARCH models
- Characterization of confidence intervals for conditional quantiles
Model Specification

- Heavy-tailed GARCH Model

\[ X_t = \sigma_t \varepsilon_t, \sigma_t^2 = c + \sum_{i=1}^{p} b_i X_{t-i}^2 + \sum_{j=1}^{q} a_j \sigma_{t-j}^2, \]

- Heavy-tail in innovation

\[ \varepsilon_t \sim G(x) \quad \text{(cdf of } \varepsilon_t) \]

\[ 1 - G(x) \sim c_1 x^{-\gamma}, \quad G(-x) \sim c_2 x^{-\gamma}, \quad \text{for } x \text{ large and } \gamma > 2 \]

- 100α% one step ahead conditional VaR

\[ x_{\alpha,n} = \inf\{x : P(X_{n+1} \leq x|X_{n+1-k}, k \geq 1) \geq \alpha\}. \]
Estimation

• Likelihood function (quasi-MLE)

\[
L_{\nu}(a, b, c) = \sum_{i=\nu}^{n} \left\{ \frac{X_t^2}{\tilde{\sigma}_t^2(a, b, c)} + \log \tilde{\sigma}_t^2(a, b, c) \right\},
\]

• Tail index

\[
\hat{\gamma} = \left\{ \frac{1}{k} \sum_{i=1}^{k} \log \frac{\hat{\epsilon}_{m,m-i+1}}{\hat{\epsilon}_{m,m-k}} \right\}^{-1},
\]

where \( \hat{\epsilon}_t = X_t / \tilde{\sigma}_t(\hat{a}, \hat{b}, \hat{c}) \) and \( \hat{\epsilon}_{m,1} \leq \cdots \leq \hat{\epsilon}_{m,m} \) denote the order statistics of \( \hat{\epsilon}_\nu, \cdots, \hat{\epsilon}_n \)

• Estimator by method I:

\[
\hat{x}_\alpha^0 = (1 - \alpha)^{-1/\hat{\gamma}} \left( \frac{k}{m} \right)^{1/\hat{\gamma}} \hat{\epsilon}_{m,m-k},
\]

\[
\hat{x}_{\alpha,n} = \tilde{\sigma}_{n+1}(\hat{a}, \hat{b}, \hat{c}) \hat{x}_\alpha^0
\]
Theorem 1

- Suppose regularity conditions hold and

\[ k = k(m) \to \infty, \frac{k}{m} \to 0, \sqrt{k}A(m/k) \to 0, \]
\[ n^{-1}\lambda_n/A(m/k) \to 0, \log\left(\frac{k}{m(1 - \alpha)}\right)/\sqrt{k} \to 0 \]

as \( n \to \infty \). Then

\[
\frac{\hat{\gamma}\sqrt{k}}{|\log\left(\frac{k}{m(1 - \alpha)}\right)|} \left\{ \frac{x_{\alpha, n}}{x_{\alpha, n}} - 1 \right\} \xrightarrow{d} N(0, 1).
\]
Confidence Intervals

- **Method I**: Normal approximation method.
  - Based on Theorem 1, a confidence interval with level $\beta$ for $x_{\alpha,n}$ is
  
  $$I^n_\beta = (\hat{x}_{\alpha,n}(1 + \frac{z_\beta}{\hat{\gamma}\sqrt{k}}\log \frac{k}{m(1-\alpha)})^{-1}, \hat{x}_{\alpha,n}(1 - \frac{z_\beta}{\hat{\gamma}\sqrt{k}}\log \frac{k}{m(1-\alpha)})^{-1}),$$

  with $z_\beta$ satisfies $P(|N(0,1)| \leq z_\beta) = \beta$.

- **Method II**: Data tilting method.
  
  $$(2m)^{-1}L(x_{\alpha,n}) = \min_w D_t(w)$$

  - A confidence interval with level $\beta$ for $x_{\alpha,n}$ is
    
    $$I^t_\beta = \{x_{\alpha,n} : L(x_{\alpha,n}) \leq u_\beta\},$$

    where $u_\beta$ is the $\beta$ level critical point of $\chi^2(1)$. 

\[\text{Georgia Institute of Technology}\]
\[\text{School of Industrial & Systems Engineering}\]
Data Sets and Their Autocorrelation

- Daily electricity price return and load change in PJM

![Log returns of PJM Real-time LMP](image1)

![Daily adjusted load](image2)
Data Sets and Their Autocorrelation

- 1-Month PJM forward price vs. SP 500
Comparison with Gaussian GARCH

<table>
<thead>
<tr>
<th>k=30, alpha=0.95</th>
<th>Method I</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.115</td>
<td>0.074</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.033</td>
<td>0.019</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.042</td>
<td>0.009</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.067</td>
<td>0.069</td>
</tr>
<tr>
<td>SP500</td>
<td>0.041</td>
<td>0.056</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k=60, alpha=0.95</th>
<th>Method I</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.118</td>
<td>0.074</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.033</td>
<td>0.019</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.042</td>
<td>0.009</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.065</td>
<td>0.069</td>
</tr>
<tr>
<td>SP500</td>
<td>0.048</td>
<td>0.056</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k=100, alpha=0.95</th>
<th>Our method</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.122</td>
<td>0.074</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.028</td>
<td>0.019</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.033</td>
<td>0.009</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.067</td>
<td>0.069</td>
</tr>
<tr>
<td>SP500</td>
<td>0.054</td>
<td>0.056</td>
</tr>
</tbody>
</table>
Comparison with Gaussian GARCH

<table>
<thead>
<tr>
<th>k=30, alpha=0.99</th>
<th>Method I</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.030</td>
<td>0.024</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.018</td>
<td>0.030</td>
</tr>
<tr>
<td>SP500</td>
<td>0.014</td>
<td>0.017</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k=60, alpha=0.99</th>
<th>Method I</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.024</td>
<td>0.024</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.016</td>
<td>0.030</td>
</tr>
<tr>
<td>SP500</td>
<td>0.014</td>
<td>0.017</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k=100, alpha=0.99</th>
<th>Method I</th>
<th>Conditional Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>daily load</td>
<td>0.017</td>
<td>0.024</td>
</tr>
<tr>
<td>1-m PJM forward</td>
<td>0.005</td>
<td>0.014</td>
</tr>
<tr>
<td>3-m PJM forward</td>
<td>0.000</td>
<td>0.005</td>
</tr>
<tr>
<td>PJM real time LMP</td>
<td>0.016</td>
<td>0.030</td>
</tr>
<tr>
<td>SP500</td>
<td>0.015</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Research Questions

• Characteristics of Power price and load
  – Extremely high volatility
  – Jumps and spikes

• Quantile-based GARCH Models.
  – Parameter inference
  – Applications
    • Electricity derivatives pricing
    • Risk management measures computation

• Risk-neutralized processes corresponding to the quantile-based GARCH models.

• Efficient simulation and dynamic programming algorithms for asset pricing problems.