Singular Perturbation Method For Smart Building Temperature Control Using Occupant Feedback

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ABSTRACT

In this work we propose a framework for incorporating occupant feedback towards temperature control of multi-occupant spaces, and analyze it using singular perturbation theory. Such a system would typically have to accommodate occupants with different temperature preferences, and incorporate that with thermal correlation among multiple zones to obtain optimal control for minimization of occupant discomfort and energy cost. In current practice, an acceptable temperature set point for the occupancy level of the zone is estimated, and the control law is designed to maintain temperature at the corresponding set point irrespective of the changes in occupancy and the preferences of multiple occupants. Proposed algorithm incorporates active occupant feedback to minimize aggregate user discomfort and total energy cost. Occupant binary feedback in the form of hot/cold or thermal comfort preference input is used by the control algorithm. The control algorithm also takes the energy cost into account, trading it off optimally with the aggregate occupant discomfort. For convergence to the optimal, sufficient separation between the occupant feedback frequency and the temperature dynamics of system is necessary; in absence of which, the occupant feedback provided do not correctly reflect the effect of current control input value on occupant discomfort. Under sufficient time scale separation, using singular perturbation theory, we establish the stability condition of the system and show convergence of the proposed solution to the desired temperature that minimizes energy cost plus occupant discomfort. The occupants are only assumed to be rational in that they choose their own comfort range to minimize individual thermal discomfort. Optimization for a multi-zone building also takes into account the thermal correlation among different zones. Simulation study with parameters based on our test facility, and experimental study conducted in the same building demonstrates performance of the algorithm under different occupancy and ambient conditions.

Key Words: Smart building temperature control, singular perturbation method, occupant comfort feedback, human-centered control algorithm.

I. INTRODUCTION

As a multitude of electronic gadgets make their way into our day-to-day lives, technological innovations are redefining the notion of human comfort, requiring it to be more personalized and adaptive. At the same time, changes in our general living style and
expectations are accompanied with a surge in our individual energy demands. This calls for the design of smart systems that can achieve desired human comfort levels without putting additional pressure on energy resources. Energy usage in buildings, both residential and commercial, account for one of the major sources of energy consumption both within the US and worldwide. Data suggests that nearly 40% of the total energy consumption in US, and 20% of the total energy consumption worldwide, is attributed to the residential and commercial building usage [1]. More than 75% of current electricity consumption is also due to the usage in buildings. Hence, global energy efficiency cannot be obtained without devising ways for energy efficient operation of buildings. Heating, ventilation, and air conditioning (HVAC) system is one of the major energy consumers in buildings. Numerous design and solution approaches have been proposed for the control and efficiency of HVAC systems. The approaches taken so far can be broadly classified into those focusing on optimal energy usage through variable electricity rates [2], [3], [4], [5], active and passive thermal energy storage [4], [5], and more recently model predictive control (MPC) approach exploiting information through weather forecast [6], [7], [8], [9], [10], [11]. The building and its occupants combined is a single entity for which the HVAC control needs to be optimized. More recent work, based on this philosophy, have focused on using occupant feedback at binary/multiple levels towards determining the direction of temperature adjustment and perform thermal management using average user vote [12], [13], [14].

A smart building energy control system that seeks and takes into account the comfort level feedback of its occupants to determine the HVAC control input is likely to be more efficient. This can however be a challenging task in large commercial buildings such as schools, libraries, offices etc. which have a very diverse collection of occupants. Achieving energy efficiency in a building taking into account the comfort levels of its occupants needs a new and unique approach towards problem solution. Our work is loosely related to Thermovote [12] and a more recent work [13] that seek user feedback at binary/multiple levels, and determines the direction of temperature adjustment based on the average user vote. Some prior work [15] have tried to link the cost of building environment control to the occupant efficiency, health and satisfaction. Other recent work [16], [17] have evaluated thermal complaint behavior using one-class classifier. In a recent work [18], Jazizadeh et al. implemented a complementary control strategy for the HVAC control based on a fuzzy predictive model to learn occupant comfort profiles. Zhao et al. used a simulation study, against a baseline rule-based algorithm, to tie occupant subjective thermal comfort feedback with MPC control algorithm for the active HVAC system [19]. Vellei et al. [20] conducted a six week study to establish the feasibility of enhanced occupant and energy savings based on comfort feedback. Chen et al. [21] developed an MPC algorithm based on the Actual Mean Vote (AMV) approach developed in [22], in order to achieve a balance between thermal comfort and energy savings.

In this work, unlike most of the existing studies, we pose the effective temperature control of the building formally as an optimization problem that takes into account both the temperature dynamics as a function of the energy input and the discomfort function of the building occupants [23]. We prove that our control solution converges to the optimal trade off point between energy cost and occupant discomfort. No such guarantees are known to hold for the existing approaches proposed in the related studies. To further contrast it with our recent work [24], [25], the approach in this work is much simpler for the occupants as they just have to share their upper and lower limit temperatures (or provide binary input) without having to worry about pricing or penalty feedback from the building system. The collaborative (collaboration between occupants, declaring thermal comfort range in self-interest) and coordination (the thermostat control is set in a coordinated manner, taking into account the multi-zone thermal correlations) aspect of our proposed framework also distinguishes it more other smart thermostat solutions such as those by Google-Nest [26] and [27]. In one of our related experimental work [28] we demonstrated energy savings on top of a learning thermostat.

In this paper we present a user feedback driven control mechanism to optimally trade off the overall energy cost with the aggregate occupant discomfort. Major contributions of our work can be summarized as follows. (i) A novel algorithm is developed and analyzed using gradient optimization and singular perturbation theory. Simple feedback of the form of ‘heat up’ or ‘cool down’ is generated from the occupant’s device (such as desktop or smart phone/tablet) based on the occupant’s preferences, which is incorporated into the control algorithm. The occupants can share their comfort range limit points pro actively and also provide instantaneous binary feedback as ‘hot or cold’ if needed. (ii) We consider a multi-zone building, and optimize its thermal environment taking into account the thermal correlations between
its different zones. (iii) For convergence of our control algorithm to the optimal, sufficient separation between the occupant feedback frequency and the dynamics of the system is necessary. We use singular perturbation theory to analyze the system, with temperature evolution on a faster time scale and occupant feedback on a relatively slower time scale. With such time scale separation, we establish the stability condition under which the proposed control algorithm achieves convergence to a desired temperature that minimizes the sum of total energy cost and the aggregate occupant discomfort. (iv) We run simulations (using parameters of our smart building testbed) as well as conduct experimental study in the testbed to establish viability and evaluate the performance of our proposed algorithm. The experimental study is conducted in our Watervliet test facility but it uses occupant comfort preference range based on our related experimental work [28], as the experimental facility is not equipped to host occupants in real-time.

A high level work flow of the proposed algorithm is presented in Figure 1. The flow starts with the occupant comfort perception based on which occupants can provide their upper and lower limit temperatures using a smart phone application. This enables the occupants to declare their comfort proactively. Based on the limits the system generates hot/cold feedback. At any point if an occupant wants to change their previous limits they can just provide binary input around the limit boundaries and the system incorporates that. Feedback corresponding to each of the occupant, based on their zone is consolidated and the control input is adequately determined. This is then used to update the HVAC system of the building accordingly. Note that the proposed algorithm is intended for any multi-occupant building, whether residential or commercial. The key contribution of our work around collaboration among multiple occupants and coordination among multiple thermal zones makes it applicable to all multi-occupant spaces.

The rest of the paper is structured as follows. In Section II we develop the problem and propose the singular perturbation based solution. In Section III we establish the condition for stability of our proposed solution. In Section IV we present results corresponding to the simulation and experimental study based on our test bed, and finally conclude in Section V.

II. SINGULAR PERTURBATION APPROACH

In this section, we first describe our optimization objective (Section 2.1) and occupant discomfort modeling (Section 2.2). We then present the widely popular RC building model used to express our control law (Section 2.3), and finally introduce our singular perturbation based solution in Section 2.4.

2.1. Objective Function

Our overall minimization objective (overall cost) is the sum of two terms: (i) energy cost (i.e., cost of heating/cooling), and (ii) aggregate discomfort cost of the occupants. For the development of objective function consider a building with \( m \) zones. The energy cost \( E(u) \) is assumed to be a convex function of the control input vector \( u \). For the sake of definiteness we express \( E(u) \) in the following quadratic form:

\[
E(u) = \frac{1}{2} u^T \Gamma u, \tag{1}
\]

where \( \Gamma \) is a positive definite matrix, referred to as energy cost parameter in our analysis, and \( u \in \mathbb{R}^m \) is the vector of heating (cooling) inputs into different zones of the building. Note that our framework and analysis also extends to other convex energy costs (heat cost could have been taken as the 1-norm or 2-norm of the heat input as well). Next, let \( S_j \) denote the set of all occupants in zone \( j \), and \( \rho = \sum_{j=1}^{m} |S_j| \) be the total number of occupants in the building. Also let \( G_s \) denote the (convex) discomfort function of occupant \( s \) in zone \( j \). Then the aggregate occupant discomfort cost is expressed as \( \sum_{j=1}^{m} \sum_{s \in S_j} G_s(y_j) \), where \( y_j \) denotes the \( j^{th} \) element of temperature vector \( y \), or the temperature of zone \( j \). Our minimization objective can then be expressed as:

\[
U(u, y) = \frac{1}{2} u^T \Gamma u + \gamma \sum_{j=1}^{m} \sum_{s \in S_j} G_s(y_j). \tag{2}
\]
In (2), $\gamma$ is a scalar constant that defines the relative weight provided to the aggregate occupant discomfort, as compared to the energy cost and is referred to as user weight parameter in our analysis. The matrix $\mathbf{I}$ and scalar $\gamma$ together determine the weight of the energy cost relative to the aggregate discomfort cost of the occupants. It can be based on the actual energy cost, or can alternately be explicitly defined by the building operator (home owner) based on the building type to relatively weigh the aggregate occupant comfort and overall energy cost.

2.2. Occupant Discomfort Modeling

Occupant thermal comfort modeling has been extensively researched and can be summarized into the following three major approaches: (1) the chamber study model, based on mapping thermal comfort from environmental and personal factors to a 7-level comfort value scale, viz. the Predicted Mean Vote - Predicted Percent Dissatisfied (PMV-PPD) [29], [30]; (2) human body physiology based models such as Gagge’s core to skin model [31], Stolwijk’s comfort model for multi-human segments [32], and Zhang et al.’s sensation on human body segments [33]; and (3) adaptive comfort models developed in field study, viz. Humphreys [34] and [35]. Recent work based on thermal complaint behavior using one-class classifier [36], [37] have also been presented. However, existing work mainly focus on average thermal comfort models instead of individual comfort modeling. Such group comfort models only capture average behavior and are not particularly useful in maximizing aggregate comfort for multi-occupant spaces, with individual thermal preferences differing from each other.

In the current work, therefore, we take into consideration discomfort functions of the occupants individually - modeled as convex quadratic functions of temperature variation based on the PMV-PPD model. Our model captures the difference across occupants in their comfortable temperature range. For simulation and experimental study we adopt occupant discomfort function of the form:

$$G_s(y_j) = \begin{cases} (y_j - y_s^U)^2 & \text{if } y_j > y_s^U, \\ 0 & \text{if } y_s^L \leq y_j \leq y_s^U, \\ (y_s^L - y_j)^2 & \text{if } y_j < y_s^L, \end{cases}$$

(3)

where $y_s^U$ and $y_s^L$ are the upper and lower limit temperatures respectively of the user $s$ located in zone $j$.

Note that these values are not explicitly indicated by the user, but conveyed implicitly to the system through user feedbacks. In our simulation we assume that a user is required to submit feedback in a simple binary form, indicating whether he/she is feeling hot or cold in the current setting. The lack of feedback from the user is assumed to be an indication that the user is currently comfortable. Based on the received user feedback the system constantly estimates the comfortable range (discomfort function) for each user and controls the temperature accordingly. We also recognize the fact that user feedback around the boundaries of its comfortable temperature range can be somewhat imprecise. This is taken into account in our simulation by associating a probability distribution with the user feedback in a $\delta$ range around the comfort range boundaries. More precisely, if $y_s^U$ is the upper limit of the comfort range of user $s$ in zone $j$, the feedback of the user changes from 0 (no feedback, or the user feels comfortable) to 1 (user feels hot) linearly as $y_j$ varies in the range $[y_s^L, y_s^U + \delta]$. A similar probability distribution is also associated for user feedback in the range $[y_s^L - \delta, y_s^U]$, where $y_s^L$ is the lower limit of the users comfort range. For this simulation study we have used the value of $\delta$ as 1. Note that this practical consideration would also account for slight changes in the upper and lower limit of comfortable temperature with the users mood and attire. This is depicted in Figure 2.

![Fig. 2. Algorithm for estimating thermal comfort limits from occupant binary feedback.](image)

2.3. Building RC Model

Several building modeling strategies have been proposed in past, which include the finite element method based model [38], lumped mass and energy transfer model [39], graph theoretic model based on electrical circuit analogy [40], [41], [42], [43] and more recently thermal networks [44]. The electrical
analogy approach to modeling multiple interconnected zones reduces the heat transfer model to an equivalent electrical circuit network. The model can be further modified to include building occupancy, room and heating equipment dynamics [43], [45]. In this paper we take this electrical circuit analogy approach, and combine it with occupant discomfort feedback modeling.

A building is modeled as a collection of interconnected zones, with temperature dynamics evolving according to a lumped heat transfer model. In the lumped heat transfer model, a single zone is modeled as a thermal capacitor and a wall is modeled as an RC network. This results in the standard lumped 3R2C wall model [41]. The heat flow and thermal capacitance model can be written for all the thermal capacitors in the system, with \( T \) as the temperature of the \( i \)th capacitor. Consider the system to have \( n \) thermal capacitors and \( l \) thermal resistors. Taking the ambient temperature \( (T_\infty) \) into account, we can write the overall heat transfer model of the system with \( m \) zones as [46]:

\[
CT = -DR^{-1}D^T T + B_0 T_\infty + Bu,
\]

where \( T \in \mathbb{R}^n \) is the temperature vector (representing the temperature of the thermal capacitors in the 3R2C model), \( u \in \mathbb{R}^m \) (as mentioned previously) is the vector of heating (cooling) inputs into different zones of the building, and \( B \in \mathbb{R}^{n \times m} \) is the corresponding input matrix. Also, note that \((T, u)\) are functions of time \((T(t), u(t))\) and accordingly \( \dot{T} = \frac{dT}{dt} \). Note that positive values of \( u \) correspond to heating the system while negative values of \( u \) correspond to cooling. In the above equation, \( C \in \mathbb{R}^{n \times n} \) consists of the wall capacitance and is a diagonal positive definite matrix; \( R \in \mathbb{R}^{n \times l} \) consists of the thermal resistors in the system and is a diagonal positive definite matrix as well. Also, \( D \in \mathbb{R}^{n \times l} \) is the incidence matrix, mapping the system capacitance to the resistors, and is of full row rank, and \( B_0 = -DR^{-1}d_0^T \in \mathbb{R}^n \) is a column vector with non-zero elements denoting the thermal conductance of nodes connected to the ambient.

In our experimental facility all the zones are actuated, which in turn implies that \( B \) is of full row rank. Also, since matrix \( D \) is of full row rank the product \( DR^{-1}D^T \) is a positive definite matrix. The vector of zone temperatures, denoted by \( y \) (which is a function of \( T \)) can be expressed as,

\[
y = BT, \tag{5}
\]

For simulation and experimental runs we obtain the model parameters from our Watervliet, NY based indoor test facility.

### 2.4. Singular Perturbation Solution

Using equilibrium condition (setting \( \dot{T} = 0 \) in (4)) we obtain:

\[
T = h(u) = (DR^{-1}D^T)^{-1}(B_0 T_\infty + Bu). \tag{6}
\]

Define, \( J(u) = U(u, h(u)) \), \( i.e., J(u) \) is obtained by plugging in \( T = h(u) \) from (6) into (2). Note that energy cost term in (2) is strictly convex in \( u \) and the aggregate occupant term is convex in \( T \), and therefore convex in \( u \) when \( T \) is set to \( h(u) \), since \( h(u) \) is affine in \( u \). This implies that \( J(u) \) is strictly convex in \( u \). Therefore \( J(u) \) has a unique optimal solution \( u^* \). Define

\[
T^* = h(u^*), \tag{8}
\]

which is also unique by definition.

With the goal of driving the system to \((u^*, T^*)\), we propose the control input \( u \) be updated once every \( \Delta \) time units as

\[
u_{k+1} = u_k - \eta(\Gamma u_k + \gamma Y \Lambda F(y)), \tag{9}
\]

where \( k \) is the iteration or the time step and \( \eta \) is a scalar that can be loosely interpreted as the “feedback gain” of the system. This feedback gain (step size) parameter controls the size of change in \( u \) in each step, and is discussed in more detail at a later stage. Furthermore, \( Y \in \mathbb{R}^{m \times m} \) in the above is the Jacobian obtained using (5) and the equilibrium condition (6), expressed as

\[
Y = \left( \frac{\partial y}{\partial u} \right) = BT(DR^{-1}D^T)^{-1}B, \tag{10}
\]

Also, \( \Lambda \in \mathbb{R}^{m \times p} \) is the zone-occupant matrix that indicates which occupants are present in a zone (\( \Lambda_{js} = 1 \) if \( s \in S_j \), and 0 otherwise), and \( F(y) \in \mathbb{R}^{p \times 1} \) is the “marginal discomfort” vector of the occupants, obtained by taking partial derivative of the occupant discomfort functions with respect to \( y \). This marginal discomfort can possibly result from the zonal temperatures being outside of the occupant’s preferred comfort range. The marginal discomfort vector is obtained by taking partial derivative of the occupant discomfort functions with respect to the zonal temperature vector \( y \).

In other words, the \( s^{th} \) element of \( F(y) \), where \( s \in S_j \), is obtained as

\[
F_s(y_j) = \frac{dG_s(y_j)}{dy_j}, \quad s \in S_j. \tag{11}
\]
Comparing (9) with (2) provides the motivation of our control algorithm: roughly speaking, (9) updates \( u \) in the gradient direction of \( U(u, T) \), while taking into account the relationship between \( T \) and \( u \) at equilibrium, as given by (6). In other words, it attempts to update \( u \) in the direction of \(-\nabla J(u)\), where \( J(u) \) is defined by (7). In this interpretation, \( \eta \) represents the constant “step size” associated with the gradient descent. Note however that using (2) - (7), \( \nabla J(u) \) is expressed as:

\[
\nabla J(u) = \Gamma u + \gamma Y\Lambda F(B^T h(u)). \tag{12}
\]

From (12) we note that update of \( u \) in the gradient direction of \( J(u) \) requires user discomfort feedback at \( y = B^T h(u) \), the equilibrated zone temperatures corresponding to \( u \). In practice, however, a user \( s \in S_j \) will provide a comfort feedback at the current temperature it experiences, \( y_j = [B^T T]_j \) (different in general from the equilibrated temperature \([B^T h(u)]_j\)), which is what we incorporate into our control algorithm as stated in (9). This implies that our control algorithm as described in (9) does not exactly move \( u \) in the gradient direction \(-\nabla J(u)\). The effect of this difference (error) can be analyzed using singular perturbation theory [47], [48], which in our case requires (for convergence to optimality) that the occupant feedback be collected after long intervals (i.e. \( \Delta \) is large), allowing the temperature \( T \) to settle down close to \( h(u) \) before the next occupant feedback collection.

Towards developing a singular perturbation model of our system, we first consider a continuous approximation to the evolution of the control input \( u \):

\[
\dot{u} \approx \frac{u_{k+1} - u_k}{\Delta} = -\frac{\eta}{\Delta} \left( \Gamma u + \gamma Y\Lambda F(y) \right). \tag{13}
\]

Note that time step \( \Delta \) is the interval at which user feedback is solicited and the control input \( u \) is updated. A larger \( \Delta \) implies a slower evolution of \( u \). We next express the system evolution in the time scale of the evolution of \( u \) (slower time scale as compared to the time scale at which \( T \) evolves).

Define \( \epsilon = \frac{1}{\Delta} \) as the perturbation parameter; then \( \tau = \frac{t}{\Delta} = \epsilon t \) is the slower time scale. Then

\[
\frac{d\tau}{dt} = \epsilon \implies \dot{u} = \frac{du}{dt} = \epsilon \frac{du}{d\tau}; \quad \dot{T} = \frac{dT}{dt} = \epsilon \frac{dT}{d\tau}. \tag{14}
\]

Using the fact that \( y = B^T T \), control input equation (13) can now be expressed in terms of \( \tau \) as follows:

\[
\frac{du}{d\tau} = -\eta \left( \Gamma u + \gamma Y\Lambda F(B^T T) \right). \tag{15}
\]

Similarly, equation (4) modeling the temperature evolution of the building can now be expressed as:

\[
Ce\frac{dT}{d\tau} = -DR^{-1}D^T T + B_0 T_\infty + Bu. \tag{16}
\]

Equations (15) and (16) represent a singularly perturbed system. Note, \( \Delta \uparrow \implies \epsilon \downarrow \) leading to steady state condition for temperature evolution. In the next section we establish the global asymptotic stability of our system as given by equations (15) and (16).

### III. STABILITY ANALYSIS

The system evolution is governed by the set of equations (15) and (16). In equation (16) the coefficient \( DR^{-1}D^T \) is positive definite which makes the unforced system (with \( u = 0 \)) exponentially stable. We use singular perturbation analysis [48] to establish the condition for stability of the system.

**Theorem 1** There exists an \( \epsilon^* > 0 \) such that \((u^*, T^*)\) is a globally asymptotically stable equilibrium of the system given by (15) and (16) for all \( \epsilon < \epsilon^* \).

**Proof:** We first introduce Lyapunov functions \( V(u) \) and \( W(u, T) \) that will be used in our stability analysis:

\[
V(u) = J(u) - J(u^*), \tag{17}
\]

\[
W(u, T) = (T - h(u))^T P(T - h(u)), \tag{18}
\]

where \( P \) is a symmetric positive definite matrix (the exact choice of matrix \( P \) will be determined at a later stage). We now define a combined Lyapunov function \( L(u, T) \):

\[
L(u, T) = (1 - \alpha)V(u) + \alpha W(u, T), \tag{19}
\]

where \( \alpha \) satisfies \( 0 < \alpha < 1 \).

We start by evaluating the conditions to establish stability using Theorem 2.1 and Corollary 2.1 from chapter 7 of [48]. We propose the following comparison functions for the analysis:

\[
\Psi(u) = \|\nabla J(u)\|, \tag{20}
\]

\[
\Phi(T - h(u)) = \|T - h(u)\|. \tag{21}
\]

We assume that the user discomfort function \( G_s(y_j) \) has bounded second derivative i.e. there exists a \( \tilde{\kappa} < \infty \):

\[
\frac{d^2G_s(y_j)}{dy_j^2} \leq \tilde{\kappa}, \forall y_j, s \in S_j. \tag{22}
\]
Note that from equation (11) above:

\[ F'_s(y_j) = \frac{dF_s(y_j)}{dy_j} = \frac{d^2G_s(y_j)}{d^2y_j} \leq \kappa, \quad s \in S_j. \]  

(23)

We can now use the Mean Value Theorem to assert:

\[ F_s(y_j) - F_s(\beta^T h(u)_j) \leq F'_s(\beta_j)(y_j - \beta^T h(u)_j), \]

for some \( \beta_j \in (y_j, [B^T h(u)]), \ s \in S_j. \)  

(24)

Applying Cauchy-Schwartz inequality on (24), for \( s \in S_j: \)

\[ \|F_s(y_j) - F_s(\beta^T h(u)_j)\| \leq \|F'_s(\beta_j)\|\|y_j - \beta^T h(u)_j\|. \]  

(25)

Define \( \kappa = \kappa\|B^T\|. \) Then from (5), (23) and (25) we have:

\[ \|F(y) - F(\beta^T h(u))\| \leq \kappa\|(T - h(u))\|. \]  

(26)

Now we evaluate the conditions in Assumption 2.3, chapter 7 of [48] on \( V(u) \) to obtain:

\[ \frac{\partial V}{\partial u} \left( \frac{du}{dT} |_{h(u)} \right) = -\eta(\nabla J(u))^T(\nabla J(u)) \leq -\eta \Psi^2(u), \]  

(27)

and

\[ \frac{\partial V}{\partial u} \left( \frac{du}{dT} |_{T - h(u)} \right) = -\eta_y(\nabla J(u))^T Y \Lambda (F(y) - F(\beta^T h(u)) \leq \eta\gamma \kappa \|Y\|\|\Lambda\|\|\Psi(u)\|\Phi(T - h(u)). \]  

(28)

The above expressions are obtained by taking partial derivative of \( V(u) \) as defined in (17) and substituting \( \frac{du}{dT} \) from (15). Next, evaluating conditions from Assumption 2.2, chapter 7 of [48] on \( W(u, T) \) yields (29) and (30):

\[ \frac{\partial W}{\partial T} \left( \frac{dt}{d\tau} \right) = -2(T - h(u))^T P \Lambda^{-1} A(T - h(u)) \leq -2\lambda_{min} \Phi^2(T - h(u)), \]  

(29)

where \( A = DR^{-1}D^T \) and \( \lambda_{min} \) is the minimum eigenvalue of the symmetric part of the matrix \( (PC^{-1}A) \), assumed to be positive definite. Note that the symmetric positive definite matrix \( P \) must be chosen such that \( (PC^{-1}A) \) is positive definite. One choice would be \( P = C \) since \( A \) is positive definite. Another choice is \( P = I \); it is reasonable to assume that the symmetric part of the matrix \( (C^{-1}A) \) has positive eigenvalues, as we have verified to hold for the data set used in our simulation study presented in the next section.

\[ \frac{\partial W}{\partial u} \left( \frac{du}{dT} |_{T} \right) = 2\eta(T - h(u))^T P A^{-1} B \nabla J(u) + 2\eta(G - h(u))^T P A^{-1} BY A(F(B^T T) - F(B^T h(u))) \leq 2\eta \|P\|\|A^{-1}\|\|B\|\|\Psi(u)\|\Phi(T - h(u)) \]

\[ + 2\eta \gamma \kappa \|P\|\|A^{-1}\|\|B\|\|Y\|\|\Lambda\|\Phi^2(T - h(u)). \]  

(30)

Equation (29) is obtained using (16) and the definition of \( W(u, T) \) in (18). Equation (30) is obtained using (18) as well as (15), (12) and (26). Given the Lyapunov functions \( V(u) \) and \( W(u, T) \) satisfy the conditions (27) - (30) and are radially unbounded by definition, Theorem 2.1 with Corollary 2.1 of [48] states that for every \( \alpha, L(u, T) \) as given by equation (19), is a Lyapunov function for \( \epsilon < \epsilon^* \). For our system \( \epsilon^* \) can be obtained in terms of the conditions derived in equations (27) - (30):

\[ \epsilon^* = \frac{\lambda_{min}}{2\eta \gamma \kappa \|P\|\|A^{-1}\|\|B\|\|Y\|\|\Lambda\|}. \]  

(31)

Note that \( L(u, T) \) is minimized uniquely at \( (T^*, u^*) \). It follows therefore that \( (T^*, u^*) \) is a globally asymptotically stable equilibrium point for all \( \epsilon < \epsilon^* \), or all \( \Delta > \Delta^* \).

\[ \square \]

Note, that in our simulation study we observed that our algorithm converges even when the occupant feedback is provided and incorporated at fast time scales (much faster than that suggested by our time-scale bound in Theorem 1). However, it would be hard to technically establish that the system moves in an acute angle with the gradient direction without assuming sufficient time-scale separation.

IV. SIMULATION AND EXPERIMENT

In this section we first briefly present the layout of our test bed facility located in Watervliet, NY in section 4.1. The simulation study of different scenarios using the model parameters from our test facility is presented in section 4.2. Finally we present our experimental study results in section 4.3 conducted on the same test facility.

4.1. Testbed Layout

We consider our six zone physical testbed of an intelligent building located in Watervliet, NY for
simulation and experimental study of our proposed algorithm and validating performance of the same. Figure 3 represents the dimensions of the facility as generated using the Building Resistance-Capacitance Modeling (BRCM) toolbox \[49\]. BRCM toolbox is used to generate the RC model of the six zone test facility, mapping it to 31 building elements resulting in a total of 93 capacitive elements.

Each zone of the testbed (except for zone 2, which is the hallway) is actuated with thermo electric coolers. Real time temperature sensing is enabled through J-type thermocouples spread across the test facility. Sensor data is acquired through wireless communication in real time to a central server, which also runs the control loops to operate the coolers and achieve the desired ambient condition. Further details of the test bed layout, instrumentation, and software architecture can be referred to in \[50\].

The simulated occupancy of the building is represented in Figure 4. Zones 1 and 6 are occupied by two occupants each and the other zones 3, 4 and 5 have one occupant each. Occupants U1 and U2 are in Zone 1, U3 in Zone 3, U4 in Zone 4, U5 in Zone 5, and finally U6 and U7 in Zone 6. All the occupants have their own specific temperature preference as depicted in Figure 5. Occupancy mapping with the corresponding preference range is also presented in Table 1. Note that the occupants U1 and U2 co-located in Zone 1 have no common range of comfort preferences, whereas the occupants U6 and U7 co-located in Zone 6 have an overlapping region of comfort preference. This distribution enables us to capture all possible scenarios in terms of conflicting and common preferences among co-located occupants.

**4.2. Simulation Study**

The control input $u$ in our study refers to the power input to the system (in watts). For simulation purpose we consider both heating and cooling actuation in each zone with a typical power rating of 1000 W. The experimental setup has a limitation of cooling input only, limited to 600 W in the larger zones and 400 W in the smaller ones. The system dynamics and the control algorithm are simulated using MATLAB and SimuLink. Using the occupancy distribution as
Table 1. Occupant preferred temperature range with occupancy mapping.

<table>
<thead>
<tr>
<th>Location</th>
<th>Occupant</th>
<th>Lower Limit (°C)</th>
<th>Upper Limit (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>U1</td>
<td>24</td>
<td>24.5</td>
</tr>
<tr>
<td>Zone 2</td>
<td>U2</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Zone 3</td>
<td>U3</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>Zone 4</td>
<td>U4</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>Zone 5</td>
<td>U5</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>Zone 6</td>
<td>U6</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Zone 7</td>
<td>U7</td>
<td>19</td>
<td>22</td>
</tr>
</tbody>
</table>

per Figure 4, occupant preference as per Table 1 and the model parameters for the Jacobian generated using BRBCM toolbox, we first simulate temperature dynamics for a 48 hour period with a fixed ambient condition of 30°C. Figure 6 represents the temperature dynamics for the 48 hour period simulation run.

The temperature of zone 6 settles at 21°C, which is acceptable to both the occupants U6 and U7, and simultaneously energy optimal being closer to the ambient temperature. Note that anything between 19°C to 21°C would have been comfortable for both U6 and U7 based on their comfort preferences, with 21°C being optimal for the given ambient condition. Zone 1 settles around 25.5°C, which tends to minimize the aggregate discomfort of both users U1 and U2 and simultaneously minimizes energy consumption considering the thermal correlation among all the zones. Similarly, the temperature of other zones also settle at a point to minimize aggregate occupant discomfort and the energy cost. In this simulation we also set the initial condition of each zone to the ambient temperature as that represents the extreme scenario, thus providing a good performance evaluation of the proposed algorithm. In most practical condition the initial zonal temperatures would be much closer to the corresponding desired temperatures.

To demonstrate the energy saving in this scenario, we compare it to the prevalent set point based method of temperature control in buildings. With the given occupant preferences, set point method would consider the mid point of each occupants comfort range as the zonal set point. In case of multiple occupants an average of the occupant set point can be used. Using this approach the corresponding zonal set points are presented in Table 2. Compared to the set point based approach, our algorithm achieves energy optimal temperature for the zones and in this particular case results in an energy saving of 12.1%. In Table 3 we also show the final zonal temperature when using set point versus the proposed optimal approach. Note that this for the case with ambient temperature at 30°C. The proposed algorithm becomes more effective and energy efficient with varying ambient conditions and changes in occupancy as demonstrated in the later cases.

Table 2. Temperature set point for each zone when using the set point based method of building temperature control.

<table>
<thead>
<tr>
<th>Location</th>
<th>Occupant set points (°C)</th>
<th>Zonal set points (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>24.25, 26.5</td>
<td>25.4</td>
</tr>
<tr>
<td>Zone 3</td>
<td>21.5</td>
<td>21.5</td>
</tr>
<tr>
<td>Zone 4</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Zone 5</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>Zone 6</td>
<td>20.5, 19.5</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 3. Temperature set point for each zone when using the set point based method of building temperature control versus the proposed algorithm.

<table>
<thead>
<tr>
<th>Location</th>
<th>Set point approach (°C)</th>
<th>Optimal approach (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>25.4</td>
<td>25.5</td>
</tr>
<tr>
<td>Zone 3</td>
<td>21.5</td>
<td>23</td>
</tr>
<tr>
<td>Zone 4</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>Zone 5</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Zone 6</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

Next we simulate another case with fixed ambient temperature at 15°C. The corresponding temperature
dynamics are represented in Figure 7. Note that in this case the temperature of Zone 6 settles at 19°C, compared to 21°C in Figure 6 as 19°C is more energy optimal for the ambient condition of 15°C.

Fig. 7. Temperature dynamics for a 48 hour period simulation with ambient condition lower than the occupant preferences, occupancy as per Table 1.

Fixed ambient temperature is an over simplification and does not represent true variations in ambient conditions. Next, we consider a sinusoidal variation of the ambient with \( T_\infty = 20°C + 5°C\sin(2\pi t/t') \), with \( t' = 24 \) hr. The corresponding temperature dynamics are presented in Figure 8. The control algorithm lets the zonal temperature vary with ambient till it hits the comfort limit, at which point appropriate heating/cooling input is applied. This approach is more optimal than maintaining a fixed set point as it harnesses the ambient variation without the need to constantly re-adjust the fixed set point. For our particular setup we observed a relative energy saving of 5.3%. Note that the relative energy savings obtained would depend on the ambient condition. Harsher ambient condition that creates load on the HVAC system would lead to higher percentage of relative energy saving using our algorithm compared to set point based approach.

Another simplification that has been applied to our study so far has been the un-interrupted occupancy of the occupants throughout the period of 48 hours. The occupants would be moving around and in case of non occupancy no occupant feedback would be generated. In practice, this occupancy information can be obtained using motion sensors or blue tooth low energy (BLE) beacons that can detect the presence of smart phone. We next consider a typical work environment occupancy schedule of the occupants entering their respective zones at 9 am in the morning and departing at 5 pm in the evening. Further, we also consider lunch time and simulate non occupancy during 12 to 1 pm. In the evening after 5 pm we increase the heat cost factor \( \Gamma \). For the lunch break heat cost factor remains unchanged but due to non-occupancy no occupant feedback is generated. The results are presented in Figure 9.

Fig. 8. Temperature dynamics for a 48 hour period simulation where ambient condition follows a sinusoidal variation with period of 24 hours, occupancy as per Table 1.

Due to increase in energy cost factor after 5 pm, the zonal temperatures tend to follow ambient variation resulting in immense energy savings. The kink in Zone 1 and Zone 6 temperatures during lunch time is attributed to thermal correlation as no occupant feedback is generated resulting in temperature variation as per prevailing thermal conditions. Compared to a
set point based approach (with predetermined energy saving set point after 5pm) this approach can result in energy savings of 6.1%. Note that if the set point based approach does not implement energy saving mode after 5pm then the relative saving through our approach would be even higher. Compared to scenario in Figure 8 additional energy saving can be attributed to efficiency during lunch break.

4.3. Experimental Study

Although the RC model parameters for the simulation study has been obtained using the test facility information, we conduct an experimental test run on the facility to further validate our model parameters and the proposed algorithm. The main purpose of these experimental run is to establish that the algorithm can be used in a real building to achieve the zonal temperatures that optimizes the group comfort of the occupants and the energy cost, based on occupant feedback mechanism. Currently we only have cooling capacity in the test bed for each zone, with power limited to 600 W in the larger zones (1 and 6) and 400 W in the smaller ones (3,4, and 5) [50]. The occupancy simulation for experimental study is as per Figure 4. Owing to the cooling power limitation of the test bed we change the occupant preferences slightly for occupants U3, U6 and U7 so as to reduce the maximum difference between the ambient temperature and desired zone temperature. The occupancy mapping with occupant comfort preference for experimental study is listed in Table 4. The control algorithm and the real-time temperature data collection was implemented using NI LabView software.

Table 4. Occupant comfortable temperature range with occupancy mapping for experimental study.

<table>
<thead>
<tr>
<th>Location</th>
<th>Occupant</th>
<th>Lower Limit (°C)</th>
<th>Upper Limit (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>U1</td>
<td>24</td>
<td>24.5</td>
</tr>
<tr>
<td>Zone 1</td>
<td>U2</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Zone 3</td>
<td>U3</td>
<td>22.5</td>
<td>24.5</td>
</tr>
<tr>
<td>Zone 4</td>
<td>U4</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>Zone 5</td>
<td>U5</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>Zone 6</td>
<td>U6</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Zone 6</td>
<td>U7</td>
<td>22</td>
<td>25</td>
</tr>
</tbody>
</table>

The temperature dynamics corresponding to a 10 hour experimental study is presented in Figure 10. The ambient is being maintained at 28.5°C (note that ambient is maintained inside the building using standard set point based thermostat). The building is pre-cooled to 26°C prior to occupant’s arrival at 9am. This is in line with the current practice of pre-cooling/pre-heating in most of the commercial building. At 9am once the occupants arrive the temperature of each zone settles as per occupant comfort preferences in Table 4. Zone 1 with users U1 and U2 having conflicting preferences settles at 25.5°C. Zone 3 settles at 24.5°C, Zone 4 at 25°C, and Zones 5 and 6 at 24°C. Note that these zonal temperatures correspond to energy optimal set points given the preference of corresponding occupants.

Between 12pm to 1pm the occupants take a lunch break. During this time no occupant feedback is generated resulting in the zonal temperatures increasing under ambient effect till they hit the pre-cool limit of 26°C. Note that Zone 6 being larger and having settled at the lowest relative temperature of 24°C, has much slower dynamics and takes the longest to reach close to 26°C during lunch hour. As the occupants get back at 1pm the zonal temperatures once again settle in accordance to their respective preferences.

At 5pm (post eight hours of work) the energy cost factor \( \Gamma \) is doubled with the occupants still in their respective zones. This results in the zonal temperatures settling at a relatively higher point slightly out of the occupant comfort range to compensate for the rise in energy cost. At 6pm the energy cost factor \( \Gamma \) is doubled once again, with the occupants still in their respective zones, resulting in further discomfort to the occupants to compensate for the additional rise in energy cost. At 7pm when all the occupants leave their respective zones, the zonal temperatures are once again maintained at the pre-cooling limit of 26°C.

The energy cost parameter \( \Gamma \) can be tuned to real time energy pricing coming directly from a utility service company. Thus our model facilitates very easy adaptation to energy price signals through adapting the \( \Gamma \) parameter appropriately. The experimental study in Figure 10 demonstrated a preliminary case of the same, as the zonal temperatures moved towards ambient after 5pm due to increase in energy cost. The overall evolution and convergence of our system is dependent on the values of three major model parameters, feedback gain (step size) parameter (\( \eta \)), user feedback weight parameter (\( \gamma \)), and the energy cost parameter (\( \Gamma \)). User weight parameter (\( \gamma \)), signifies the weight given to the penalty associated with the building occupant discomfort. A higher \( \gamma \) would result in a high value of control input for a given user comfort feedback vector. This in turn would cause the temperature to change sharply as a result of occupant discomfort, but...
may also result in large overshoots as well as higher energy costs. Note that the final relative weight of energy cost and occupant discomfort is determined by the ratio of the energy cost parameter ($\Gamma$) and user weight parameter ($\gamma$), and could as well be incorporated through appropriate scaling of just one parameter. The feedback gain (step size) parameter controls the size of change in $u$ in each step. Small values of $\eta$ would lead to a very slow convergence of the system to the desired temperature. With a high value of $\eta$ we can hit the desired range of temperature much faster, but at the cost of high temperature overshoots resulting in much higher total energy input. Also, a sufficiently high value of $\eta$ could violate the time scale separation assumption that is needed for stability. In our study, $\eta$ was tuned to obtain a reasonable trade-off. The parameter values used for the simulation and experimental study is presented in Table 5.

![Diagram of temperature dynamics](image)

**Fig. 10.** Temperature dynamics for a typical work day from 9am to 7pm for the experimental study at our test facility. The zonal temperatures adjust to occupants preferences when they walk in at 9am. During lunch break there is no occupant feedback generated. Energy cost is doubled at 5pm and then again at 6pm, and finally the occupants leave at 7pm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability range</td>
<td>$\delta$</td>
<td>1°C</td>
</tr>
<tr>
<td>Time step</td>
<td>$\Delta$</td>
<td>10 min</td>
</tr>
<tr>
<td>Feedback gain</td>
<td>$\eta$</td>
<td>0.1</td>
</tr>
<tr>
<td>User weight parameter</td>
<td>$\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>Energy cost parameter</td>
<td>$\Gamma$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5. Parameter values used for the simulation and experimental study.

6 pm, owing to lower energy consumption or in other words the building starts to operate in energy saving mode beyond the regular work hours.

V. CONCLUSION

In this work we have demonstrated that the building temperature and energy usage can be controlled successfully and efficiently through dynamic feedback from the occupants based on their comfort levels. Under the reasonable assumption that occupant feedback is provided at a slower time scale as compared to the building temperature dynamics, our analysis shows that the proposed control algorithm results in the desired (optimal) trade-off between energy usage and occupant discomfort. The simulation and experimental
studies presented further establish that with effective tuning of a few parameters, the control algorithm can be effectively combined with a demand response mechanism that reacts to energy price signals.

While we have relied on the RC model of a building to develop and analyze our optimal control solution, it is worth noting that the control law described in (9) is reasonably ‘model independent’, allowing for practical implementation. The only term in (9) that depends on the building model is the Jacobian matrix $Y$ given by (10). In practice the matrix $Y$ would be sparse owing to low coupling between zones far apart in a building, and could be easily estimated through measurements. Results from various scenarios considered under the simulation and experimental study of this work seem promising. Further implementation on a full scale building with real occupants is an ongoing effort.

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37. Q. C. Zhao, Y. Zhao, F. L. Wang, Y. Jiang, F. Zhang, Preliminary study of learning individual thermal
complaint behavior using one-class classifier for indoor environment control, Building and Environment, Vol. 72, pp. 201-211, Feb. 2014.