Collaborative Energy and Thermal Comfort Management Through Distributed Consensus Algorithms

Santosh K. Gupta, Koushik Kar, *Member, IEEE,* Sandipan Mishra, *Member, IEEE* and John T. Wen, *Fellow, IEEE*

***Abstract*—Buildings with shared spaces such as corporate office buildings, university dorms, etc. are occupied by multiple occupants who typically have different temperature preferences. Attaining a common temperature set-point that is agreeable to all users (occupants) in such a multi-occupant space is a challenging problem. Furthermore, the ideal temperature set-point should optimally trade off the building energy cost with the aggregate discomfort of all the occupants. However, the information on the comfort range (function) is held privately by each occupant. Using occupant-differentiated dynamically-adjusted penalty factor as feedback signals, we propose a distributed solution which ensures that a consensus is attained among all occupants upon convergence, irrespective of their ideal temperature preferences being in coherence or conflicting. Occupants are only assumed to be rational, in that they choose their own temperature set-points so as to minimize their individual energy cost plus discomfort. We establish the convergence of the proposed algorithm to the optimal temperature set-point vector that minimizes the sum of the energy cost and the aggregate discomfort of all occupants in a multi-zone building. Simulations with realistic parameter settings illustrate validation of our theoretical claims and provide insights on the dynamics of the system with a mobile user population.**

***Note to Practitioners*—This paper was motivated by the problem of computing an optimal commonly-agreeable temperature set-point in spaces with multiple occupants. Consider office floors with cubicles, conference rooms, student dorms, homes, and other multi-occupant spaces where temperature set-points on thermostats are chosen irrespective of the number of occupants and their individual preferences. This existing approach is not only non user-centric but also sub-optimal from both energy consumption and occupant satisfaction/productivity perspectives. It is thus highly desirable for such multi-occupant spaces to have a mechanism that would take into account each occupant's individual comfort preference and the energy cost, to come up with an optimal temperature set-point. Individual occupant's feedback and preference can be obtained through wearable sensors or smart phone applications. In this work we propose an algorithm that takes into account each occupant's preferences along with the thermal correlations between the different zones in a building, to arrive at optimal temperature set-points for all the zones of the building in a coordinated manner. This approach is therefore also more performance-efficient than controlling the temperature set-point of each zone in the building in an isolated manner. We establish convergence of our algorithm and use the parameters of an experimental facility to simulate and evaluate the proposed algorithm. The proposed solution can be deployed in multi-occupant spaces to make the occupants of the entire space more comfortable, and simultaneously facilitate energy efficient operation of the space. The occupants can provide their temperature preference one time (or when it changes) using smart phone or desktop application or wearables, which is then utilized to calculate the optimal temperature set-point of various zones in real time. The optimal set-point would be updated in real time based on change in occupancy and or occupant preference.**

***Index Terms*—Temperature consensus, human-centered building environment control, collaborative comfort management, smart building energy management.**

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I. Introduction

Buildings that can maintain the indoor conditions as per the comfort level of its occupants, irrespective of the variations in the external weather, is one of the minimal expectations of a developed society. With changes in general living style and consumer expectations over the past decade, the demand for comfort levels have grown more and more personalized. This personal comfort level expectations pose a conflicting situation in multi-occupant spaces such as corporate office buildings, student dorms, buses and airplanes etc., where each occupant has its own range of comfortable temperature distribution. This range also depends on the occupant's individual characteristics, including metabolism rate, age and external factors such as attire, physical and mental condition, and level of tolerance; this individual range can also vary depending on other environmental factors. In shared multi-occupant spaces personal comfort levels are affected both by the presence of co-occupants and the correlation between the temperatures in the different zones and rooms occupied. Arriving at a consensus among all the occupants of different rooms and zones in a building is therefore an important but challenging problem. With the rising energy cost and emphasis on energy conservation, the total energy cost also needs to be accounted for when trying to achieve consensus among the occupants of a building.

Energy usage in buildings, both residential and commercial, accounts for one major source of energy consumption both within the US and worldwide. Data suggests that nearly 40% of the total energy consumption in US, and 20% of the total energy consumption worldwide, is attributed to residential and commercial building usage [1]. Numerous design and solution approaches have been proposed for efficient control and operation of building heating, ventilation, and air-conditioning (HVAC) systems. The approaches taken so far can be broadly classified into those focusing on optimizing energy usage by utilizing variable electricity rates [2], [3], [4], [5], active and passive thermal energy storage [4], [5], and model predictive control approach exploiting information through weather forecast [6], [7], [8], [9]. More recently there have also been focus on using occupant feedback at binary/multiple levels to determine the direction of temperature adjustment based on the average user vote [10], [11], [12] and achieving energy optimization along with occupant discomfort minimization [13].

With the development of HVAC systems, there has been a multitude of studies on occupant thermal comfort modeling. Thermal comfort has been studied extensively for many years, and the existing models can be combined into three different categories: the chamber study model, models based on the human body physiology and the field study comfort models. Chamber study model is based on averaging a large number of data points to map thermal comfort from environmental and personal factors to a 7-level comfort value scale. The Predicted Mean Vote - Predicted Percent Dissatisfied (PMV-PPD) [14], [15] is one of the popular chamber study based comfort models. Some of the thermal comfort models based on human body physiology are: Gagge's core to skin model [16], Stolwijk's comfort model for multi-human segments [17], and Zhang et al.'s sensation on human body segments and for the whole body [18]. Some adaptive comfort models have been developed in field study, viz. Humphreys [19] and [20]. Note that these works focus on average thermal comfort models instead of personalized comfort modeling. Some more recent works have conducted experimental study for a group of occupants [21], and have presented thermal comfort model for a single person [22]. However, these are based on thermal complaint behavior using one-class classifier. In this work we model the initial estimate of the individual occupant thermal discomfort function as a convex quadratic curve of temperature variation, based on the PMV-PPD model. Note that such group comfort models only capture average behavior, and are not particularly useful in maximizing aggregate comfort for a specific set of occupants in a shared space, whose individual thermal preferences may differ from each other. In our work, therefore, we take into consideration discomfort functions of the occupants individually - modeled as convex functions. Our model not only captures the differences across occupants in their comfortable temperature range, but also individual differences in their sensitivities (degree of discomfort) for temperature variations beyond their range of comfort.

Achieving a common temperature set-point that is both energy optimal and acceptable to the occupants requires consensus among all the occupants and the central building management system (building operator). Achieving this in a distributed framework, where the exact discomfort functions are held privately by each occupant, remains an open question which we seek to address in this paper. We pose the collaborative building temperature control problem as a convex optimization question, and develop a distributed solution approach by utilizing a consensus algorithm framework. The minimization objective is an aggregate of all the occupant discomfort functions and the total energy cost, subject to the constraint of common zonal temperatures. Penalty factor per unit temperature change serves as the feedback signal to the occupants, to drive them to a consensus on zonal temperatures that optimize the overall discomfort plus energy cost objective as mentioned above. The consensus algorithm that we develop, through the use of the *alternating direction method of multipliers* (ADMM), is amenable to distributed implementation and has the following appealing properties. Firstly, occupants (or their agents) are only assumed to be rational, in that they choose their preferred temperature set-points so as to minimize their personal discomfort plus energy cost, given the pricing signals. In other words, the occupants are not required to explicitly declare their discomfort functions (which can be held privately), but only react rationally to the pricing signals by choosing their preferred temperature set-point. The occupants can also provide their feedback in the form of 'hot/cold', which would be captured into modifying their discomfort functions accordingly. On the other hand, the building thermal management system (BTMS) chooses the zonal temperature set-points to maximize the overall profit of the building operator (for the current cost); the penalty factor signals are then updated so as to attain consensus among the occupants, and with the building operator, on the zonal temperatures. Finally, as we formally show, the algorithm converges to the optimal zonal temperatures, from which rational occupants would not have any incentive to deviate. In terms of practical implementation, occupant feedback could be obtained through a smart phone application, and the zonal temperature set-points could be calculated by the building operator on a central server. Note that this functionality of the application might as well be implemented on the central server itself for the entire building. The smart phone application in that case would provide an interface for capturing user response. A high level work flow of the proposed algorithm is depicted in Figure 1.

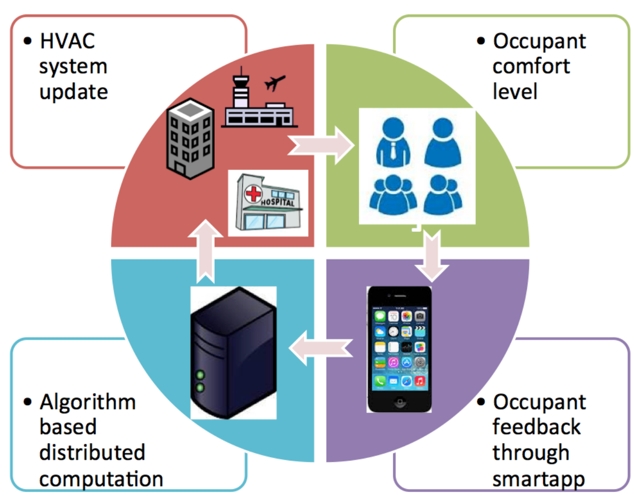

Fig. 1. High level process flow depiction of the proposed algorithm.

The paper is structured as follows. The consensus algorithm, which constitutes the main contribution of this work, along with the optimization objective, solution approach and convergence analysis is presented in Section II. Section III provides the building thermal model and the control law we use to simulate our algorithm. We evaluate our approach through simulations and also present a sensitivity study in Section IV, and finally conclude in Section V.

II. Consensus Algorithm and Its Analysis

In this section, we first describe our optimization objective (Section II-A) and solution approach (Section II-B). This is followed by the distributed consensus algorithm (Section II-C) and its convergence analysis (Section II-D), and finally profit analysis in Section II-E.

A. Optimization Objective

Consider a building with m zones, and let S_j represent the set of occupants located in zone j of the building. Let D_i represent the discomfort function of occupant i, and function E the overall energy cost. Then a reasonable objective is to attain (in steady state) the zonal temperature vector y that achieves the following objective:

$$\text{minimize} \sum_{j=1}^{m} \sum_{i \in S_j} D_i(y_j) + E(u). \tag{1}$$

where y_j is the temperature of zone j, and u is the heat input vector that is required to attain those zonal temperatures. Note that an occupant i located in zone j (i.e., $i \in S_j$) experiences temperature y_j, and therefore its discomfort can be represented as $D_i(y_j)$. We assume the discomfort function $D_i(y_j)$ as convex in its argument y_j. It is worth noting that the discomfort function *need not be* "strictly" convex. This allows for the occupants to be insensitive to temperature fluctuations over a certain range; or in other words, the discomfort function could be flat over the occupant's "comfort range".

In the above, $E(u)$ is assumed to be a convex function of the control input vector u. For the sake of definiteness, we use $E(u)$ to be of the following quadratic form (although other convex forms of the function $E(u)$ are also allowed by our framework):

$$E(u) = u^T \Gamma u, \tag{2}$$

where Γ is a positive definite matrix. The Γ matrix captures the weight of the energy cost relative to the total discomfort cost. In practice, it could be determined by the actual cost of energy, as well as additional input from the building operator to determine how much relative weight to associate with the energy cost as compared to the occupant discomfort cost.

Finally, since the optimization variable in the objective function (1) is only the zonal temperature vector y, the relationship between the heat input vector u and the zonal temperature vector y needs to be stated to make the formula meaningful. We can express $u = g(y)$ and using that write the energy cost $E(u)$ as $G(y) = E\big(g(y)\big)$, where the function $G(y)$ is convex in y. Taking the case of an RC model, we would express function $g(.)$ in terms of model parameters for the purpose of simulation later in section III-A.

B. Solution Approach

Before we describe the distributed consensus algorithm, we provide an overview of the solution approach. Note that if the individual occupant (user) discomfort functions are assumed to be known to the building operator, the optimal zonal temperature vector y^* could be computed directly. Such a centralized approach suffers from several practical limitations, however. Firstly, reporting the entire discomfort function to the building operator is complex, and the occupant may not even be able to correctly estimate its discomfort function. Secondly, even if we assume that the occupant knows its discomfort function exactly, there is no incentive for it to report the same truthfully. In practice, therefore, it may be more desirable to have a mechanism through which the building operator indirectly learns about the true discomfort functions of the occupants, who are providing their temperature preference feedback in a simple and convenient format, acting in best response to some penalty factor signals provided by the building operator as depicted in Figure 2. Furthermore, the penalty factor signals should be such that it guides the occupants towards a *consensus*, i.e., rational users (acting in self-interest) in a zone will end up agreeing on their temperature choice for each zone. The distributed consensus algorithm that we describe in the next subsection works according to the above principles.

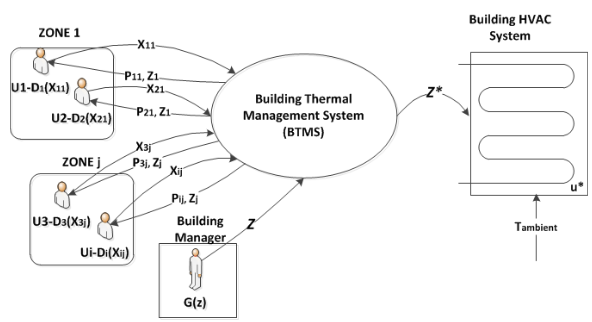

Fig. 2. Distributed consensus algorithm with the building HVAC system.

To provide an overview of our approach, we first introduce new notations to denote the choice of zonal temperatures by the occupants and the building thermal management system (BTMS); these temperature choices will in general be different from the actual (current) zonal temperatures. Let x_{ij} denote the desired temperature of occupant $i \in S_j$ located in zone j. Let z_j denote the target temperature of zone j as set by the BTMS. Then vector z represents the target temperature of the entire building consisting of m zones. In general, x_{ij} for any occupant $i \in S_j$ can differ from z_j; the actual zonal temperature y_j could also differ from these temperatures. On convergence however, the consensus algorithm ensures that x_{ij} for all occupants $i \in S_j$ equals z_j, which optimizes the objective function in (1) subject to (26). The zonal temperatures obtained through consensus is then attained in the building by utilizing some temperature set-point based HVAC control system. This results in the decomposability of the problem into two parts: (i) the derivation of the optimal zonal temperatures through the consensus between the occupants and the BTMS;

(ii) attaining the temperature set-points resulting from (i) in the actual building. The key novelty of this work is in developing a distributed consensus algorithm for (i), which we describe in Section II-C. Some standard or existing control laws that could be utilized to solve (ii) is discussed in the subsection (Section III-B).

Using a smart phone/tablet application, that serves as an interface, the occupants provide their comfort preference as a *one time input*. Once the occupant's comfort preference is provided, the smart application constructs a discomfort function for the occupant. It takes part (on behalf of the occupant) in the consensus algorithm that the Building Thermal Management System (BTMS) runs by providing a temperature set-point (x_ij), in response to the BTMSs choice of penalty factor signal (p_ij) and preferred zonal temperature (z_j). Thus the consensus algorithm runs through iterative (back-and-forth) communication between the BTMS and the smart applications associated with the individual occupants. Note that the occupant is not directly involved in this iterative process. The occupant is only involved when it wants to provide additional input or update its comfort preferences (which could simply be in the form of hot/cold feedback, or a comfortable temperature range), which is used by the smart application associated with the occupant to recompute discomfort function of the occupant. The occupants (humans) themselves are not required to perform any computational task or take part in iterative communication with the BTMS. Also, the occupants are not required to re-enter their preferences even when there is a change in occupancy (users move in/out) or ambient conditions, unless they actually want to change their preference. The algorithm would automatically compute the adjusted optimal set-points based on changes in occupancy and environmental conditions.

Note that the smart application could also be residing in a central server (maintained by the BTMS) as well. In that case, each iteration of the consensus algorithm would involve communication between two processes (or servers) within the BTMS, rather than the occupants smart phone and the BTMS.

Upon convergence of the distributed consensus algorithm (which we establish later in the paper), let x^*, z^* and p^* respectively denote the (vectors of) occupant temperature choices, zonal target temperatures, and penalty factor signals. Then we desire that x^*, z^*, p^* satisfy the following properties:

- *(Individual Rationality)* Each occupant agent (smart application associated with an individual occupant) chooses its desired temperature so as to minimize its total cost function, represented as the sum of its discomfort plus the energy price paid to the building operator (considering penalty factor as equivalent to pricing per unit temperature change for the occupant):
$$x_{ij}^* = \arg\min_{x_{ij}} \left\{ D_i(x_{ij}) + p_{ij}(x_{ij} - z_j) \right\}.$$
- *(Consensus)* For each zone, the temperature choices of the occupant agents of the zone agree with each other, and with the target zone temperature set by the BTMS:
$$x_{ij}^* = z_j^*, \ \forall i \in S_j.$$
- *(Optimality)* The target zone temperatures minimize the aggregate occupant discomfort plus the building energy cost, given by (1):
$$z^* = \arg\min_z \sum_{j=1}^{m} \sum_{i \in S_j} D_i(z_j) + E(g(z)).$$

It is easy to argue that such a solution (x^*, z^*, p^*) - one that satisfies the above three properties - exists. Note however that for general convex discomfort functions D_i, the x_{ij}^* that satisfies the individual rationality property may be non-unique. In this paper, we make the reasonable assumption that while the discomfort functions are convex, they are not necessarily *strictly* convex. For example, each occupant may have a "comfortable" temperature range, over which the discomfort function is essentially flat, i.e., the occupant is insensitive to temperature changes within that range. Despite this non-uniqueness of the optimal z^* (and therefore the non-uniqueness of the optimal solution (x^*, z^*, p^*)), the distributed consensus algorithm that we describe next ensures that the system *converges* to *one* of the optimal solutions.

C. Distributed Consensus Algorithm

To develop the consensus algorithm, we re-write the minimization objective in (1) in terms of the zonal temperature choices of the occupants and the BTMS, as:

$$\begin{aligned} \text{minimize} \ & \sum_{j=1}^{m} \sum_{i \in S_j} D_i(x_{ij}) + G(z) \\ \text{subject to} \ & x_{ij} = z_j, i \in S_j \end{aligned} \tag{3}$$

where function $G(z) = E\big(g(z)\big)$ represents the total energy cost in terms of the target zonal temperature vector z.

We can now solve (3) through the ADMM approach as described in [32]. The ADMM approach blends the decomposability of dual ascent with the superior convergence properties of the method of multipliers, to develop an algorithm that is amenable to distributed implementation, and also has good convergence properties.

To motivate the ADMM based consensus algorithm, let us consider the augmented Lagrangian:

$$\begin{aligned} L_\rho(x, z, p, \rho) = \sum_{j=1}^{m} \sum_{i \in S_j} \Big(& D_i(x_{ij}) + p_{ij}(x_{ij} - z_j) + \\ & (\rho/2)|x_{ij} - z_j|^2 \Big) + G(z) \end{aligned} \tag{4}$$

where p_{ij} (penalty factor) is the dual variable, $\rho > 0$ is a constant. The ADMM based consensus algorithm can then be derived as iterations of coordinate-wise optimization of this augmented Lagrangian along each x_{ij} and z directions, followed by update of the dual variable in a gradient direction. More precisely, in our consensus algorithm, in iteration $k = 1, 2, \ldots$, the variable vector z, and the variables x_{ij}, p_{ij} for all $i \in S_j, \ j = 1, \ldots m$, are updated as follows:

$$x_{ij}^{k+1} := \operatorname*{argmin}_{x_{ij}} \Big(D_i(x_{ij}) + p_{ij}^k x_{ij} + (\rho/2)|x_{ij} - z_j^k|^2 \Big) \tag{5}$$

$$z^{k+1} := \underset{z}{\text{argmin}}\Big(G(z) + \sum_{j=1}^{m}\big(-\sum_{i\in S_j} p_{ij}^k z_j + \sum_{i\in S_j}(\rho/2)|x_{ij}^{k+1} - z_j|^2\big)\Big) \quad (6)$$

$$p_{ij}^{k+1} := p_{ij}^k + \rho(x_{ij}^{k+1} - z_j^{k+1}) \quad (7)$$

The above set of update equations has a nice game theoretic (price-driven rational-response) interpretation, as follows. The BTMS iteratively communicates to each occupant i in any zone j two parameters, p_{ij} and z_j, based on which the occupant's cost (price paid) for a chosen temperature set-point x_{ij} would be computed as $p_{ij}x_{ij} + (\rho/2)|x_{ij} - z_j|^2$. A rational occupant then chooses its personal temperature preference x_{ij} to minimize their individual cost function:

$$\text{minimize } D_i(x_{ij}) + p_{ij}x_{ij} + (\rho/2)|x_{ij} - z_j|^2. \quad (8)$$

The BTMS, acting on behalf of the building operator, would choose the target building temperature vector z so as to minimize

$$\text{minimize } G(z) - \sum_{j=1}^{m}\sum_{i\in S_j} p_{ij}z_j + \sum_{j=1}^{m}\sum_{i\in S_j}(\rho/2)|x_{ij} - z_j|^2, \quad (9)$$

which on convergence (when consensus is attained) would equate to the total energy cost incurred by the building operator, when the payments made by the occupants are taken into account. Finally, the penalty factor p_{ij} (equivalent of price for per-unit temperature change) are updated in a way that helps in the consensus, i.e., in bringing x_{ij} and z_j close to each other in each zone j, for each occupant $i \in S_j$.

In Section II-D we present a convergence proof for the consensus algorithm described above, following the general line of analysis on the convergence of the ADMM algorithm as provided in [33].

It is worth noting here that in practice, it may take several hundred iterations or more for the consensus algorithm to converge, as we will see in the simulation results presented later in the paper (Section IV). Involving humans to carry out the task in (5) and communicating the temperature preference to the BTMS would therefore lead to impractically long convergence times. To implement the consensus algorithm in practice, the user (occupant) could input its comfort range (function) into a software agent (running on the user's smartphone, or a PC in the user's room/office); this user agent could then be involved in the interactive communication with the BTMS, and setting the temperature set-point preference (for a given pricing signal) in the best interest of the individual user (occupant).

D. Convergence Analysis of the Consensus Algorithm

The convergence proof presented in this section assumes that the functions $D(.)$ and $G(.)$ are closed, proper, and convex, and the UN augmented Lagrangian L_o in (10) below has a saddle point.

$$L_o(x, z, p) = \sum_{j=1}^{m}\sum_{i\in S_j}\Big(D_i(x_{ij}) + p_{ij}(x_{ij} - z_j)\Big) + G(z). \quad (10)$$

Based on these assumptions we establish the objective convergence, the residual convergence, and the convergence of the dual variables, for our consensus algorithm as described in Section II-C. In doing so, we utilize the convergence analysis of the ADMM approach as described in [33], suitably adapted to our model. Consider the objective,

$$O^* = \text{minimum} \sum_{j=1}^{m}\sum_{i\in S_j} D_i(x_{ij}) + G(z) = \sum_{j=1}^{m}\sum_{i\in S_j} D_i(x_{ij}^*) + G(z^*), \quad (11)$$

where x_{ij}^* and z^* denote the corresponding optimal values of temperature choices. Note that for any zone j, $x_{ij}^* = z_j^*$ for all $i \in S_j$. Also, define residual for zone j as:

$$r_{ij} = x_{ij} - z_j \quad (12)$$

We prove our result through a sequence of lemmas, each involving an inequality (refer to appendix for complete proof of the lemmas).

Lemma 1.

$$O^* - O^{k+1} \le \sum_{j=1}^{m}\sum_{i\in S_j} p_{ij}^* r_{ij}^{k+1}. \quad (13)$$

Lemma 2.

$$O^{k+1} - O^* \le -\sum_{j=1}^{m}\sum_{i\in S_j}\Big(p_{ij}^{k+1} r_{ij}^{k+1} + \rho(z_j^{k+1} - z_j^k)(-r_{ij}^{k+1} - (z_j^{k+1} - z_j^*))\Big). \quad (14)$$

Next, define Lyapunov function V for the ADMM algorithm as:

$$V^k = (1/\rho)\sum_{j=1}^{m}\sum_{i\in S_j}|p_{ij}^k - p_{ij}^*|^2 + \rho\sum_{j=1}^{m}|z_j^k - z_j^*|^2 \quad (15)$$

This Lyapunov function satisfies the inequality as stated in the lemma below.

Lemma 3.

$$V^{k+1} \le V^k - \rho\sum_{j=1}^{m}\sum_{i\in S_j}|r_{ij}^{k+1}|^2 - \rho\sum_{j=1}^{m}|z_j^{k+1} - z_j^k|^2 \quad (16)$$

Now, since $V^k \le V^0$, p_{ij}^k and z_j^k are bounded. Iterating (16) gives:

$$\rho\sum_{k=0}^{\infty}\big((r_{ij}^{k+1})^2 + |z_j^{k+1} - z_j^k|^2\big) \le V^0, \quad (17)$$

which implies $r_{ij}^k \to 0$ and $|z_j^{k+1} - z_j^k| \to 0$ as $k \to \infty$. Further, from inequalities (13) and (14) we have $\lim_{k\to\infty} O^k = O^*$ or the objective convergence.

Hence, the inequalities (13), (14) and (16) implies the convergence of our algorithm.

E. Profit Analysis for the Building Operator

For this analysis we consider the penalty factor p^* as equivalent to optimal pricing for per unit change of temperature. Let the vector y^* denote the optimal zonal temperatures. Note that the pricing feedback signals p_{ij}'s naturally reflect the marginal energy cost saved by the occupants (users). It is easy to establish that the optimal pricing feedback signal p^*_{ij} satisfies:

$$p^*_{ij} = -D'_i(y^*_j). \tag{18}$$

We can further show that,

$$\sum_{i \in S_j} p^*_{ij} = \frac{\partial G(y^*)}{\partial y_j}. \tag{19}$$

Now, we can obtain an expression for the difference of the net revenue from occupants and the operational cost of maintaining the building at the consensus temperature set-point. Note, that this expression represents the operational profit of the building operator and is given by:

$$\sum_j \sum_{i \in S_j} p^*_{ij}(y^*_j - \hat{T}_\infty) - (G(y^*) - G(\hat{T}_\infty)), \tag{20}$$

where $\hat{T}_\infty$ is a vector of size m of T_∞ values. Using Taylor series:

$$\begin{aligned} G(\hat{T}_\infty) &= G(y^*) + (\hat{T}_\infty - y^*)^T \nabla G(y^*) \\ &+ \frac{1}{2}(\hat{T}_\infty - y^*)^T \nabla^2(y^*)(\hat{T}_\infty - y^*), \end{aligned} \tag{21}$$

the building operator's profit can be further expressed as:

$$\frac{1}{2}(\hat{T}_\infty - y^*)^T \nabla^2(y^*)(\hat{T}_\infty - y^*). \tag{22}$$

Since $G(y)$ is convex in y, $\nabla^2(y^*) \geq 0$, and (22) is upper bounded by $\leq \frac{\lambda_{max}}{2}||y^* - \hat{T}_\infty||^2$, where λ_{max} is the maximum eigenvalue of $\nabla^2(y^*)$. From (22) we can assert that if the penalty factor signals were to be translated to real money (or equivalent credit) transaction between the building operator and the occupants, the building operator does not lose money, and instead may end up making a small profit that is bounded by the convexity of the energy cost function $G(y)$. The expression in (22) equals zero ($= 0$) when $G(y)$ is affine in y. Therefore, when $G(y)$ is affine in y, perfect budget balance is attained, i.e. payments (credits) of the users are just redistributed between themselves, and the building operator does not make any profit or loss.

III. Application To RC Model

The consensus algorithm and its convergence, proposed in this work, does not depend on the thermal model of the building so much. Only assumption needed is the convexity of energy cost function $E(u)$. In this section we use the widely popular RC model just for the sake of definiteness, in expressing our control law and for the purpose of simulations. The results can be extended to other models as well. We present an RC model of a building (Section III-A) and then develop an adaptive control law (Section III-B) for simulation purpose on the model.

A. Building Heat Transfer Model

Multiple building modeling strategies have been proposed in the literature, which include the finite element method based model [23], lumped mass and energy transfer model [24], and graph theoretic model based on electrical circuit analogy [25], [26], [27], [28], [29]. For the purpose of simulating our algorithm we take this electrical circuit analogy approach, and combine it with the distributed consensus algorithm to achieve collaborative temperature control of buildings. A building is modeled as a collection of interconnected zones, with energy/temperature dynamics evolving according to a lumped heat transfer model. In the lumped heat transfer model, a single zone is modeled as a thermal capacitor and a wall is modeled as an RC network. This results in the standard lumped 4R3C wall model [26].

The heat flow and thermal capacitance model can be written for all the thermal capacitors in the system, with T_i as the temperature of the ith capacitor. Consider the system to have n thermal capacitors and l thermal resistors. Taking the ambient temperature (T_∞) into account, and neglecting any "thermal noise" in the system, we can write the overall heat transfer model of the system with m zones as [10]:

$$C\dot{T} = -DR^{-1}D^T T + B_0 T_\infty + Bu \tag{23}$$

where $T \in \mathbb{R}^n$ is the temperature vector (representing the temperature of the thermal capacitors in the model), $u \in R^m$ is the vector of heat inputs into the different zones of the building, and $B \in \mathbb{R}^{n \times m}$ is the corresponding input matrix. Also, note that (T, u) are functions of time $(T(t), u(t))$ and accordingly $\dot{T} = \frac{dT}{dt}$. Note that positive values of u correspond to heating the system while negative values of u correspond to cooling. In the above equation, $C \in \mathbb{R}^{n \times n}$ consists of the wall capacitances and is a diagonal positive definite matrix; $R \in \mathbb{R}^{l \times l}$ consists of the thermal resistors in the system and is a diagonal positive definite matrix as well. Also, $D \in \mathbb{R}^{n \times l}$ is the incidence matrix, mapping the system capacitances to the resistors, and is of full row rank [31], and $B_0 = -DR^{-1}d_0^T \in \mathbb{R}^n$ is a column vector with non-zero elements denoting the thermal conductances of nodes connected to the ambient.

In our model, the zones are picked such that each of them has a heating/cooling unit, which in turn implies that B is of full row rank. Also, since matrix D is of full row rank the product $DR^{-1}D^T$ is a positive definite matrix. The vector of zone temperatures, denoted by y (which is a function of T) can be expressed as,

$$y = B^T T. \tag{24}$$

B. Control Law Design

The ADMM algorithm generates consensus among the building occupants and the BTMS, and converges to the minimum cost temperature vector z^* for the building. The objective of the control law design is then to drive all the zones in system (23) to their corresponding consensus temperature z^*_j in steady state. Using the steady state condition we can obtain the steady state temperature y_{ss} and the corresponding steady state input u_{ss} as

$$u_{ss} = g(y_{ss}) \tag{25}$$

Using (24) and the steady state condition in (23) (i.e., setting $\dot{T} = 0$), the steady state mapping can be further expressed in terms of model parameters as:

$$u_{ss} = g(y_{ss}) \doteq (B^T A^{-1} B)^{-1}(y_{ss} - B^T A^{-1} B_0 T_\infty), \quad (26)$$

where $A = DR^{-1}D^T$. We add a passive feedback component to design the control law as:

$$u = u_{ss} - K(y - z^*), \quad (27)$$

and present the corresponding simulation results in Section IV-B and IV-C.

To further remove the model dependency from the control input and incorporate time varying conditions we use the passive controller with adaptive feed forward from [30] (with z^* replacing the y_{des}):

$$\begin{aligned} u &= \bar{F}_0 z^* + \bar{F}_1 T_\infty - K(y - z^*) \\ \dot{\bar{F}}_0 &= -\Gamma_0 (y - z^*) z^{*T} \\ \dot{\bar{F}}_1 &= -\Gamma_1 (y - z^*) T_\infty \end{aligned} \quad (28)$$

where Γ_0 and Γ_1 are both > 0, and present the result in Section IV-D. Though model information is not used in this inner loop temperature controller, the steady state model (26) is needed to solve for z^* in the outer optimization loop.

IV. SIMULATIONS

In this section we present simulation results of our proposed algorithms. We also present a sensitivity study (Section IV-E) of our algorithm with respect to modeling error in RC parameters.

A. Simulation Runs

For the purpose of simulation we consider our six zone experimental facility located in Watervliet, NY. The experimental facility dimensions are as per Figure 3 that has been generated using the BRCM toolbox [34]. For this example model of six zones, the BRCM toolbox generates a total of 31 building elements resulting in a total of 93 capacitive elements. We use the resistance and capacitance matrices as generated by the BRCM toolbox, volumetric heat capacity values and thermal resistance values as per [10] to simulate the temperature dynamics of our linear model in equation (23). Using this information we simulate the model with ambient temperature at $T_\infty = 18°C$. However, it should be noted that the algorithm successfully converges with a wide range of ambient conditions. The presented value of T_∞ was chosen as it offers interesting perspective into the convergence and penalty factor signals corresponding to the preferences of occupants and building operator. The occupancy of the building is as depicted in Figure 4. Zones 1 and 6 are occupied by two occupants each and the other inhabitable zones 3, 4 and 5 have one occupant each. All the occupants have their own specific temperature preference.

The first step in the problem simulation is obtaining the consensus temperature for each zone. We capture the temperature preferences of the occupant of each zone and the corresponding temperature preference of the building operator

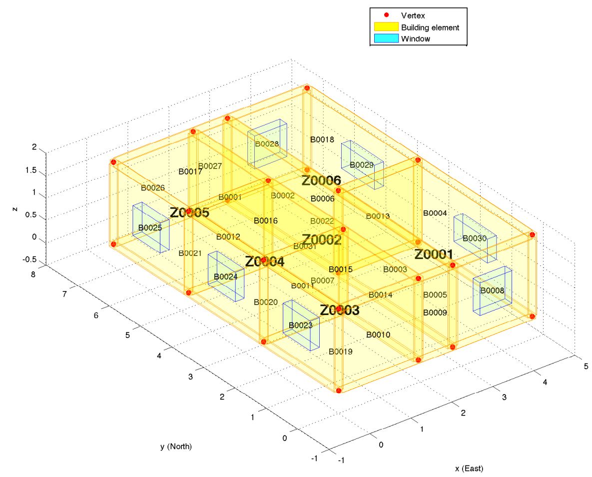

Fig. 3. Test bed with the building elements as generated by the BRCM Toolbox.

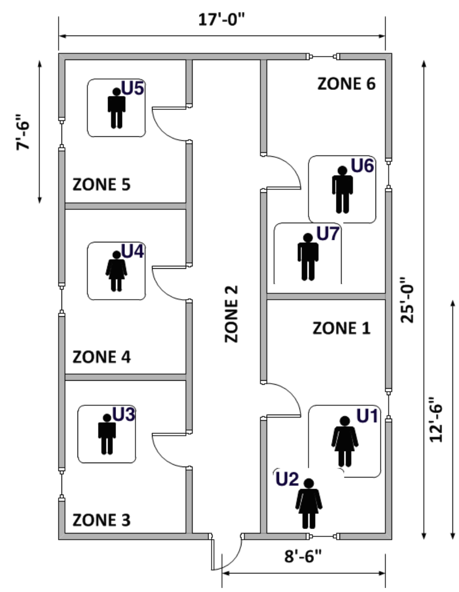

Fig. 4. Test bed layout with occupancy.

for the zones in Table I. Based on the PMV-PPD model [14], [15] we start with a quadratic occupant discomfort function of the form: $\beta(y_j - \alpha_{ij})^2$, where y_j is the temperature of the zone j and α_{ij} is the ideally preferred temperature of the occupant i in zone j as captured in Table I. Note that the convex quadratic thermal discomfort function of an occupant can be defined uniquely using the preferred temperature point (α_{ij}) and the curvature of the quadratic function (β). As and when an occupant indicates a change in the preferred temperature, their thermal discomfort function changes accordingly. In this work we have considered thermal discomfort function with constant curvature β. However, it can be extended to include functions where β can differ across occupants, or the discomfort function is modeled as a more general (complex) convex function.

TABLE I
IDEAL TEMPERATURE SETTING IN $^{\circ}C$ OF EACH ZONE AS PER ITS OCCUPANT AND THE BUILDING OPERATOR

Zone	**Occupant(s) pref**	**Building Operator pref**
Zone 1	18.5 & 19°C	15°C
Zone 3	20°C	15°C
Zone 4	21°C	15°C
Zone 5	22°C	15°C
Zone 6	22.5 & 23°C	15°C

In Figure 5 we present the result of the distributed consensus algorithm using ADMM approach. Each zone (room) occupant agent starts with the ideally preferred temperature set-point of the corresponding occupant as per Table I and the BTMS with the preferred set-point of the building operator for the corresponding zones.

Each iteration in Figure 5 represents one round of communication (computation) between (at) the smart applications and the BTMS. So 100 iterations would represent 100 round-trip communication (100 computation) rounds, and typically evaluate to a few seconds. The occupant at their end would only see the final (converged) temperature and penalty factor signal, the intermediate values being internal to the algorithm would not be visible to the occupants.

With each iteration of the algorithm, the difference between the corresponding zonal temperature preference of the occupant and that of the BTMS narrows and finally compromise is attained in all the zones. Note that in Figure 5 the consensus temperature of zone 6 comes down to that of zone 5, irrespective of the zone 6 occupants preferring much higher temperature than the occupant of zone 5. This is in accordance with the energy cost attributed to maintaining zone 6 relative to zone 5. The trend can also be reasoned from the penalty factor curve in Figure 6, as the penalty factor feedback to zone 6 occupants is much higher compared to the occupant of zone 5. Further, the consensus temperature for both the occupants of zone 1 and zone 6 converge to the respective consensus zonal temperature.

The penalty factor for unit change in temperature varies with each iteration, as shown in Figure 6. The penalty factor increases for the zone occupant if the temperature choice is away from the BTMS' preference and the ambient temperature. In Figure 6 the per-unit penalty factor for occupant 1 (located in zone 1) turns negative. This can be attributed to the fact that on consensus, the temperature for that zone moves away from the ambient and building operator's preferred temperature for the zone, even beyond the occupant's preferred value.

The penalty factor signals in the form of notifications/information can serve as a means for the building operator to communicate with rational occupants into selecting a lower/higher preferred temperature. An occupant of a particular zone might not be aware of their zone's thermal correlation and the temperature preference of the occupants of their neighboring zones. However, the building operator based on the energy cost data can share that picture with the occupants,

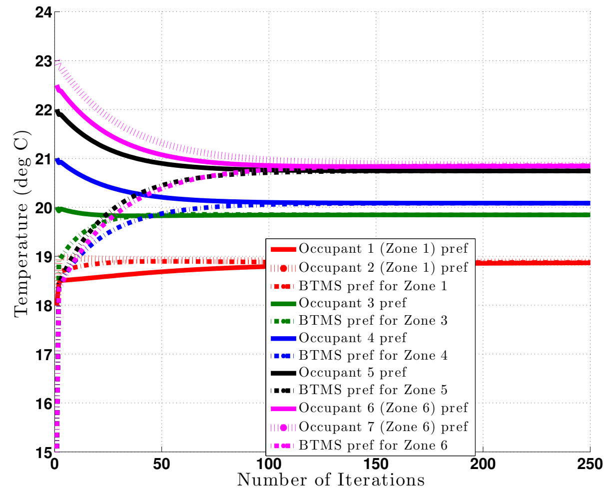

Fig. 5. Convergence of temperature set-point preferences in each zone, for the occupants and the BTMS. The solid lines depict the occupant temperature preferences, and the dashed ones the BTMS' corresponding preferences.

through penalty factor signal, without disclosing any private information. Using this information, rational occupants and the building operator can work together to modify their preference and accommodate users with different and at times extreme (which in general incurs greater overall operating energy cost to the building) thermal preferences.

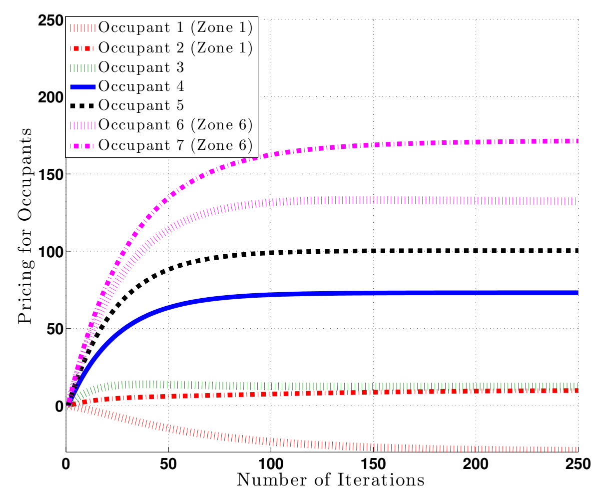

Fig. 6. Variation in penalty factor for the zone occupants for desired change in the zonal temperatures. A negative penalty factor indicates the corresponding occupant receiving reward from the building operator.

B. Uninterrupted user occupancy

Next we use the consensus temperature of the zones as the target temperatures in the building dynamics model in (23) to simulate the temperature variation of the building for a 48 hour period. We present our simulation results with the control law proposed in (27). The corresponding temperature dynamics for a 48 hour period is presented for the six zone model in Figure 7. The temperature of each zone converges to the corresponding component of the consensus temperature vector z^*, as can be observed in the figure.

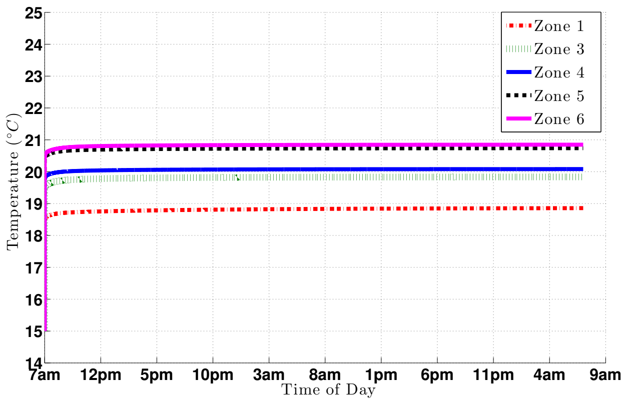

Fig. 7. Temperature dynamics of the six zone model for a 48 hour period with the proportional feedback and model based feed forward controller input.

C. Scheduled user occupancy

However, the uninterrupted occupancy of users presented in the results so far is not a real world scenario as the occupants would be moving in and out. In the next set of simulation results in Figures 8 and 9, we present the temperature dynamics with a real world working environment schedule. The occupants of each room/zone walk-in at 7 am on day 1 (start of the simulation), take an hour long lunch break at 12 pm and leave for the day at 5 pm The following day the occupants get in at 8 am, take the lunch break at noon and leave at 5 pm. When the zones are unoccupied we go into an energy saving mode during which the zonal temperatures start sliding to the ambient temperature. The occupancy of the zones can be obtained through an online occupancy sensor or can be an offline system scheduler.

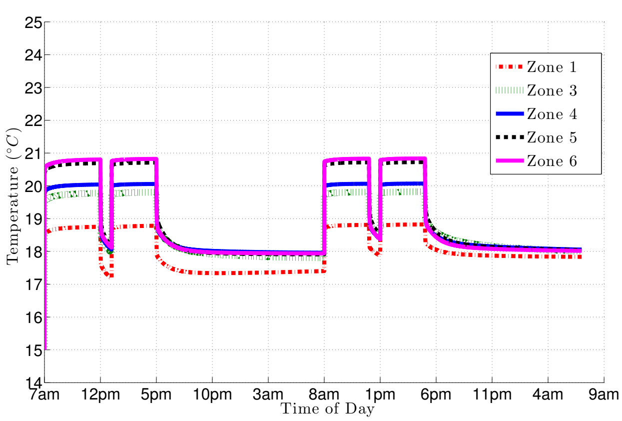

Fig. 8. Temperature dynamics of the six zone model for a 48 hour period, with real-world user occupancy schedule.

The corresponding heat input variation is presented in Figure 9. The short duration burst of high heat inputs correspond to the occupants returning from a break (lunch or next day morning), ensuring fast convergence to the optimum temperatures when the rooms get re-occupied. When the temperatures get close to the desired values, then a much lower heat input (closer to the steady state value) suffices.

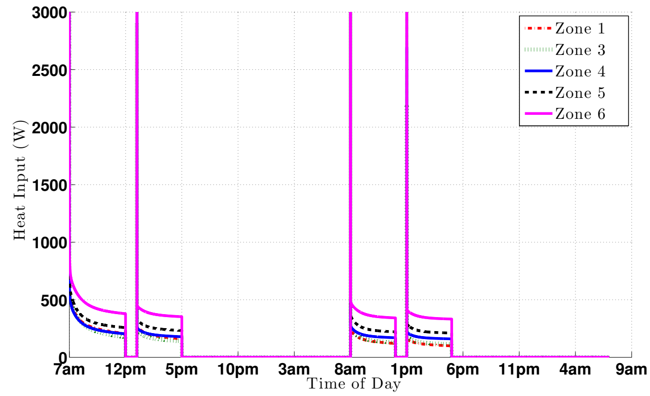

Fig. 9. Heat input variation of the six zone model for a 48 hour period, with real-world user occupancy scheduling.

D. Adaptive Controller with time varying T_∞

So far we have presented simulation results with our model based feed forward controller in (27) considering a fixed value of T_∞. Next we consider a time-varying ambient temperature of the form $T_\infty = 18°C + 5°C\sin(2\pi t/T)$, with $T = 24$hr. The model based feed forward controller cannot adapt to the time varying T_∞ as can be seen in Figure 10. We next use the adaptive feed forward controller as proposed in (28) with time varying T_∞. Convergence of the temperature for each zone is obtained as can be seen in Figure 11. The zonal temperature no longer fluctuates with T_∞ as the control input in Figure 12 is out of phase with T_∞.

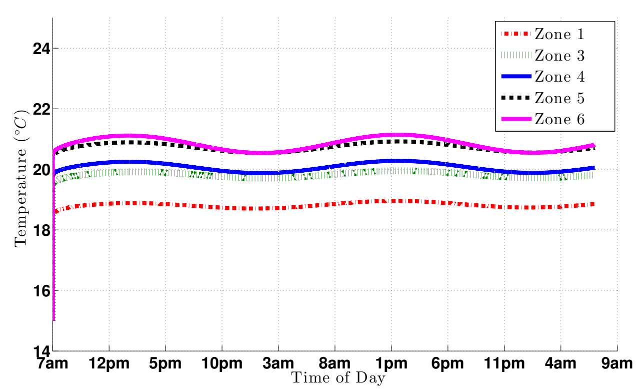

Fig. 10. Zonal temperature dynamics for time varying sinusoidal T_∞ with the model based feed forward controller.

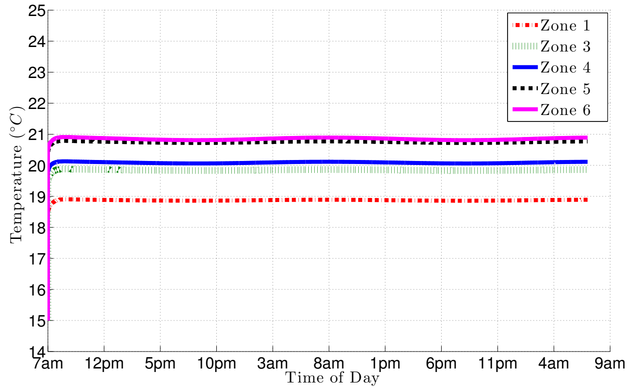

Fig. 11. Zonal temperature dynamics for time varying sinusoidal T_∞ with the adaptive feed forward controller.

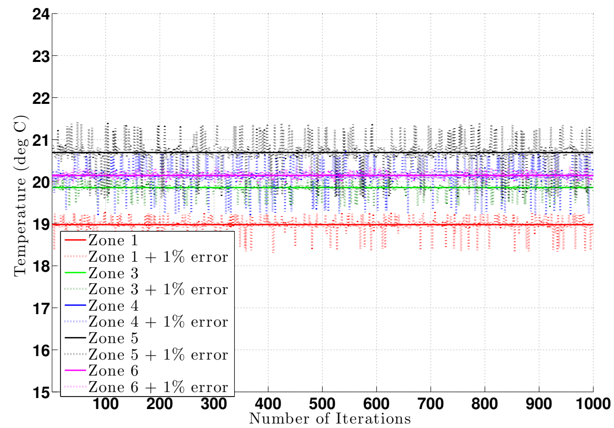

Fig. 13. Monte Carlo simulation with 1000 iterations and 1% error in model.

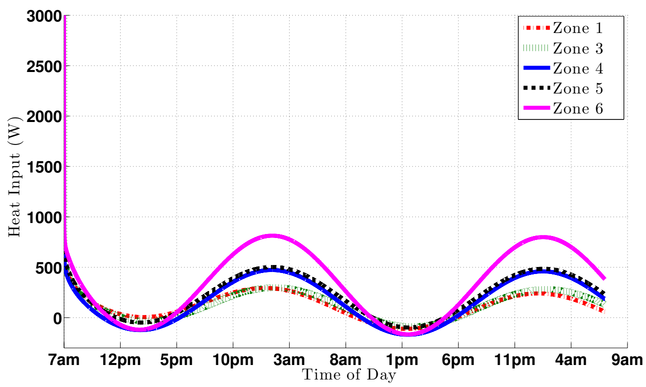

Fig. 12. Heat input into the zones corresponding to adaptive feed forward controller for time varying sinusoidal T_∞.

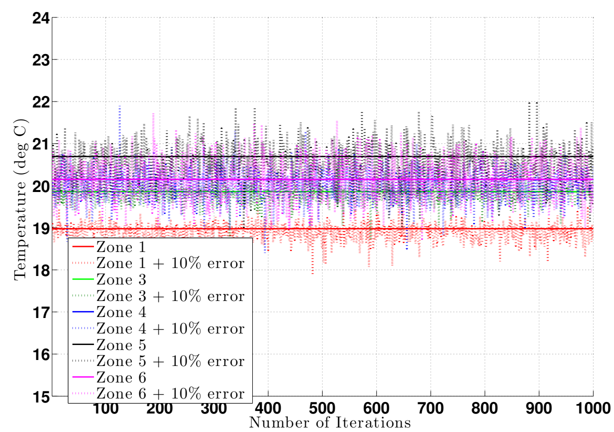

Fig. 14. Monte Carlo simulation with 1000 iterations and 10% error in model.

E. Sensitivity Analysis

The adaptive feed forward controller and the consensus algorithm is fairly model independent. However, for this study we relied on RC model parameters for the energy cost function $G(z)$. In this section we present results on sensitivity of the consensus algorithm to the percentage error in the modeling parameters of the test facility. We run Monte Carlo simulation with 1000 iterations each for 1% to 10% randomly induced error. Figures 13 and 14 show the results for 1% and 10% induced error respectively. In figure 15 we present the error bar graph with increasing percentage error. Simulation data suggests that the algorithm is fairly robust to errors in modeling. Next, we also simulate temperature dynamics for the six zone model for the consensus temperature obtained with 10% modeling error. Figure 16 represents the results and establishes that the consensus temperatures with modeling error can also be successfully achieved.

V. Conclusion

In this work, we have proposed an approach for collaborative temperature control in multi-occupant spaces, that uses pricing feedback to attain a consensus between the rational occupants (interested in minimizing their individual discomfort plus energy cost) and the building operator (thermal management system). Upon convergence, the consensus algorithm attains temperature set-points that minimize the sum of the aggregate discomfort of the occupants and the total energy cost in the building. The temperature set-points attained on consensus is then used by a control law with proportional feedback and an adaptive feed forward component, to drive the building to the desired (optimal) temperature. Through simulations, we have demonstrated the convergence of the consensus algorithm, as well as the control law, to the desired (optimal) temperatures. We have further included a study on the sensitivity of the algorithm to potential errors in the estimation of model parameters. This establishes robustness of our algorithm to modeling and estimation errors.

APPENDIX

A. Proof of Lemma 1

Since L_o has a saddle point:

$$L_o(x_{ij}^*, p_{ij}^*, z_j^*) \leq L_o(x_{ij}^{k+1}, p_{ij}^*, z_j^{k+1}) \qquad (29)$$

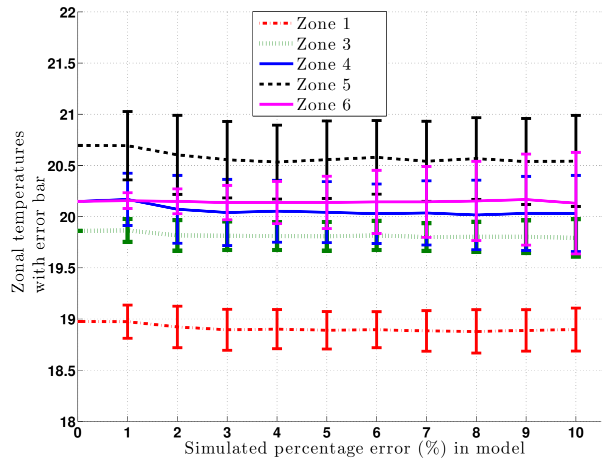

Fig. 15. Monte Carlo simulation error bar graph for 1% to 10% modeling error. Error bar represents standard deviation of the values.

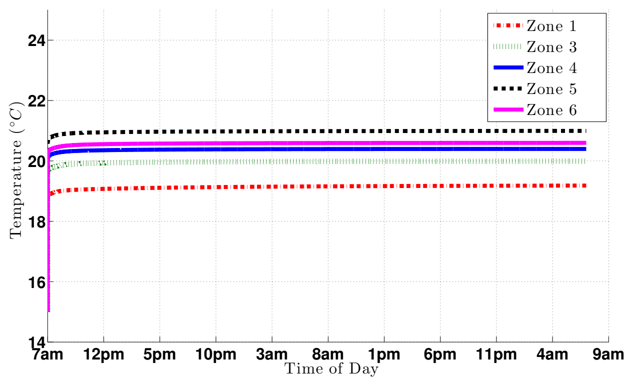

Fig. 16. Temperature dynamics for consensus zonal temperatures obtained with 10% modeling error.

Since, the constraint is satisfied at optimality:

$$x_{ij}^* = z_j^* \implies L_o(x_{ij}^*, p_{ij}^*, z_j^*) = O^* \tag{30}$$

Also,

$$\begin{aligned} L_o(x_{ij}^{k+1}, p_{ij}^*, z_j^{k+1}) &= \sum_{j=1}^{m}\sum_{i\in S_j} D_i(x_{ij}^{k+1}) + G(z^{k+1}) \\ &\quad + \sum_{j=1}^{m}\sum_{i\in S_j} p_{ij}^*(x_{ij}^{k+1} - z_j^{k+1}) \\ &= O^{k+1} + \sum_{j=1}^{m}\sum_{i\in S_j} p_{ij}^* r_{ij}^{k+1} \end{aligned} \tag{31}$$

Hence,

$$O^* \leq O^{k+1} + \sum_{j=1}^{m}\sum_{i\in S_j} p_{ij}^* r_{ij}^{k+1} \tag{32}$$

which completes the proof of inequality (13). ■

B. Proof of Lemma 2

From the augmented Lagrangian in (4) and re-writing the update equation in (7) as:

$$p_{ij}^{k+1} = p_{ij}^k + \rho r_{ij}^{k+1}, \tag{33}$$

it can be shown that x_{ij}^{k+1} minimizes

$$D_i(x_{ij}) + p_{ij}^{k+1} x_{ij} + \rho x_{ij}(z_j^{k+1} - z_j^k), \tag{34}$$

and similarly z_j^{k+1} minimizes

$$G(z) - p_{ij}^{k+1} z_j. \tag{35}$$

Hence,

$$\begin{aligned} D_i(x_{ij}^{k+1}) + p_{ij}^{k+1} x_{ij}^{k+1} + \rho x_{ij}^{k+1}(z_j^{k+1} - z_j^k) \leq \\ D_i(x_{ij}^*) + p_{ij}^{k+1} x_{ij}^* + \rho x_{ij}^*(z_j^{k+1} - z_j^k), \end{aligned} \tag{36}$$

and

$$G(z^{k+1}) - p_{ij}^{k+1} z_j^{k+1} \leq G(z^*) - p_{ij}^{k+1} z_j^*. \tag{37}$$

Adding (36) and (37) across all zones and all occupants, and re-arranging we obtain the inequality (14). ■

C. Proof of Lemma 3

Adding inequalities (13) and (14) and multiplying by 2 we obtain:

$$\begin{aligned} &\sum_{j=1}^{m}\sum_{i\in S_j}\left(2r_{ij}^{k+1}(p_{ij}^{k+1} - p_{ij}^*) + 2\rho r_{ij}^{k+1}(z_j^{k+1} - z_j^k)\right) \\ &+ \; 2\rho\sum_{j=1}^{m}\left((z_j^{k+1} - z_j^k)(z_j^{k+1} - z_j^*)\right) \leq 0. \end{aligned} \tag{38}$$

Using update relation (33) in (38) and re-arranging terms we can obtain:

$$\begin{aligned} &(1/\rho)\sum_{j=1}^{m}\sum_{i\in S_j}\Big((|p_{ij}^{k+1} - p_{ij}^*|^2 - |p_{ij}^k - p_{ij}^*|^2) \\ &\qquad + \rho|r_{ij}^{k+1} + (z_j^{k+1} - z_j^k)|^2\Big) \\ &+ \rho\sum_{i\in S_j}\left(((z_j^{k+1} - z_j^*)^2 - (z_j^k - z_j^*)^2)\right) \leq 0. \end{aligned} \tag{39}$$

From the definition of Lyapunov function in (15) this gives:

$$V^{k+1} - V^k + \rho\sum_{j=1}^{m}\sum_{i\in S_j}|r_{ij}^{k+1} + (z_j^{k+1} - z_j^k)|^2 \leq 0, \tag{40}$$

which can be re-written as:

$$\begin{aligned} V^{k+1} \leq V^k - \rho\sum_{j=1}^{m}\sum_{i\in S_j}|r_{ij}^{k+1}|^2 - \rho\sum_{j=1}^{m}|z_j^{k+1} - z_j^k|^2 \\ - 2\rho\sum_{j=1}^{m}\sum_{i\in S_j} r_{ij}^{k+1}(z_j^{k+1} - z_j^k). \end{aligned} \tag{41}$$

The last term in (41) can be shown to be positive, which proves the third inequality (16). ■

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