Extended Second Price Auctions for Plug-in Electric Vehicle (PEV) Charging in Smart Distribution Grids

Saptarshi Bhattacharya, Koushik Kar, Joe H. Chow and Aparna Gupta

***Abstract*— Large scale deployment of Plug-in Electric Vehicles (PEVs) in the smart grid environment necessitates the use of appropriate charging control algorithms that handle the additional PEV based load effectively. While ensuring grid stability, these PEV charging control algorithms must ensure that the available energy is delivered to where it is needed the most. In this paper, we explore the question of efficient allocation of energy (charging rates and schedules) to PEVs by the aggregator (electricity utility) through an auction mechanism. Recognizing the practical limitations of the Vickrey-Clark-Groves (VCG) mechanism which would be natural to apply in this context, we investigate two practical mechanisms that can be viewed as extensions of second price auction mechanisms, and have limited message (bid) complexity. In the first mechanism, the *multi-level second price (MSP)*, each PEV agent submits a number of price bids, one for each of a given set of energy levels (energy quantities). In the second mechanism, the *progressive second price (PSP)*, the PEV agents submit a two-dimensional bid indicating the price as well as the desired energy quantity. Taking into account differences across PEV-owners in terms of their willingness-to-pay values and charging time constraints, we analyze the *social optimality* and *incentive compatibility* properties of the two auction mechanisms.**

I. Introduction

Effective management of the PEV based electricity demand will be crucial for maintaining the stability and the operational efficiency of the power grid in the near future [1], [2], [3]. Fortunately, PEVs also provide significant flexibility in terms of their energy consumption rates and schedules, which can be utilized towards reducing the variability of the aggregate demand over time. Additionally, it can also help to partially absorb the variability associated with the supply side, particularly when a significant fraction of the energy is being supplied from variable rate renewable energy sources. Coordinated charging of PEVs is necessary for optimizing electricity dispatch over a temporal scale, hence ensuring that undesirable demand peaks are not created in the power grid [4]. The problem of charging electric vehicles has been dealt with by several researchers in the past from different perspectives including game theory [5], [6], gradient optimization [7], sequential quadratic optimization [8], [9], dynamic programming [10] and other heuristic methods [11]. A price driven charging mechanism that results from non-linear pricing of PEV demand and results in load variance minimization is reported in [12].

In this paper, we study the use of *auction mechanisms* for *efficient* charging control (scheduling) problem for PEVs. We realize that PEV agents (PEV owners) can differ in terms of their charging constraints and their willing-to-pay values (for the energy given to them), and a desirable goal would be maximize the total valuation of the resulting energy allocation (or equivalently, the charging rates and schedules of the agents). Since the valuation and charging constraints are private information to the agents, the auction mechanism must induce the agents to be *truthful* in declaring that information to the aggregator, or in making its bids as required by the auction process. A natural candidate for this auction is a *Vickrey-Clark-Groves (VCG)* mechanism [13] which requires the agents to pay according to the their *opportunity cost* values. It easily follows that VCG payments induce agents to declare their valuation functions truthfully, based on which the aggregator can compute and assign an energy allocation (charging solution) that is *socially optimal*, i.e., maximizes the aggregate valuation function over all PEVs, given their charging constraints as well as the energy availability constraints of the distribution network. In other words, the socially optimal solution can be realized in *dominant strategies*, which is a very desirable property for the auction.

The VCG mechanism is associated with some practical limitations however, which poses difficulties in implementing it directly in our problem context. Firstly, direct implementation of the VCG mechanism requires the agents to declare their entire valuation function, which is typically not exactly known to (or may be hard to estimate by) the agents (PEV owners) themselves. Secondly, even if the valuation functions are known explicitly, declaring them to the aggregator (exactly, or to a close degree of approximation) requires a very high message complexity, as the function is defined over a continuous space of real numbers. To address these limitations of VCG, in this paper we apply and study two extensions of this auction mechanism to the PEV charging context, that require the agent to declare only a small number of price and/or quantity values to the auctioneer (aggregator). Despite this, we show that the proposed auction mechanisms retain the desirable incentive compatibility and social optimality properties, at least to a reasonable/desired degree of accuracy, or when equilibrium is attained. Since the two auction mechanisms proposed can be viewed as extensions of VCG, or more generally extensions of the second price auctions, we will refer them to as *extended second price auction* mechanisms.

In the first mechanism, which we call the *multi-level second price (MSP)* auction, each agent is required to declare a set of prices that it is willing to pay for certain (given)

The authors are associated with the Rensselaer Polytechnic Institute, Troy, NY - USA. {bhatts5, kark, chowj, guptaa}@rpi.edu

levels of energy (quantities of charge). The energy allocation to the agents can occur at any value up to the maximum requested quantity (no necessarily at the quantities for which the agent declares the price-bids), and the agent is required to pay the opportunity cost computed based on a piecewise linear approximation of their true valuation functions, as computed by the aggregator based on the declared price bids. This mechanism can rightly be seen as a discretized approximation of the VCG mechanism. Not surprisingly, therefore, we show that this mechanism guarantees incentive compatibility and social optimality unto an additive approximation factor that depends on the degree of discretization, expressed as the maximum difference between the energy quantities for which the agent is required to declare the price bids. Since this maximum difference can be controlled by the number of price bids that the agent is required to declare, the aggregator can trade-off the incentive compatibility and social optimality of the auction mechanism with the message complexity, to any desired degree.

In the second mechanism, which we call the *progressive second price (PSP)* auction, each agent is required to declare a two-dimensional bid comparing of a price (willingness to pay) and a requested quantity. The energy allocation to the agents can occur at any value up to the requested quantity, and the agent is required to pay the opportunity cost computed based on the price bids and the requested quantities. We argue, following/extending the results/analysis of PSP auctions in [15], [16] that social optimality is attained *at Nash equilibrium*; a weaker notion of incentive compatibility (i.e. truthful declaration of the two-dimensional bid) also holds. It is worth noting that the MSP mechanism is meant to work at a single-step auction process. More specifically, the aggregator requests agents for the multilevel price bids, which the agents declare truthfully (up to a degree of approximation); the energy allocation is done by the aggregator in response, and the auction concludes. For the PSP mechanism, however, since the social optimality is attained at equilibrium, the auction process may need to be implemented over a reasonable window of time, allowing agents to re-bid (possibly many times) and attain equilibrium.

MSP and PSP auction mechanisms or their analogues have been considered in prior work, mostly in the context of bandwidth allocation in the Internet [14]-[20]. The MSP mechanism is related to the notion of multi-bid auctions investigated in [19], [20]. The application context, and the network model is significantly different however; in particular our system reduces to a bipartite network graph whereas [19], [20] studies a single node (single block of divisible resource) or resources connected as a tree network topology. While the broad nature of our approximation results (for incentive compatibility and social optimality) are similar to those in these prior work, our mechanism is slightly different - which we believe not only makes it more practical to implement it (in our application context at least), but also allows much simpler proofs of the results. The PSP mechanism was proposed in [14], and further analyzed in [15]-[18], all in the context of bandwidth auctions. The proposed PSP mechanism can be viewed an a generalization of the one suggested in [14], and a special case of that considered (for a general network topology) in [16]. The result on "social optimality at equilibrium" for our PSP mechanism is implied by [16], albeit for a slightly stronger assumption on the valuation functions. For the valuation function types we consider, this result can however be obtained by extending the proof of the analogous result obtained in [14], [15] for the single-resource case; a result showing the incentive compatibility of the PSP mechanism (although the notion of truthfulness here is weaker than than what can be shown for a VCG) can also be obtained similarly. Very recently, PSP auctions have been applied to the problem of coordination of large scale elastic loads (which may include PEVs) as well [22]. The main difference in our treatment/application of the PSP mechanism in this context, as compared to this recent work, is that we explicitly take into account differences across agents (PEV owners) in terms of their charging time constraints.

The paper is structured as follows. The next section (Section II) describes the system model. Sections III and IV describe and analyze the MSP and PSP auction mechanisms, respectively. We evaluate these two auction mechanisms through simulations in Section V.

II. System Model

Let $\mathcal{K} = \{1, 2, ..., k, ..., K\}$ denote the set of PEVs that are attempting to obtain energy from the grid. The charging time constraints of the PEV k are captured by the set T_k that contains all permissible time slots for PEV agent k in the next charging period. In case of contiguous permissible time slots, T_k has the special form $T_k = \{t_s^k, ..., t_f^k\}$ where t_s^k and t_f^k are the charging start and finish times for agent k, respectively.

In practice, the total time for which the energy is being auctioned could vary - it could be the next day or the next hour, for example. Let the total time available for charging (for which the auction is being conducted) be divided into T slots. Let the set $\mathcal{T} = \{1, 2, ..., T\}$ represent the set of these (contiguous) time slots. We assume that due to the variability in the generation rate (that may arise due to the renewable energy generation units, for example), each time slot is associated with a specific amount of energy g_t, as indicated in Figure 1. The transformer distribution capacity is assumed to W. Assuming that every time slot $t \in \{1, 2, ..., T\}$ has a non-PEV inelastic demand associated with it which is designated as d_t, the amount of electricity which is available for dispatch to the PEVs in the distribution network, in any given time slot t is $w_t = \min(W, g_t) - d_t$. Under a feasible allocation (result of the auction), the net charge received by the PEV k is $\sum_{t \in T_k} Q_k^t$, where Q_k^t is the amount of charge received by the PEV k in time slot t. Note this total charge (energy) must be less than the remaining battery capacity of PEV k, denoted by β_k, which can be measured/estimated by the aggregator through smart metering equipment.

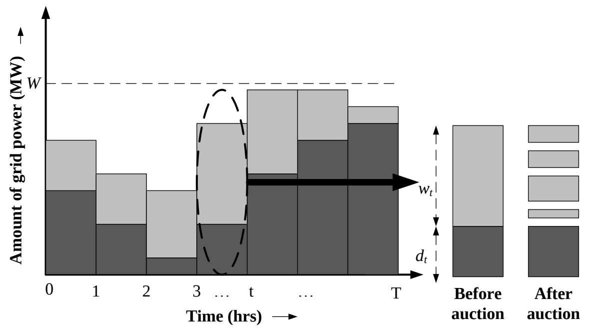

Fig. 1. Auction of electricity to PEVs in the different time slots.

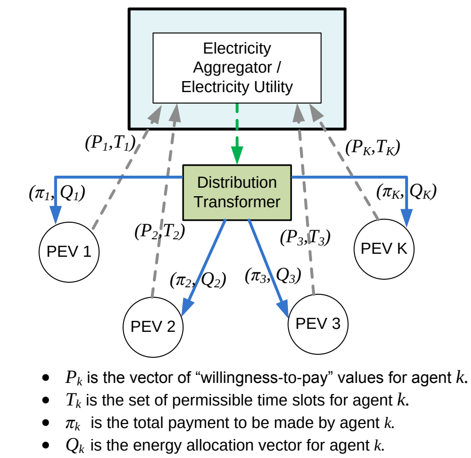

Fig. 2. Schematic diagram of the MSP auction process.

After the PEV agents submit their charging preferences, the aggregator must appropriately conduct a fair energy allocation scheme based on the available information in a manner that maximizes the entire valuation of the resource at hand for dispatch (efficient auction), as well as ensure that there is no incentive for individual agents to gain anything through untruthful declaration of their charging preferences (bids in the auction). Ensuring efficient allocation schemes would have been possible and fairly simplistic had the aggregator possessed information about the entire *valuation function* of all agents. Such a valuation function θ_k (for each vehicle $k \in \mathcal{K}$) can be assumed to be non-decreasing with diminishing returns (strictly concave). In that case, the aggregator could optimize

$$\max \sum_{k\in\mathcal{K}} \theta_k(\sum_{t\in T_k} Q_k^t), \tag{1}$$

$$\text{s.t.} \sum_{k\in\mathcal{K}} Q_k^t \leq w_t \quad \forall t \in \mathcal{T}, \tag{2}$$

$$\sum_{t\in T_k} Q_k^t \leq \beta_k \quad \forall k \in \mathcal{K}, \tag{3}$$

$$Q_k^t \geq 0 \quad \forall t \in T_k, \quad Q_k^t = 0 \; \forall t \notin T_k \quad \forall k \in \mathcal{K}. \tag{4}$$

Since on general the aggregator does not know the agent valuation functions, one option it is design a payment scheme that induces the agents to declare their valuations *truthfully*. This is what the VCG mechanism [13] attempts to achieve, by setting the payment of agent k to be the *opportunity cost* caused by the presence of an agent. Then it can be shown the agent k that seeks to maximize its utility computed as $U_k = \theta_k(\sum_{t\in T_k} Q_k^t) - \pi_k$ would not have any incentive to misreport its valuation function (incentive compatibility). However, application of VCG in our context has several drawbacks as outlined in Section I. Thus, it becomes imperative for the aggregator to obtain/estimate the individual agent preferences (valuations) as much as possible through limited messaging. In this paper, we study how this could be done through two auction processes intuitively motivated by VCG (or generalized second price) auctions: (a) the *multi-level second price (MSP)* auction, (b) the *progressive second price (PSP)* auction.

III. The Multi-level Second Price (MSP) Auction

A. Bidding Process and Electricity Dispatch

In MSP auction, the electricity aggregator selects a number of energy levels (say N_k levels for agent k, indexed as $\bar{Q}_{k,1}, \cdots \bar{Q}_{k,N_k}$, for which the agents (PEV agents) have to declare the prices they are willing to pay for that energy level. Let $P_{k,n}$ denote the willingness-to-pay value (total price bid) of agent k corresponding to energy level $\bar{Q}_{k,n}$ (for any $n = 1, \cdots, N_k$). The aggregator interprets these willingness-to-pay values as the agents valuation of that particular energy level, and constructs a piecewise linear *approximate valuation function* (which can be assumed to be monotonic with diminishing returns) $\bar{\theta}_k$ such that $\bar{\theta}_k(\bar{Q}_{k,n}) = P_{k,n}$, and $\bar{\theta}_k$ is linear in between two consecutive energy levels, $Q_{k,n}$ and $Q_{k,n+1}, n = 1, \cdots, N_{k-1}$. It is possible that the energy levels are pre-determined, and known to both the aggregator and the agents; it could also be dynamically determined based by the aggregator based on the remaining batter capacity (β_k), assumed to be measured/estimated by the aggregator. Additionally, agents also report their charging time constraints, denoted by the set T_k. Now, the aggregator solves the following optimization problem to calculate the allocation rule for all participating PEVs under constraints (2)-(4),

$$\max \sum_{k\in\mathcal{K}} \bar{\theta}_k(\sum_{t\in T_k} Q_k^t). \tag{5}$$

The aggregator also calculates the payment for PEV k as,

$$\pi_k = \sum_{j\neq k} (\bar{\theta}_j(a_{j,-k}) - \bar{\theta}_j(a_{j,k})). \tag{6}$$

Here, $\bar{\theta}_j(a_{j,-k})$ represents the approximate valuation of agent j when agent k is not present in the auction; here, $a_{j,-k}$ represents the total energy that would have been allocated to agent j in absence of agent k. Similarly, $\bar{\theta}_j(a_j)$ represents the approximate valuation of agent j when all agents (including agent k) are present. Intuitively, π_k represents the opportunity cost of agent k. It is possible that this opportunity cost evaluates to zero; typically there will be a nominal amount that must be paid by any agent to get energy, or even to participate in the auction. A schematic diagram for the overall process is given in Figure 2, and the auction process is described below as Algorithm 1.

Algorithm 1 MSP auction for PEV charging

Step 1: Each agent $k \in \mathcal{K}$ submits its price bid vector P_k and charging constraint set T_k, to the aggregator.
Step 2: Aggregator calculates the piecewise linear approximate valuation function $\bar{\theta}_k(x)\ \forall k \in \mathcal{K}$.
Step 3: Aggregator solves (5) (subject to appropriate constraints) to compute the optimal schedule $Q_k = \{Q_k^1, Q_k^2, ..., Q_k^T\}\ \forall k \in \mathcal{K}$.
Step 4: Aggregator computes $\pi_k\ \forall k \in \mathcal{K}$ using (6).
Step 5: Q_k and π_k are communicated to all $k \in \mathcal{K}$.

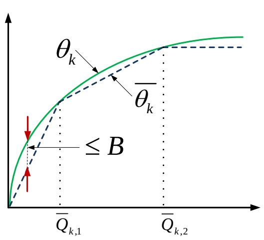

Fig. 3. Comparison of functions θ_k and $\bar{\theta}_k$.

B. Analysis

Assuming that the actual valuation function θ_k is non-decreasing, concave with $\theta_k(0) = 0$, and assuming $\bar{\theta}_k(0) = 0$, it is easy to see that $\bar{\theta}_k(x) \leq \theta_k(x),\ \forall x$. This is illustrated in Figure 3. Let $B = \max_k \max_{x \leq \beta_k} \left(\theta_k(x) - \bar{\theta}_k(x)\right)$ represent the maximum deviation of the approximate valuation function $\bar{\theta}_k$ from the actual valuation θ_k over the energy range of interest. (Note that β_k is upper bounded by battery capacity.) Then we have the following results.

Proposition 1: Under truthful bidding, the social valuation of the energy allocation resulting from the MSP auction differs from the maximum possible social valuation by at most KB.[1]

Proof: See Appendix I ■

In the above, by "truthful" bidding, we mean that all agents report their willingness-to-pay values correctly, i.e. declare $P_{k,n}$ to be $\theta_k(Q_{k,n})$, the real valuations at each level n. Proposition 1 is intuitive, as the actual valuation functions θ_k and their piece-wise linear approximations, $\bar{\theta}_k$, differ by at most B.

Proposition 2: The maximum utility gained by any PEV k by untruthful bidding in the MSP auction is upper-bounded by B.

Proof: See Appendix I ■

[1]This result appears incorrectly in [23] where the bound is given as K.

In the above, by "untruthful" bidding, we mean declaring $P_{k,n}$ values that differ from $\theta_k(Q_{k,n})$ for any (possibly multiple or all) level(s) n. Note that the result holds irrespective of the bidding assumption (truthful or untruthful) on the other agents. Thus, Proposition 1 shows that truthful bidding is an *approximate* dominant strategy, where the additive approximation factor is B. It is also easy to argue that an agent k would not have any incentive to not truthfully report the constraint set T_k.

Finally note that B can be bounded in terms of the "granularity" at which the agents are required to submit their bids, as follows. Let $\hat{\delta}$ be the maximum gap between successive energy levels for which the agents are required to declare their willingness-to-pay values, i.e., $\hat{\delta} = \max_k \max_{n=1}^{N_k} (Q_{k,n} - Q_{k,n-1})$, where $Q_{k,0}$ is assumed to be zero for all k. Also, assume that the valuation function θ_k is differentiable and *strictly* concave, and $-\theta_k^{''}(x) \geq \nu > 0$ for all $x \leq \beta_k$. Then, it can be shown through the following lemma that the degree of approximation in Proposition 1 (social optimality) and Proposition 2 can be made as small as desired by reducing $\hat{\delta}$.

Lemma 1: Assume that the valuation function θ_k is differentiable and *strictly* concave, and $-\theta_k^{''}(x) \geq \nu > 0$ for all $x \leq \beta_k$. Then $B \leq \frac{\nu\hat{\delta}^2}{2}$.

Proof: See Appendix I ■

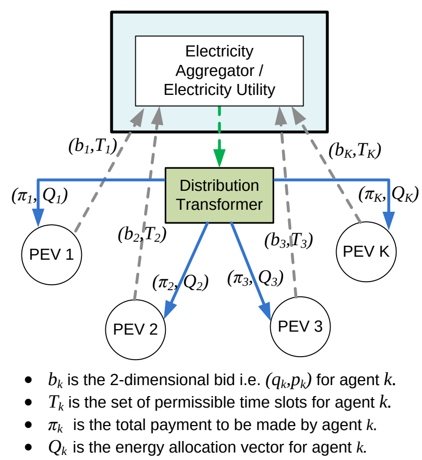

Fig. 4. Schematic diagram of the PSP auction process.

IV. The Progressive Second Price (PSP) auction

A. Bidding Process and Electricity Dispatch

In this scheme, which is is derived from the auction process described and analyzed in [15], [16] for bandwidth resources, the electricity aggregator asks the PEV agents to submit a 2-dimensional bid (q_k, p_k) where q_k is the amount of energy that the PEV k is willing to obtain at a *per unit* price of p_k; agents also submit their charging time constraints. Based on these bids, the aggregator tries to solve the following optimization problem,

$$\max \quad \sum_{k\in\mathcal{K}} \sum_{t\in T_k} p_k Q_k^t(p_k), \tag{7}$$

$$\text{s.t.} \sum_{t\in T_k} Q_k^t \ \leq \ q_k, \tag{8}$$

and constraints (2), (4). Note that the constraint (8) supercedes the constraint (3), since $q_k \leq \beta_k$. The total payment for the PEV k is calculated as,

$$\pi_k = \sum_{j\neq k} p_j \left(\sum_{t\in T_k} \tilde{Q}_j^t - Q_j^t \right), \tag{9}$$

where Q_j^t, $\forall j = 1, 2, .., K$ is obtained as optimal solution of (7), subject to the appropriate constraints. Moreover, $\tilde{Q}_j^t$, $\forall j = 1, 2, .., K$ is obtained as an optimal solution to the same problem, but assuming that PEV k is absent in the auction process (with all other PEV agents and their bids remaining unchanged). Unlike MSP auctions however, here the agents are allowed to rebid (hence the term *progressive* in PSP) to improve their utility computed as $U_k = \theta_k(\sum_{t\in T_k} Q_k^t) - \pi_k$. It is assumed that the bidding process continues until an equilibrium is reached, i.e., no agent can gain utility by more than ϵ (cost of bid) by rebidding. Figure 4 provides an overview of the PSP auction; a synchronous iterative implementation of the auction process is outlined in Algorithm 2.

Algorithm 2 PSP auction for PEV charging

Step 1: $n \leftarrow 1$ (Start of PSP auction).
Step 2: Each agent $k \in \mathcal{K}$ submits its price bid vector P_k and charging constraint set T_k, to the aggregator, in the n^{th} round.
Step 3: Aggregator solves (7) (subject to appropriate constraints) to compute the optimal schedule $Q_k = \{Q_k^1, Q_k^2, ..., Q_k^T\}$ $\forall k \in \mathcal{K}$.
Step 4: Aggregator computes π_k $\forall k \in \mathcal{K}$ using equation (9).
Step 5: Q_k and π_k are communicated to all $k \in \mathcal{K}$ (along with possibly the bids and constraints of other agents).
Step 6: Every agent k computes to see if the if it can improve its utility by rebidding (assuming the bids of others remain unchanged).
Step 7: If the improvement is more than ϵ (cost of bid) for any $k \in \mathcal{K}$, increase n by 1 and go to Step 2; else stop auction.

B. Analysis of the PSP auction

This section starts with a graph theoretic interpretation of the charging allocation problem. Subsequently, we state a set of lemmas that are instrumental in arriving at the main results related to this mechanism. These main results, that comprise of the incentive compatibility and efficiency properties of the PSP auction mechanism, conclude this section. The detailed proofs of the lemmas can be found in Appendix II. The allocation of charge to the PEVs by the aggregator can be modeled as a max-cost flow problem (maximum weighted multi-commodity flow) whereby the objective of the aggregator would be to maximize its revenue, i.e., maximizing $\sum_{k\in\mathcal{K}} p_k \mathcal{Q}_k(p_k)$, given the bids of the users (refer to step 2 of the auction algorithm description in the previous subsection). Note that $\mathcal{Q}_k = \sum_{t\in T_k} Q_k^t$. This is in tune with the objective of the aggregator which is to maximize the term $\sum_{k\in\mathcal{K}} p_k \mathcal{Q}_k(p_k)$. Let us consider that the set of PEVs be represented by K nodes. Without loss of generality it is assumed that these nodes are arranged in top-down manner in increasing order of price bids (p_k). Each of these $k \in \{1, 2, ..., K\}$ nodes has a node capacity (q_k) associated with them which represents the maximum possible charge it can obtain from all feasible time slots. Let the T time slots be represented by another set as shown in Figure 5, where node (time slot) t is associated with a nodal capacity of w_t. We assume that the edge between source node $\mathbf{S}$ and PEV node k has capacity q_k and the edges between time slot node t and sink node $\mathbf{T}$ has capacity w_t. In this network, the flow between source $\mathbf{S}$ and sink $\mathbf{T}$ represents the total amount of electricity (energy) used for PEV charging. If we assume that the flow through PEV node k is weighted by p_k, then the maximum weight (multi-commodity) flow from $\mathbf{S}$ to $\mathbf{T}$ represents the optimal charging solution that solves the linear program in equation (7). In other words, this flow maximizes $\sum_{k\in\mathcal{K}} p_k \mathcal{Q}_k(p_k)$, and the flow on the edge (k, t) (for PEV node k and time slot node t) in the optimal flow solution, Q_k^t represents the charge that PEV k obtains at time slot t.

In the following, we will assume that (PEV, time) bipartite graph is *connected*, in the following sense. Consider just the bipartite graph of the PEV and time slot nodes by removing the source and sink nodes (and all edges attached to them). In this reduced graph, if we assume all the (k, j) edges are undirected, then the entire graph constitutes a *single connected component*, i.e., there exists a path from any node of the graph to another. It is easy to see that if the network consists of multiple such connected components, the different components are disjoint, and therefore the optimal charging solutions in the different connected components can be computed independently of one another. Therefore, there is no loss of generality in analyzing a single connected component in isolation; the results that we present next for a single connected component do generalize if the network in Figure 5 consists of multiple connected components. Note that this node capacitated network model can easily be converted to the standard edge capacitated network model, as shown the Appendix II (Figure 9).

Lemma 2: Assuming all the price bids are non zero, the the total amount of energy allocated to all PEVs over all time slots is the same in all optimal charging solutions.

The above result states that under any optimal charging solution, the total energy allocated to PEVs over the entire

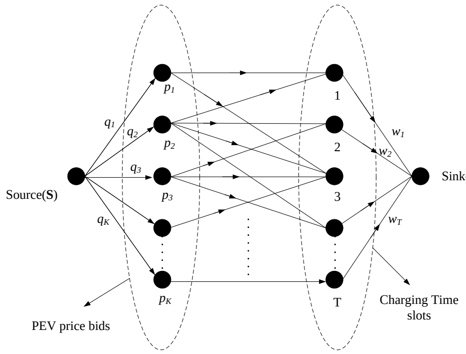

Fig. 5. Network flow representation of the optimal charging problem.

allocation time window is the same. What is significant is that the equality of the total energy flow at optimality holds across different price bid vectors as well, as long as all the bids are non-zero.

Proof: See Appendix II. ■

Lemma 3: Assuming all bids are distinct, the total amount of charge supplied to the individual PEVs is unique under any optimal allocation for a given set of bids.

Now let $\mathcal{Q}_k(y, p_{-k})$ denote the total amount of charge allocated to PEV k when it has a price bid of y but the price bids of the other PEVs $k' \neq k$ are kept fixed at $p_{k'}$. In other words, the $\mathcal{Q}_k(y, p_{-k})$ represents the total charge allocated to PEV k as a function of its price bid y while the rest of the system parameters remain unchanged. Next we will prove some properties of the function $\mathcal{Q}_k(y, p_{-k})$ which will be necessary to prove the main results.

Proof: See Appendix II. ■

Lemma 4: $\mathcal{Q}_k(y, p_{-k})$ is a non-decreasing function of y, i.e. $\forall y_1,y_2$ such that if $y_1 < y_2$, $\mathcal{Q}_k(y_2, p_{-k}) \geq \mathcal{Q}_k(y_1, p_{-k})$.

Proof: See Appendix II. ■

Assume that the $(K-1)$ PEVs other than PEV k are ordered according in increasing order of their bids, i.e. $p_{j-1} < p_j$ for $j = 2, 3, ..., K$.

Lemma 5: For any PEV k, if y_1, y_2 are such that $y_1 \neq y_2$, $p_j < y_1, y_2 < p_{j+1}$, then $\mathcal{Q}_k(y_1, p_{-k}) = \mathcal{Q}_k(y_2, p_{-k})$.

Proof: See Appendix II. ■

Following the analysis in [15], we assume the the valuation functions θ_k are differentiable, non-decreasing and strictly concave. Then by extending the analysis of [15], we can show the following two results.

Proposition 3: There exists an ϵ-Nash equilibrium for the PSP auction at which the social valuation of the energy allocation differs from the maximum possible social valuation by $O(\sqrt{\epsilon})$.

The above result follows from Proposition 3 of [15] along with lemmas 4 and 5 that we have proven in this paper. It is worth noting that a stronger version of the above result also follows a special case of Theorem 2 of [16], under a slightly stronger assumption of the valuation functions θ_k. In particular, this requires the functions θ_k to be strictly increasing (instead of non-decreasing), therefore not allowing the valuations to completely "flatten out".

Let $b_{-k} = (b_j, j \neq k)$ be the set of bids by all agents other than k.

Proposition 4: For any given b_{-k}, there exists a truthful ϵ-best bid for agent k, $\rho_k(b_{-k})$.

Proposition 3 implies that the Nash equilibrium becomes socially efficient as the cost of bid ϵ approaches zero and follows from Proposition 1 of [15] (along with lemmas 4 and 5 proven in this paper). Proposition 4 shows that any agent k can not loose much utility (only ϵ at most) by bidding truthfully in PSP.

V. Numerical Study

In our numerical study, we consider a residential power distribution network having a transformer constraint of 154 kW and having 24 Plug-in Electric Vehicles (PEVs) connected to the network. All energy allocation mechanisms are assumed to be taking place in the day-ahead electricity market where the inelastic non-PEV demand is assumed to be well predicted. The PEVs are assumed to have a concave valuation function of the form $\theta_k(x) = k_i(1 - e^{-a_i x})$, where x is the energy in kWh and i is used to highlight the *type* of the vehicle. It must be noted that the valuation functions of individual PEVs are monotonically increasing with diminishing returns. Parameters k_i and a_i determine the concavity and overall nature of the valuation function and is distinct for a specific *type* of vehicle. For sake of simplicity, it has been assumed that there are four *types* of vehicles, each *type* consisting of 6 vehicles each. In this numerical study, $k_1 = 10, k_2 = 9, k_3 = 11, k_4 = 7$ and $a_1 = 0.1, a_2 = 0.11, a_3 = 0.12, a_4 = 0.09$ are taken to be the distinct values for the vehicles. We have also defined the battery capacity of each PEV to be 23 kWh and assumed that every PEV is available to charge in each of 24 time slots. All simulations have been carried out in using MATLAB.

We first demonstrate the case where we consider a VCG like auction process of energy allocation in the day ahead market, where the auctioneer is aware of the entire valuation function of the PEV agents. Through such a mechanism, it is possible to calculate the socially optimal allocation in the network. Note that every agent of the same type is going to be allocated an equal amount, owing to their symmetry in charging preferences and valuation function parameters. For the VCG like auction, we assume all agents demand 23 kWh of energy (full battery capacity). In this case, only partial fulfillment of the agents' energy request through socially optimal allocation is possible, owing to insufficient resources in the network. We find that agents of *type 1*, *type 2*, *type 3* and *type 4* receive 10.9639 kWh, 9.8763 kWh, 11.4508 kWh and 7.0423 kWh of energy respectively. Owing to the impracticality of the VCG like mechanism, we proposed the MSP auction process as an alternative, where the aggregator is seen to allocate energy in a near-optimal manner. In this respect, we define $\gamma = \frac{S_{VCG} - S_{MSP}}{S_{VCG}}$ as an index representing the deviation of MSP from VCG in terms of social optimality; here S_{VCG} and S_{MSP} represent the total valuation of resource by all agents under the respective auction schemes. The number of levels in MSP determines the nearness to social optimum and as seen from the Figure 6. Note that the number of levels also determines the bid-complexity in MSP. Next we consider the PSP auction,

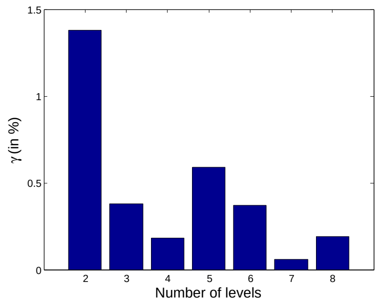

Fig. 6. MSP: Nearness to social optimality with respect to the number of levels.

and demonstrate that a social optimum solution is a Nash equilibrium. To do that, we assume study the effect of increasing the quantity of bid (q_k) for each agent k, while the bids of all other 23 agents are held constant at their socially optimal values. We assume truthful bidding, so determining the quantity q_k also determines the per-unit price p_k (which can be obtained from agent k's valuation function). We

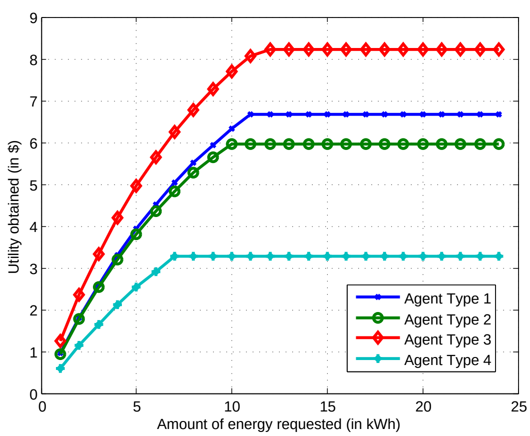

Fig. 7. PSP: Utility received by agents of different types by varying their bids unilaterally, while the other agent's are kept at their socially optimal values.

observe from Figure 7 that, agents gain maximum utility by bidding at the socially optimal values, at which point there is no incentive to unilaterally deviate and gain a greater utility. Beyond this value, agents are seen not being able to improve their utility by increasing the bid quantity. From Figure 7, one might think that since similar utility is being obtained by all agents when bidding a quantity in a range from social optimal value to maximum battery capacity, the agent of any particular type is better off by bidding the maximum quantity. However, precise simulations considering all bids fixed at 23 kWh shows that at such a bid profile, agents of the four types (1,2,3,4) respectively get utilities worth \$7.0759, \$0, \$9.4506 and \$0 whereas, if any one agent (depending on it being of type 2, 3 or 4) changes its bid to 22 kWh, it ends up increasing its utility unilaterally to \$6.37, \$10.2150 or \$4.5761 respectively. This clearly shows that in this case, bidding at the maximum quantities is not a Nash equilibrium.

VI. APPENDIX I

Proof of Proposition 1: Consider $\theta(x) \triangleq \sum_{k\in\mathcal{K}} \theta_k(x_k)$ where $x = (x_1, x_2, ..., x_K)$ is the complete allocation vector. x^* and $\tilde{x}$ are taken to be the optimal allocations to the pure VCG auction and the MSP auction process respectively. We need to show that, $\theta(x^*) - \theta(\tilde{x}) \leq KB$ where B is some constant.

From optimality conditions, $\theta(x^*) \geq \theta(x)\ \forall x$.

Similarly, $\bar{\theta}(\tilde{x}) \geq \bar{\theta}(x)\ \forall x$.

Let us assume that $\theta_k(x_k) - \bar{\theta}_k(x_k) \leq B\ \forall x_k, \forall k \in \mathcal{K}$. B is defined as shown in Figure 3. Therefore, we can write that,

$$\sum_{k\in\mathcal{K}} \Big(\theta_k(x_k) - \bar{\theta}_k(x_k)\Big) \leq KB, \tag{10}$$

$$\Rightarrow \theta(x) - \bar{\theta}(x) \leq KB. \tag{11}$$

Identifying the fact that $\theta(x) \geq \bar{\theta}(x)\ \forall x$, and noting that $\tilde{x}$ is the optimal value for auction with discretized functions $\bar{\theta}(.)$, it can be written that,

$$\theta(\tilde{x}) \geq \bar{\theta}(\tilde{x}) \geq \bar{\theta}(x^*) \geq \theta(x^*) - KB. \tag{12}$$

The first inequality in equation (12) comes from the fact that $\theta(x) \geq \bar{\theta}(x)\ \forall x$; the second inequality stems from the fact that $\tilde{x}$ is optimal for $\bar{\theta}(.)$ and the third inequality is from construction as seen in equation (11). Therefore, from the above equation, it is clearly seen that $\theta(x^*) - \theta(\tilde{x}) \leq KB$.

Proof of Proposition 2: This proof is based on the proof of incentive compatibility in VCG mechanisms which is given in Chapter 9 of [13]. Define $Q = (Q_1, Q_2, ..., Q_K)$ to be the post MSP auction allocation to all agents under assumption that agent i is bidding its discretized valuation function $\bar{\theta}_i(.)$ truthfully. Assume that the other agents $j \neq i$ bid v_j as their discretized valuation function; v_j may or may not be truthful. Let $\hat{Q}$ be the same allocation under a setting when agent i does not bid truthfully; $\hat{\theta}_i(.)$ being the untruthful valuation function. Let $Q_{-i} = (Q_{1,-i}, Q_{2,-i}, ..., Q_{i-1,-i}, Q_{i+1,-i}, ..., Q_{K,-i})$ be the allocations to all agents under a setting when agent i is not present in the auction. For notational simplicity, we will consider $u_i(Q_i) = u_i(Q)$ for the rest of the proof.

The utilities garnered by the agent i under the two different settings can be described by the following two equations,

$$u_i(Q) = \theta_i(Q) - \Big(\sum_{j\neq i} v_j(Q_{j,-i}) - \sum_{j\neq i} v_j(Q)\Big), \tag{13}$$

$$u_i(\hat{Q}) = \theta_i(\hat{Q}) - \Big(\sum_{j\neq i} v_j(Q_{j,-i}) - \sum_{j\neq i} v_j(\hat{Q})\Big). \tag{14}$$

Now, define $\Delta u_i := u_i(\hat{Q}) - u_i(Q)$. Therefore,

$$\Delta u_i = \theta_i(\hat{Q}) + \sum_{j\neq i} v_j(\hat{Q}) - \theta_i(Q) - \sum_{j\neq i} v_j(Q). \tag{15}$$

This can also be written as $\Delta u_i = \Big(\theta_i(\hat{Q}) - \bar{\theta}_i(\hat{Q})\Big) + \Big(\bar{\theta}_i(\hat{Q}) + \sum_{j\neq i} v_j(\hat{Q})\Big) - \Big(\theta_i(Q) - \bar{\theta}_i(Q)\Big) - \Big(\bar{\theta}_i(Q) + \sum_{j\neq i} v_j(Q)\Big)$. Observing that $\Big(\bar{\theta}_i(\hat{Q}) + \sum_{j\neq i} v_j(\hat{Q})\Big) - \Big(\bar{\theta}_i(Q) + \sum_{j\neq i} v_j(Q)\Big) \leq 0$ as argued from a similar discussion in Chapter 9 of [13] and then grouping and rearranging terms, it is evident that,

$$\Delta u_i \leq \Big(\theta_i(\hat{Q}) - \bar{\theta}_i(\hat{Q})\Big) - \Big(\theta_i(Q) - \bar{\theta}_i(Q)\Big), \quad (16)$$
$$\Rightarrow \Delta u_i \leq B - \Big(\theta_i(Q) - \bar{\theta}_i(Q)\Big), \quad (17)$$
$$\Rightarrow \Delta u_i \leq B. \quad (18)$$

This completes our proof.

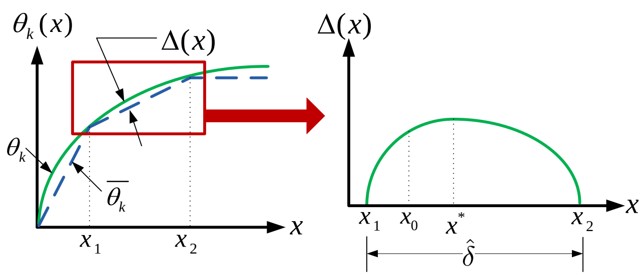

Fig. 8. Variation of the difference between resource valuation under VCG mechanism and MSP mechanism.

Proof of Lemma 1: Refer to Figure 8 for this proof. Assume that $\Delta(x) \triangleq (\theta_k(x) - \bar{\theta}_k(x))$. Define $\nu \triangleq \max|\Delta''(x)|$. Assume $x_1 \leq x_0 \leq x_2$. Using Taylor series expansion, we can write,

$$\Delta'(x_2) = \Delta'(x_1) + (x_2 - x_1)\Delta''(x_0), \quad (19)$$
$$\geq \Delta'(x_1) - \nu(x_2 - x_1). \quad (20)$$

Therefore, it can be written that,

$$\Delta'(x_1) - \Delta'(x_2) \leq \nu(x_2 - x_1), \quad (21)$$
$$\Rightarrow \Delta'(x_1) - \Delta'(x_2) \leq \nu\hat{\delta}. \quad (22)$$

Observing that $\Delta'(x_1) \geq 0$, $\Delta'(x_2) \leq 0$ (in Figure 8) and using equation (22), the following inequalities can be written,

$$\Delta'(x_1) \leq \nu\hat{\delta}, \quad (23)$$
$$-\Delta'(x_2) \leq \nu\hat{\delta}. \quad (24)$$

Inequality (24) comes from (23)and the definition of ν.

Assume x^* to be the point where $\Delta(x)$ is maximized. Assume $x_1 \leq x_{10} \leq x^*$ and $x^* \leq x_{20} \leq x_2$. Using Taylor series expansion of $\Delta(x)$ about x_1, we can write,

$$\Delta(x^*) = \Delta(x_1) + \Delta'(x_{10})(x^* - x_1), \quad (25)$$
$$\leq \Delta'(x_1)(x^* - x_1), \quad (26)$$
$$\leq \nu\hat{\delta}(x^* - x_1). \quad (27)$$

Again, expanding $\Delta(x)$ about x_2 and using a similar argument as in (25)-(27), we can write that,

$$\Delta(x^*) \leq \nu\hat{\delta}(x_2 - x^*). \quad (28)$$

Adding inequalities (27) and (28), we get,

$$2\Delta(x^*) \leq \nu\hat{\delta}(x_2 - x_1), \quad (29)$$
$$\Rightarrow 2\Delta(x^*) \leq \nu\hat{\delta}^2, \quad (30)$$
$$\Rightarrow \Delta(x^*) \leq \frac{\nu\hat{\delta}^2}{2}. \quad (31)$$

This clearly shows that $B \leq \frac{\nu\hat{\delta}}{2}$.

VII. Appendix II

Proof of Lemma 2: Assume $f = \sum_{k\in\mathcal{K}} \mathcal{Q}_k(p_k)$ and $f' = \sum_{k\in\mathcal{K}} \mathcal{Q}'_k(p'_k)$ to be two optimal flow solutions with $f \neq f'$, possibly under two different price vectors p and p'. Without loss of generality let us assume $f < f'$. Since $f < f'$, then there exists an augmenting path $\mathbf{S}$ to $\mathbf{T}$ in the residual flow graph corresponding to the smaller flow value f, with capacity $\delta > 0$. See Figure 9. Pushing a δ amount of flow on this augmenting path increases the value (total revenue) of that flow by $p_m\delta > 0$ (from $f = \sum_{k\in\mathcal{K}} \mathcal{Q}_k(p_k)$ to $f + p_m\delta = \sum_{k\in\mathcal{K}} \mathcal{Q}_k(p_k) + p_m\delta$, since $p_m > 0$. This contradicts our assumption that $f < f'$, thereby showing $f = f'$.

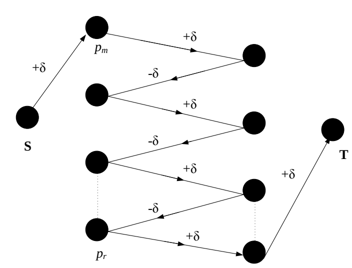

Fig. 9. An augmenting path in the residual flow graph of f.

Proof of Lemma 3: For a given price vector $\vec{p}$, let $\vec{\tilde{\mathcal{Q}}} = (\tilde{\mathcal{Q}}_1, \tilde{\mathcal{Q}}_2, ...\tilde{\mathcal{Q}}_K)$ and $\vec{\mathcal{Q}}' = (\mathcal{Q}'_1, \mathcal{Q}'_2, ...\mathcal{Q}'_K)$, be two different optimal energy allocation solutions. Let m be the smallest index (PEV with the smallest price bid) in which the two allocations differ. Without loss of generality, let us assume $\tilde{\mathcal{Q}}_m > \mathcal{Q}'_m$. Since $\sum_{k\in\mathcal{K}} \tilde{\mathcal{Q}}_k = \sum_{k\in\mathcal{K}} \mathcal{Q}'_k$ (Lemma 2), there must exist an $r > m$ (i.e., $p_r > p_m$) (note that all price bids are distinct), such that $\mathcal{Q}'_r > \tilde{\mathcal{Q}}_r$. Then, we redirect the flow (charging allocation solution) corresponding to $\vec{\tilde{\mathcal{Q}}}$ by some amount $\delta > 0$ in such a way that its total value (revenue) increases by $(p_r - p_m)\delta$. This is done by reducing the flow for PEV m by δ and increasing that for PEV r by an equal amount (Figure 10 shows the corresponding "value augmenting cycle" that corresponds to the flow redirection). Therefore, $\vec{\tilde{\mathcal{Q}}}$ can not be an optimal solution for price vector $\vec{p}$, contradicting our assumption. The result in Lemma 3 follows.

Proof of Lemma 4: Let us consider two price vectors for the PEVs given as $\vec{\tilde{p}} = (y_1, p_{-k})$ and $\vec{\hat{p}} = (y_2, p_{-k})$ which differ only in the k^{th} component. Let $\vec{\tilde{\mathcal{Q}}} = (\tilde{Q}_1^1, \tilde{Q}_1^2, ..., \tilde{Q}_1^T, \tilde{Q}_2^1, \tilde{Q}_2^2, ..., \tilde{Q}_2^T, ..., \tilde{Q}_K^1, \tilde{Q}_K^2, ..., \tilde{Q}_K^T)$ and $\vec{\hat{\mathcal{Q}}} = (\hat{Q}_1^1, \hat{Q}_1^2, ..., \hat{Q}_1^T, \hat{Q}_2^1, \hat{Q}_2^2, ..., \hat{Q}_2^T, ..., \hat{Q}_K^1, \hat{Q}_K^2, ..., \hat{Q}_K^T)$ be the corresponding optimal allocation solutions. Note from lemma 3 that $\vec{\tilde{\mathcal{Q}}}$ and $\vec{\hat{\mathcal{Q}}}$ are unique. Let $\vec{\mathcal{Q}} = (Q_1^1, Q_1^2, ..Q_1^T, Q_2^1, Q_2^2, .., Q_2^T, .., Q_K^1, Q_K^2, .., Q_K^T) \in \mathfrak{Q}$ be an arbitrary allocation vector where $\mathfrak{Q}$ is a polytope representing the set of all feasible charging vectors. From optimality criterion,

$$\sum_{k \in \mathcal{K}} \sum_{t \in T_k} \tilde{p}_k \cdot (Q_k^t - \tilde{Q}_k^t) \leq 0 \qquad (32)$$

$$\Rightarrow \sum_{k \in \mathcal{K}} \tilde{p}_k \cdot \sum_{t \in T_k} (Q_k^t - \tilde{Q}_k^t) \leq 0 \qquad (33)$$

$$\Rightarrow \sum_{k \in \mathcal{K}} \tilde{p}_k \cdot (\sum_{t \in T_k} Q_k^t - \sum_{t \in T_k} \tilde{Q}_k^t) \leq 0 \qquad (34)$$

$$\Rightarrow \sum_{k \in \mathcal{K}} \tilde{p}_k \cdot (\mathcal{Q}_k - \tilde{\mathcal{Q}}_k) \leq 0 \qquad (35)$$

Since $\vec{\hat{\mathcal{Q}}} = (\hat{\mathcal{Q}}_1, \hat{\mathcal{Q}}_2, ..., \hat{\mathcal{Q}}_K) \in \mathfrak{Q}$, we have,

$$\sum_{k \in \mathcal{K}} \tilde{p}_k \cdot (\hat{\mathcal{Q}}_k - \tilde{\mathcal{Q}}_k) \leq 0. \qquad (36)$$

Similarly, from optimality of $\vec{\hat{\mathcal{Q}}}$ over $\mathfrak{Q}$ for price vector $\vec{\hat{p}}$,

$$\sum_{k \in \mathcal{K}} \hat{p}_k \cdot (\tilde{\mathcal{Q}}_k - \hat{\mathcal{Q}}_k) \leq 0. \qquad (37)$$

Adding inequalities (36) and (37), we have $\sum_{k \in \mathcal{K}} (\tilde{p}_k - \hat{p}_k) \cdot (\hat{\mathcal{Q}}_k - \tilde{\mathcal{Q}}_k) \leq 0$. Since $\tilde{p}_i = \hat{p}_i \ \forall i \in \mathcal{K} - \{k\}$ so the previous expression reduces to $(y_1 - y_2)(\mathcal{Q}_k(y_2, p_{-k}) - \mathcal{Q}_k(y_1, p_{-k})) \leq 0$. This can be further written as

$$(y_2 - y_1)(\mathcal{Q}_k(y_2, p_{-k}) - \mathcal{Q}_k(y_1, p_{-k})) \geq 0.$$

Since $y_2 > y_1$ so $\mathcal{Q}_k(y_2, p_{-k}) \geq \mathcal{Q}_k(y_1, p_{-k})$. This completes our proof.

Proof of Lemma 5: We want to show that as the price of bid p_k (for k^{th} PEV) increases to $p_k + \chi$ but remains between p_j and p_{j+1} i.e. $p_j < p_k < p_k + \chi < p_{j+1}$, the allocation $\mathcal{Q}_k(p_k, p_{-k})$ does not change. In the rest of the proof, for simplicity, we drop p_{-k} from the notation and express $\mathcal{Q}_k(p_k, p_{-k})$ as $\mathcal{Q}_k(p_k)$.

From Lemma 4, we know that since $\mathcal{Q}_k$ is a non-decreasing function of p_k, as we increase p_k to $p_k + \chi$, $\mathcal{Q}_k(p_k + \chi)$ cannot be less than $\mathcal{Q}_k(p_k)$. Now let us assume that $\mathcal{Q}_k(p_k + \chi) > \mathcal{Q}_k(p_k)$. From Lemma 2, since the total flow is constant, this excess flow at k^{th} node (coming from the source $\mathbf{S}$) must return back to the source via some other PEV nodes. Such redirection cannot occur through nodes

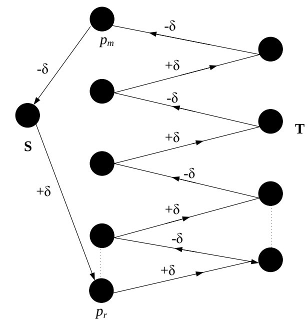

Fig. 10. A cycle that improves total value (revenue) of the flow (energy allocation).

having prices p such that $p \geq p_{j+1}$, since that contradicts optimality of the solution. So this flow will get redirected to the source $\mathbf{S}$ through nodes having prices less than p_j. Let any such node be i. Therefore, there must exist a positive value circulation of the form $\mathbf{S} \rightarrow k \rightarrow i \rightarrow \mathbf{S}$ in the residual flow graph of the optimal solution at (p_k, p_{-k}). Existence of such path contradicts the optimality of the solution at (p_k, p_{-k}). Thus our assumption was incorrect and hence, the result of lemma 5 holds.

REFERENCES

[1] G. T. Heydt, The impact of electric vehicle deployment on load management strategies, IEEE Transactions on Power Application Systems, vol. PAS-1, no. 144.

[2] A. Heider and H. J. Haubrich, Impact of wide-scale EV charging on the power supply network, Proc. IEE Colloq. Electric Vehicles - A Technology Roadmap for the Future, vol. 6, no. 262, p. 14, 1998.

[3] R. Liu, L. Dow, and E. Liu, A survey of PEV impacts on electric utilities, in Proc. IEEE PES Innov. Smart Grid Tech. Conf., January 2011.

[4] M. D. Galus and G. Andersson, Demand management of grid connected plug-in hybrid electric vehicles, in Proc. IEEE Energy 2030 Conference, 2008.

[5] Z. Ma, D. Callaway, and I. Hiskens, Decentralized charging control for large populations of plug-in electric vehicles, IEEE Tranactions on Control Systems Technology, October 2011.

[6] W. Tushar, W. Saad, V. Poor, and D. Smith, Economics of electric vehicle charging: A game theoretic approach, IEEE Transactions on Smart Grid, July 2012.

[7] L. Gan, U. Topcu, and S. Low, Optimal decenteralized protocols for electric vehicle charging, in Proc. IEEE Conference on Decision and Control, December 2011.

[8] K. Clement-Nyns, E. Haesen, and J. Driesen, The impact of charging plug-in hybrid electric vehicles on a residential distribution grid, IEEE Transactions on Power Systems, vol. 25, no. 1, p. 371-380, 2010.

[9] A. Hajimiragha, C. A. Canizares, M. Fowler, and A. Elkamel, Optimal transition to plug-in hybrid electric vehicles in Ontario, Canada, considering the electricity-grid limitations, IEEE Transactions on Industrial Electronics, vol. 57, p. 690-701, February 2010.

[10] N. Rotering and M. Ilic, Optimal charge control of plug-in hybrid electric vehicles in deregulated electricity markets, IEEE Transactions on Power Systems, August 2012.

[11] E. Sortomme, M. Hindi, S. D. MacPherson, and S. S. Venkata, Coordinated charging of plug-in hybrid electric vehicles to minimize distribution system losses, IEEE Transactions on Smart Grid, vol. 2, March 2012.

[12] A. Ghavami, K. Kar, S. Bhattacharya, A. Gupta, Price-driven charging of plug-in electric vehicles: Nash equilibrium and best-response strategy convergence, in Proc. 47th Annual Conference on Information Science and Systems, March 2013.

[13] N. Nisan, T. Roughgarden, E. Tardos, V. V. Vazirani, Algorithmic Game Theory, Chapter 9, Cambridge University Press.

[14] A.A. Lazar and N. Semret, The PSP auction mechanism for network resource sharing, In Proc. 8th International Symposium on Dynamic Games and Applications, July 1998.

[15] A.A. Lazar and N. Semret, Design and analysis of the Progressive Second Price Auction for network bandwidth sharing, (Extended version of [14]), available at http://www.ee.columbia.edu/~aurel/papers/networking_games/telecom-systems.pdf.

[16] R. Jain and J. Walrand, An efficient Nash-implementation mechanism for network resource allocation, Automatica, vol. 46, no. 8, p. 1276-1283, Oct. 2010.

[17] C. W. Qu, P. Jia and P. E. Caines, Analysis of a class of decentralized decision processes: quantized progressive second price auctions, in Proc. 46th IEEE Conf. on Decision and Control, December 2007.

[18] B. Tuffin, Revisited progressive second price auction for charging telecommunication networks, Telecommunication Systems, vol. 20, no. 3-4, p. 255-263, 2002.

[19] P. Maille and B. Tuffin, Multi-bid auctions for bandwidth allocation in communication networks, in Proc. IEEE INFOCOM, Mar. 2004.

[20] P. Maille and B. Tuffin, Pricing the internet with multibid auctions. IEEE/ACM Trans. Networking, vol. 14, no. 5, p. 992-1004, October 2006.

[21] J. Contreras, O. Candiles, J. I. de la Fuente and T. Gomez, Auction design in day-ahead electricity markets, IEEE Transactions on Power Systems, vol. 16, no. 1, p. 88-96, 2001.

[22] S. Zou, Z. Ma and X. Liu, Efficient Coordinations of large-scale elastic loads with MCP and PSP auction Mechanisms, in Proc. 32nd Chinese Control Conference, July 2013.

[23] S. Bhattacharya, K. Kar, J. H. Chow and A. Gupta, Extended second price auctions for plug-in electric vehicle (PEV) charging in smart distribution grids, to appear in Proc. of 2014 American Control Conference (ACC), June 2014.